Fundamentals of Electromagnetics

Everything You Always Wanted to Know About Dipoles*

*But Were Afraid To Ask

Part 1

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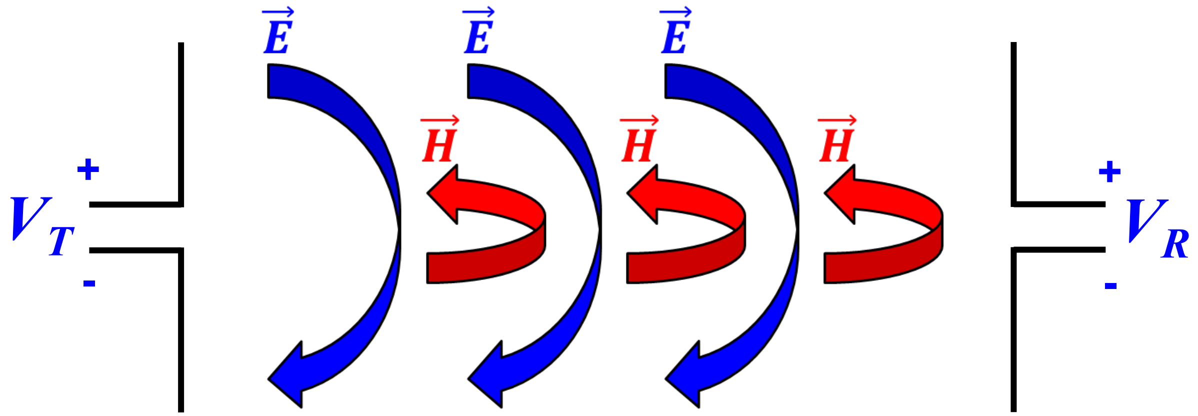
Topics

- **Intentional and Unintentional Antennas**
- **Transmission Line Model Review**
- **From Transmission Lines to Dipoles**
- **Elemental Dipoles – Electric (Hertzian) and Magnetic (Loop)**
- **Near Field, Intermediate Field, and Far Field**
- **Wave Impedance**

- ***Shameless plug: "Fundamentals of Electromagnetics" video series***
  - *Publicly available on YouTube; search for above title*
  - *Direct playlist link:*
    - https://youtube.com/playlist?list=PLtrpQ-gPvnJn2r9Mw49jij7Ky0mb6RJYF&si=UxEKqVRgsR9w6nZ7

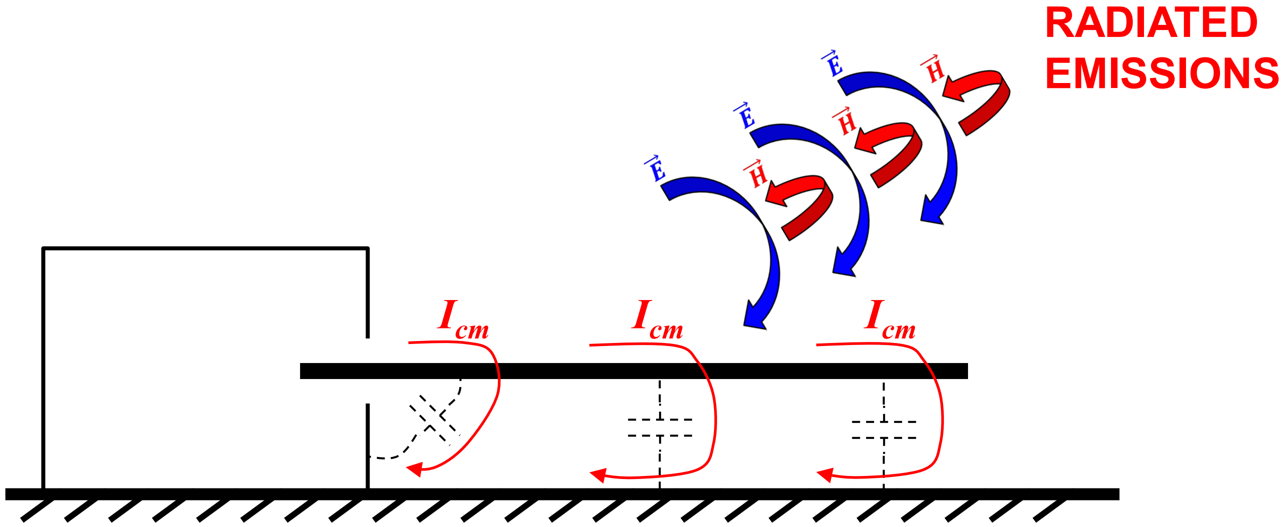
Intentional Radiators, a.k.a. Antennas

- An antenna is a transducer between conducted and radiated energy
- An antenna is made by establishing an RF potential between two conductors



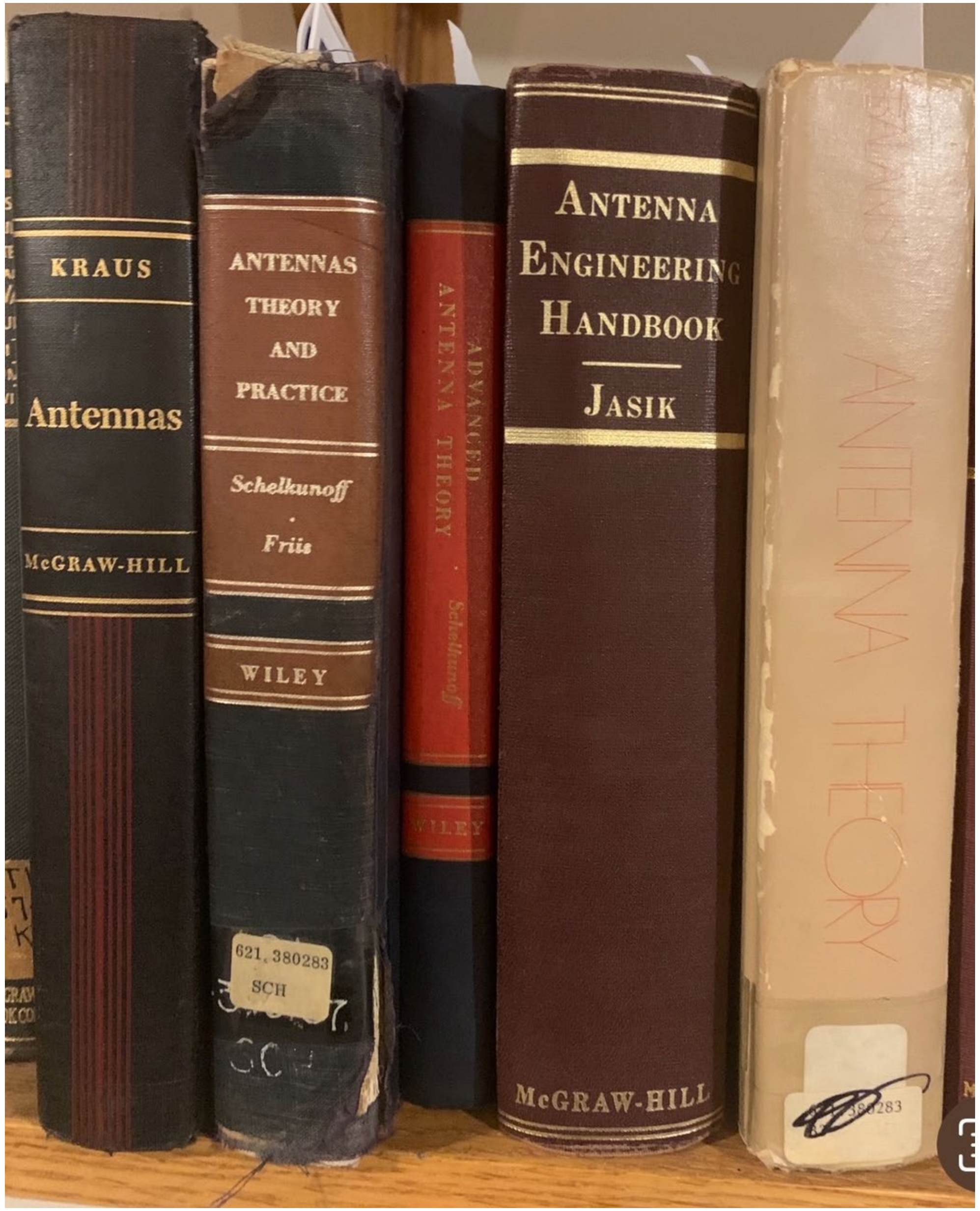
Unintentional Radiators, a.k.a. Antennas

- **If any part of your equipment radiates or is susceptible to radio frequency (RF) energy, you have created an antenna, whether or not you call it one**
- **These can be any two conductors, e.g.:**
  - Cable and chassis
  - Cable and ground plane (structure)
  - PC board trace and chassis
  - Poorly bonded connector and chassis (seams on chassis)
  - Etc.



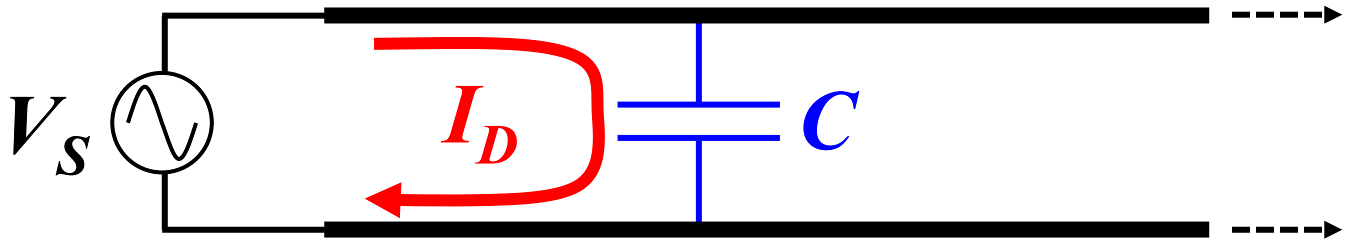
Antennas for EMC Engineers

- **Goal of this presentation:**
  - Provide basic knowledge of antenna operation in the context of addressing radiated EMI problems
  - Special emphasis on linear dipole antenna, which is a useful model for the most common type of unintentional antennas
    - Including cables
    - **ESPECIALLY cables**
- **Formal study of antennas is clearly a VERY big topic and beyond scope of this presentation**
- **Recommendations for further reading provided on "References" slide**



Review: General Transmission Line Model

Two conductors separated by a dielectric will have capacitance between them…



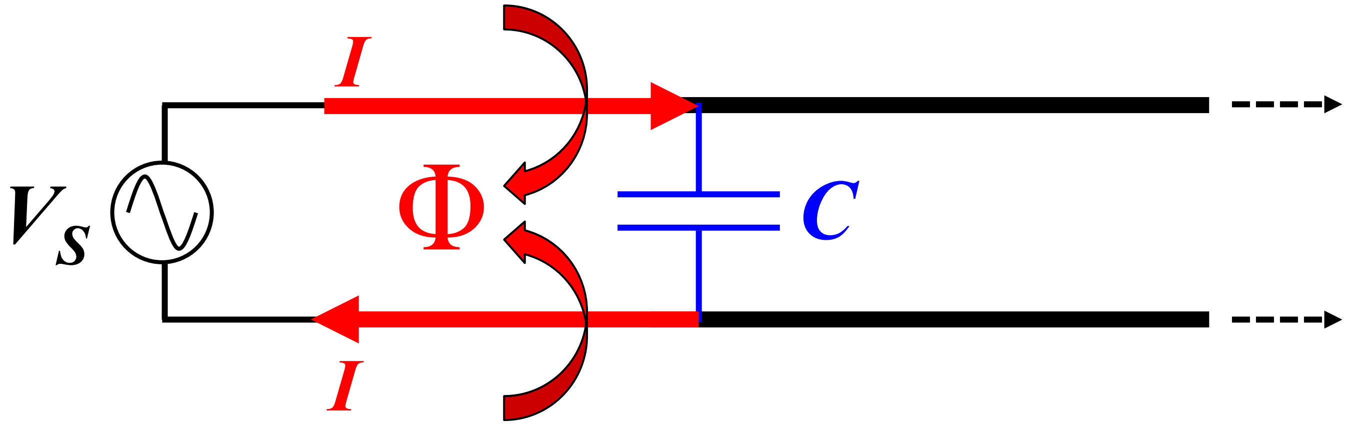
…that will allow displacement current to flow when an AC potential is applied

$$I_D = C\frac{dV_S}{dt}$$

$$X_C = \frac{1}{j\omega C}$$

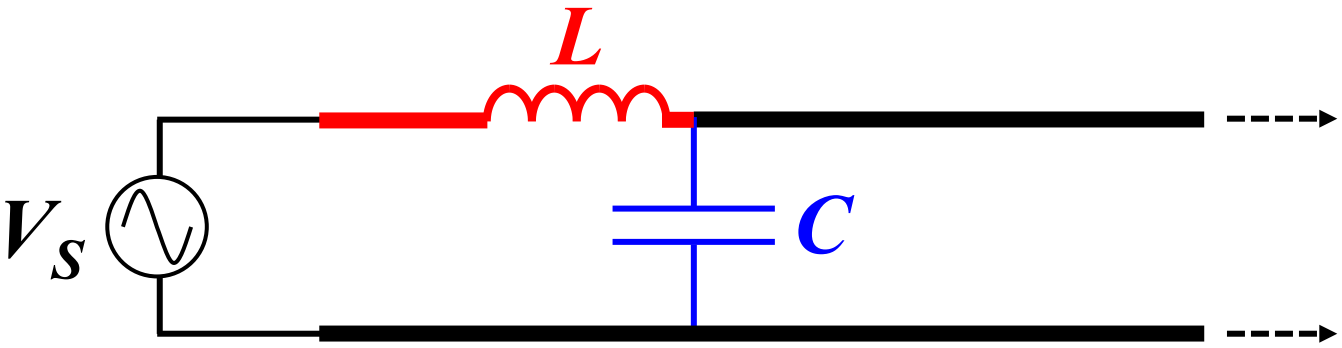
Capacitive reactance decreases with frequency
Displacement current increases with frequency

General Transmission Line Model (cont.)



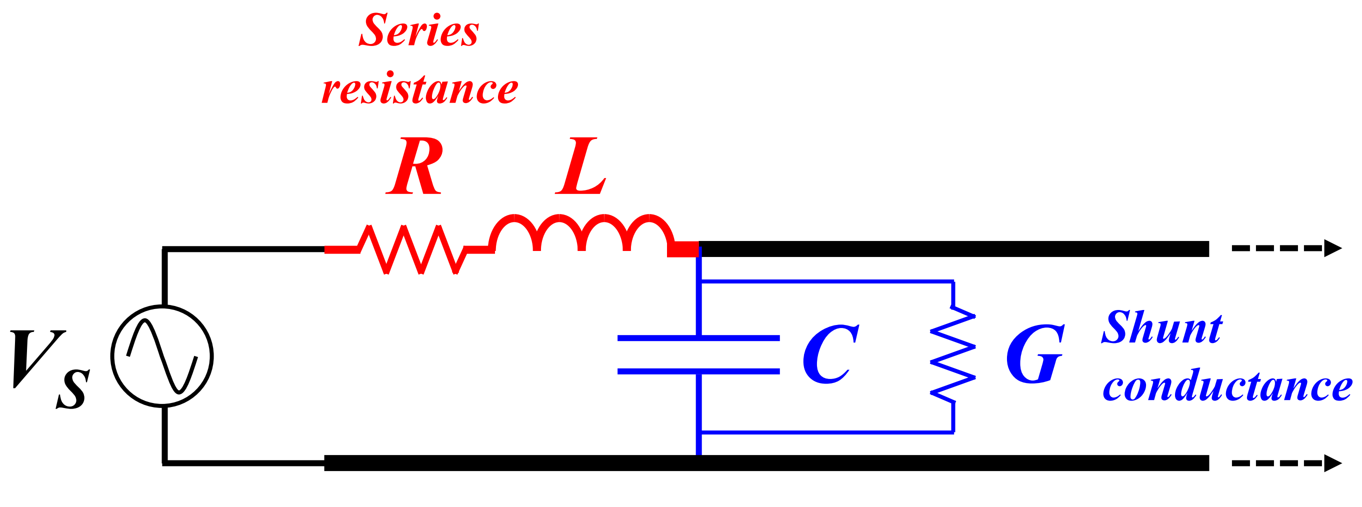
Current produces magnetic flux through loop between conductors

General Transmission Line Model (cont.)



Lossless transmission line model

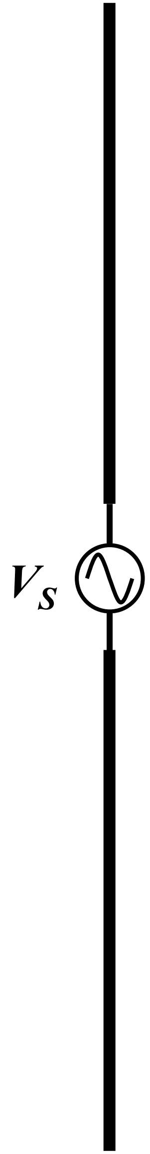
General Transmission Line Model (cont.)



"Lossy" transmission line model

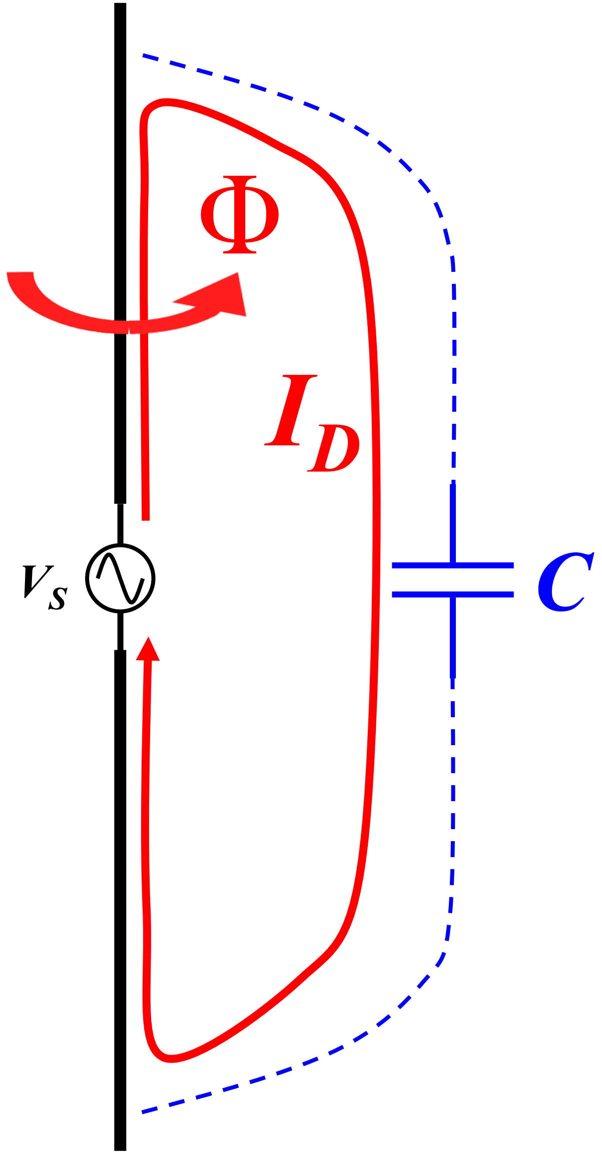
From Transmission Line to Dipole (cont.)

From Transmission Line to Dipole (cont.)

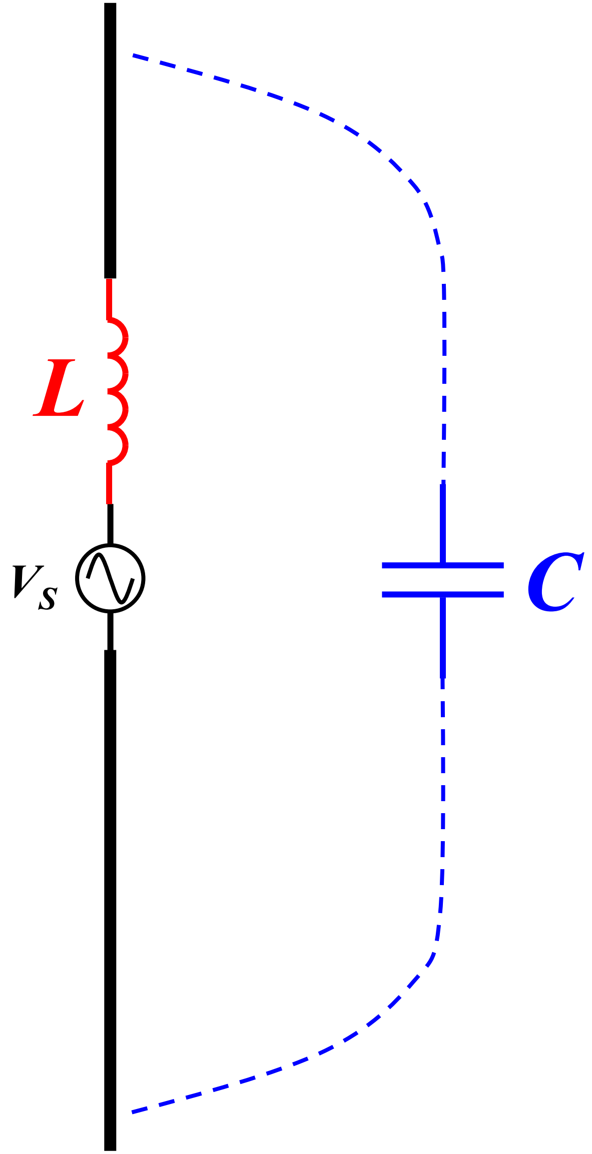


Different geometry
(and more complex math),
but physics is physics…

Simplified Dipole Circuit Model



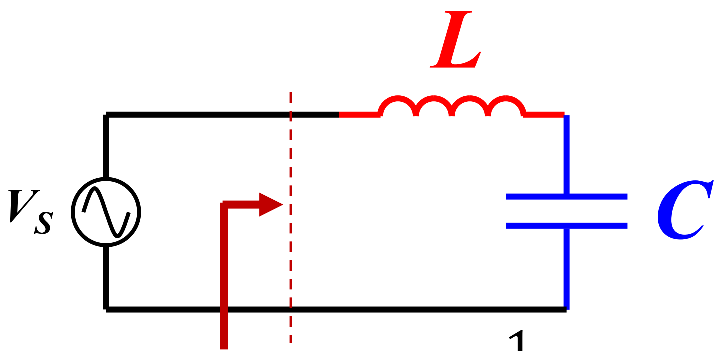
Simplified Dipole Circuit Model (cont.)



Incomplete model,
but hold that thought…

Simplified Dipole Circuit Model (cont.)

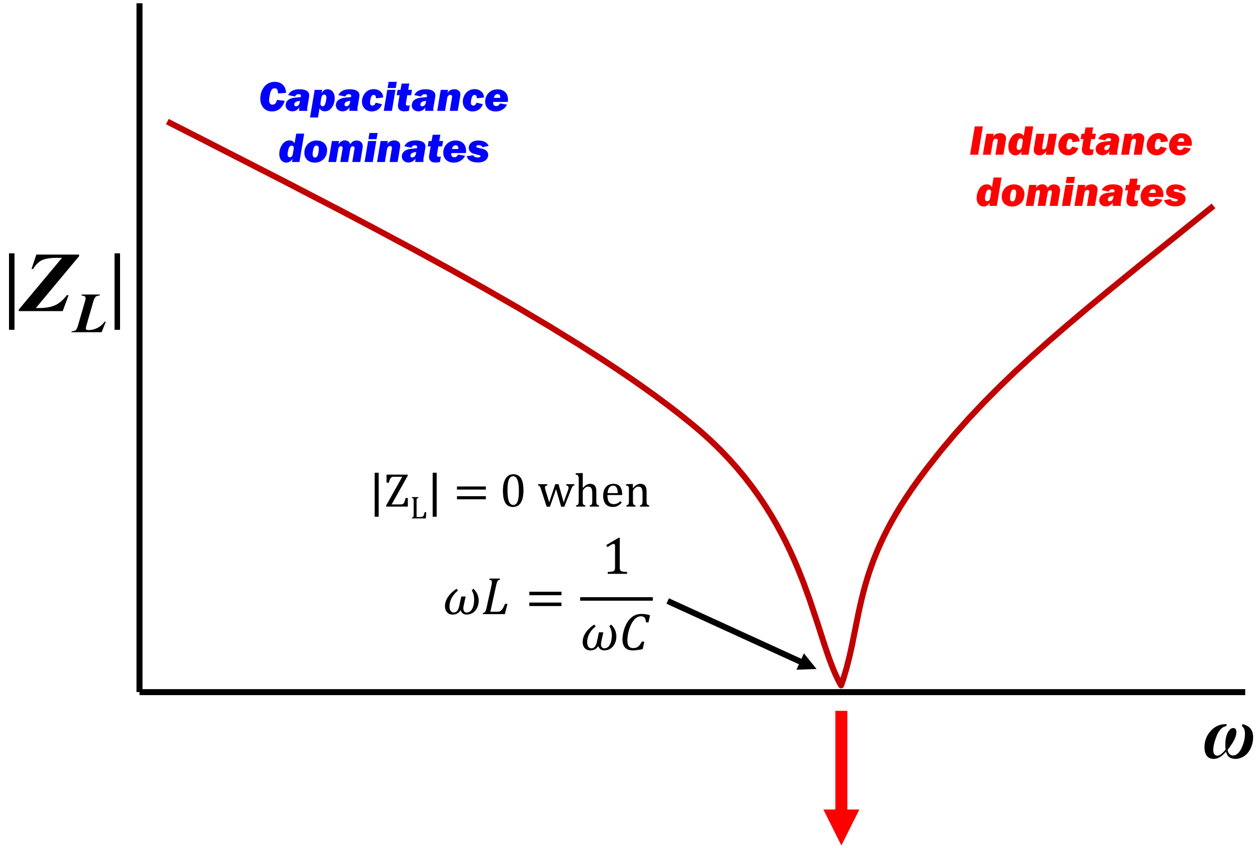
Simplified equivalent circuit



$$Z_L = j\omega L + \frac{1}{j\omega C}$$

$$= j\omega L - j\frac{1}{\omega C}$$

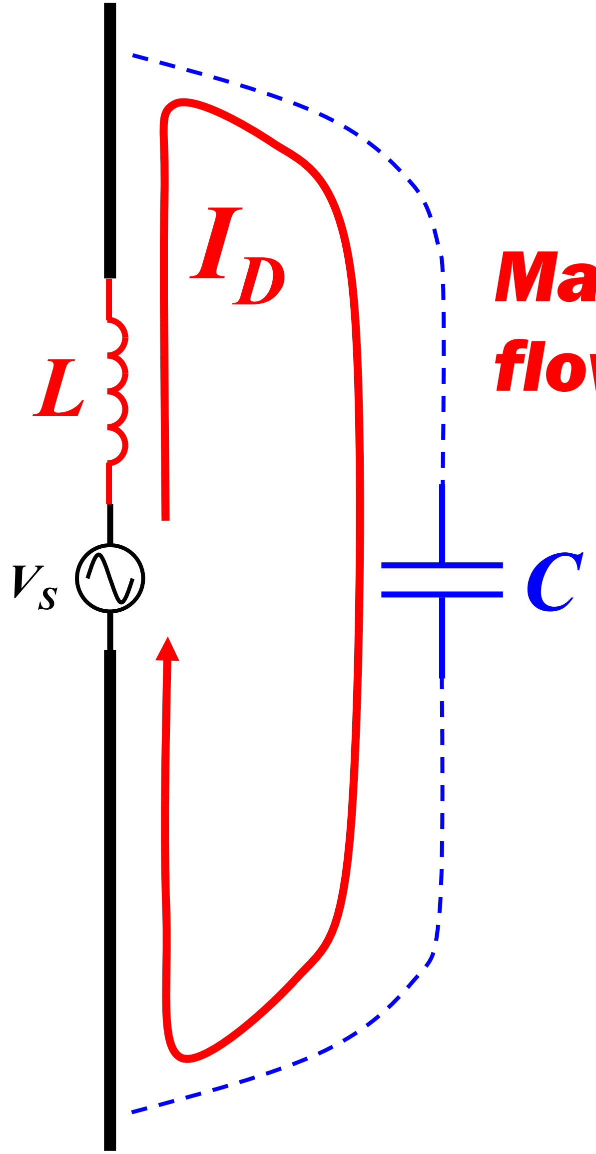
$$Z_L = j\left(\omega L - \frac{1}{\omega C}\right)$$



Lots o' current…

Simplified Dipole Circuit Model (cont.)

When $\omega L = \dfrac{1}{\omega C}$ ...



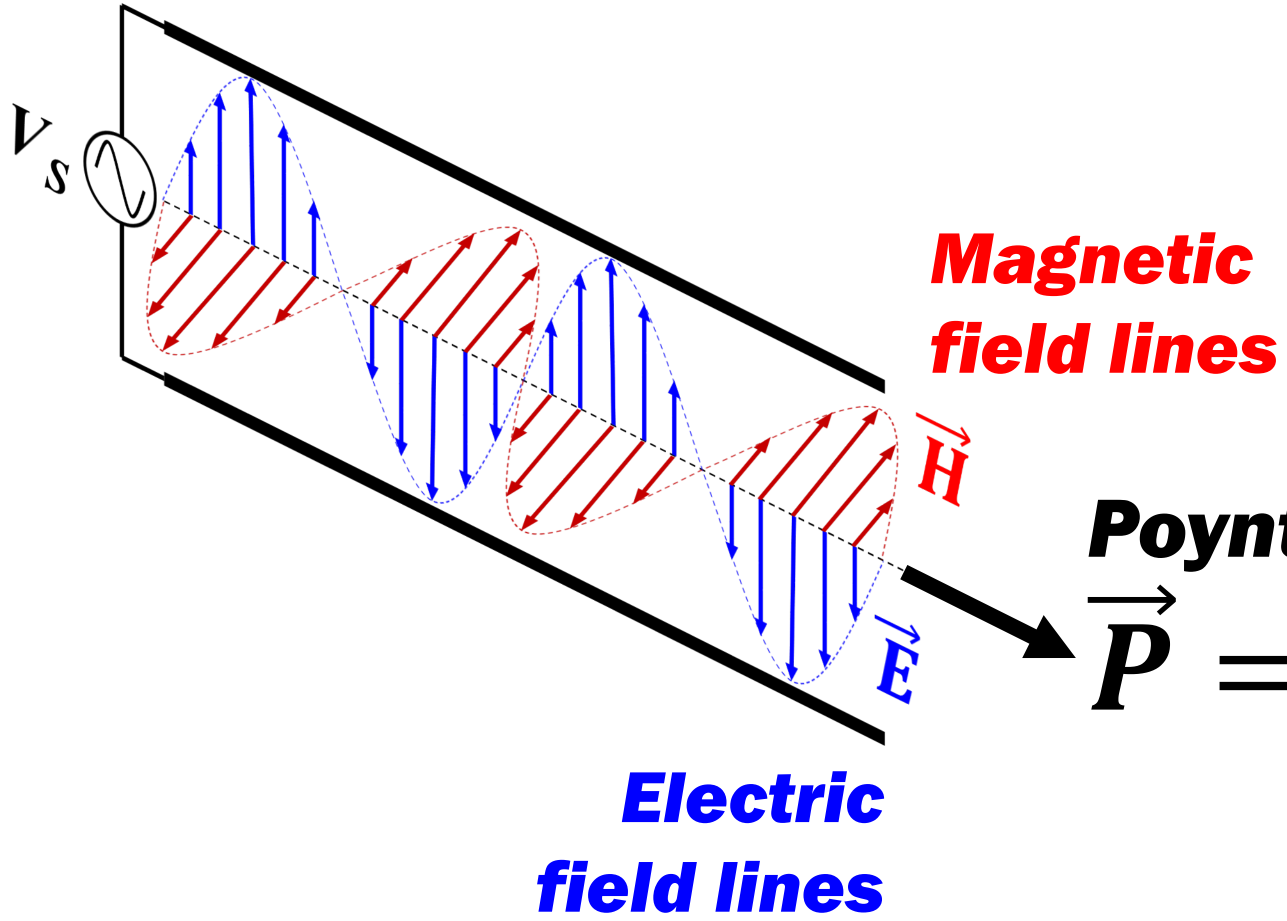
Maximum current flows in dipole…

Sneak preview:
Maximum occurs when each dipole "arm" has length = $\lambda/4$

$\downarrow$

Total dipole length = $\lambda/2$ ("half-wave dipole")

Field Lines and Poynting Vector

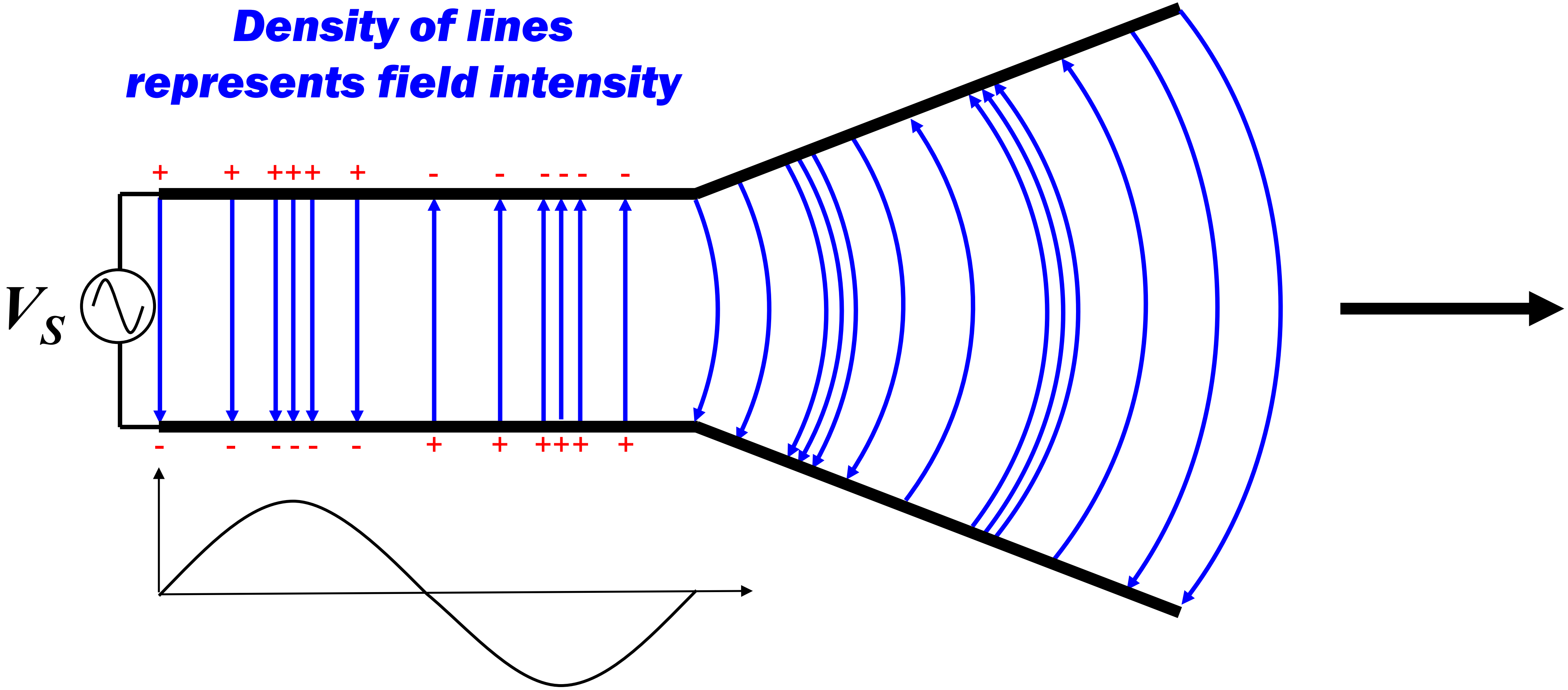


Poynting vector

$$\vec{P} = \vec{E} \times \vec{H}$$

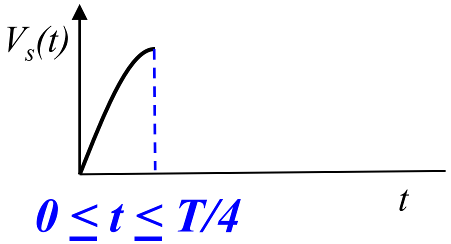
Electric Field Lines

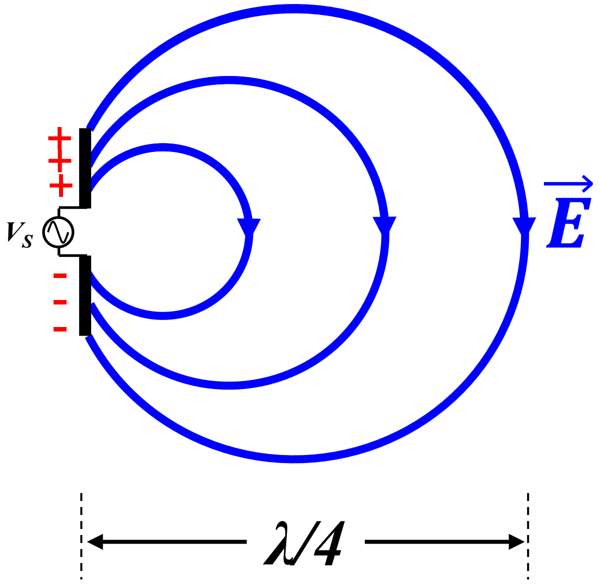
Density of lines represents field intensity



Simplified Radiation Mechanism – Short Dipole Example

Following discussion adapted from:
Balanis, "Antenna Theory: Analysis and Design," chapter 1

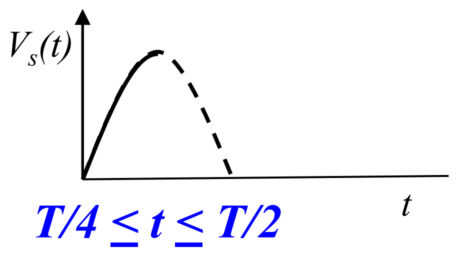


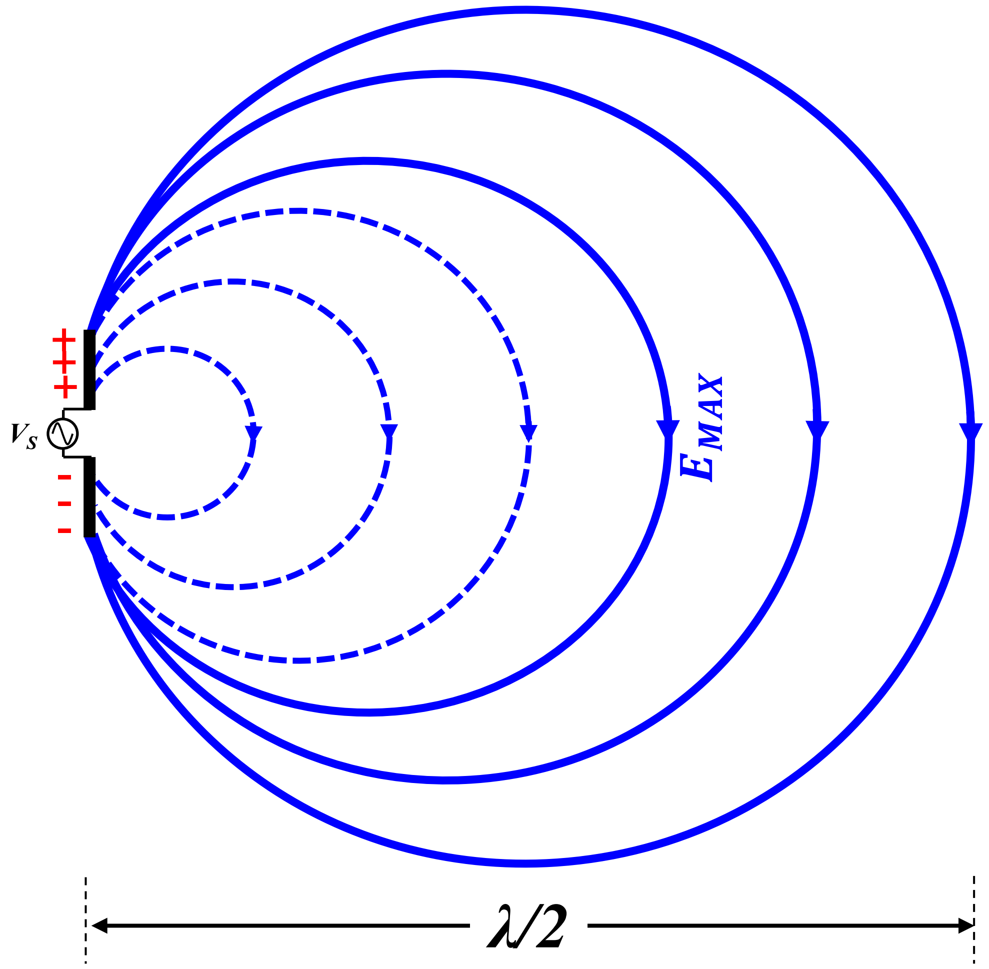


In first quarter (temporal) period,

charges deposited on conductors set up electric field lines that travel outward a distance of a quarter wavelength ($\lambda/4$) from dipole axis

Simplified Radiation Mechanism – Short Dipole Example (cont.)





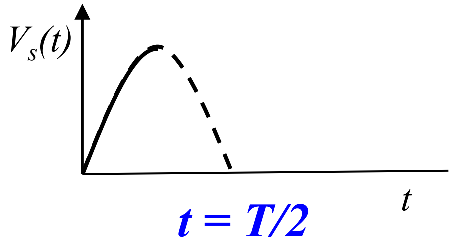
In 2nd quarter period,

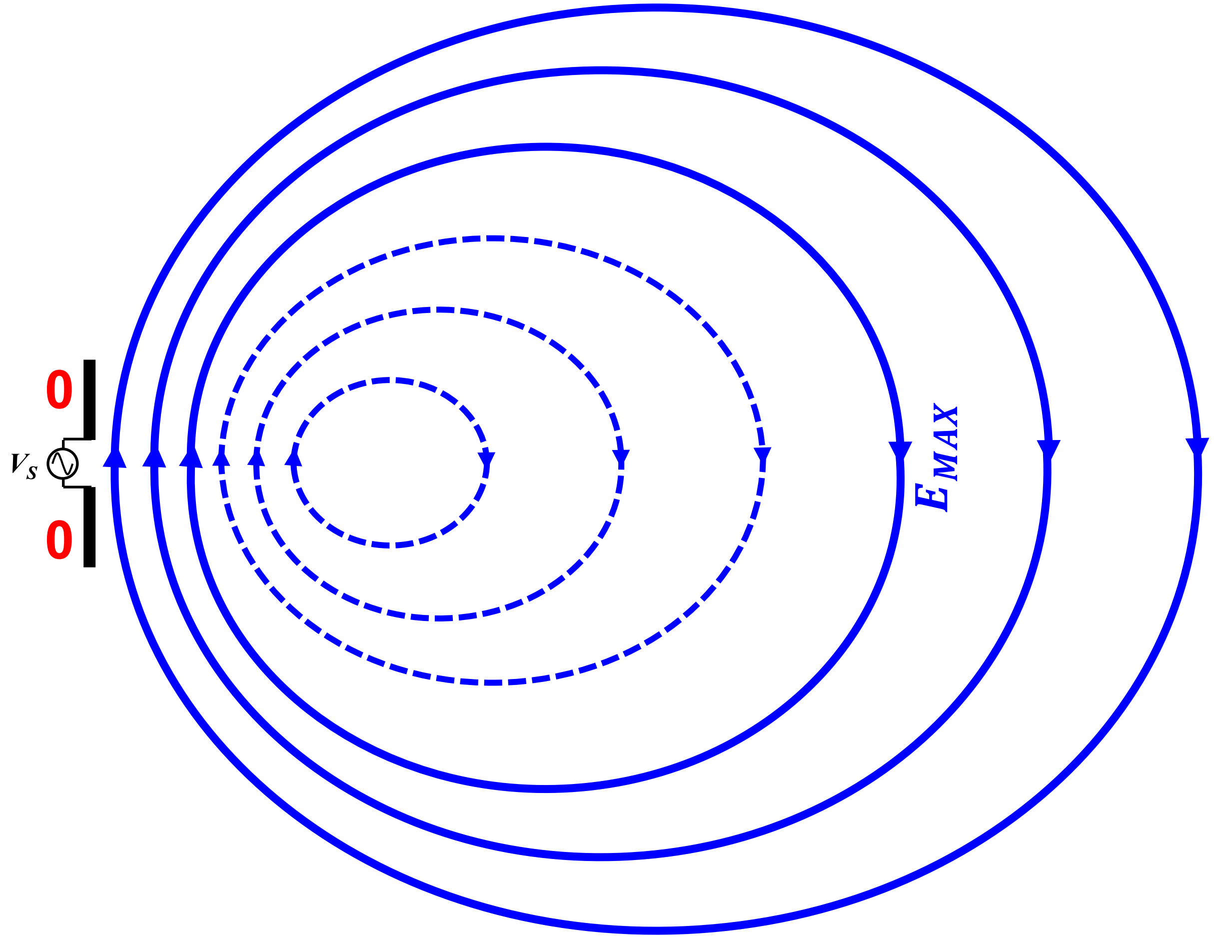
1st group of electric field lines travel outward an additional distance of $\lambda/4$ from dipole axis ($\lambda/2$ total)

New group of field lines forms in first $\lambda/4$ from dipole axis

Potential and charge on conductors decrease but still positive (decreasing but still positive E)

Simplified Radiation Mechanism – Short Dipole Example (cont.)





At t = T/2:

Net potential and charge on conductors = 0 (momentarily)

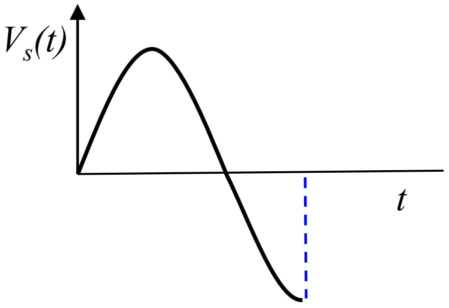
Electric field lines close on themselves and separate from dipole conductors

Simplified Radiation Mechanism – Short Dipole Example (cont.)

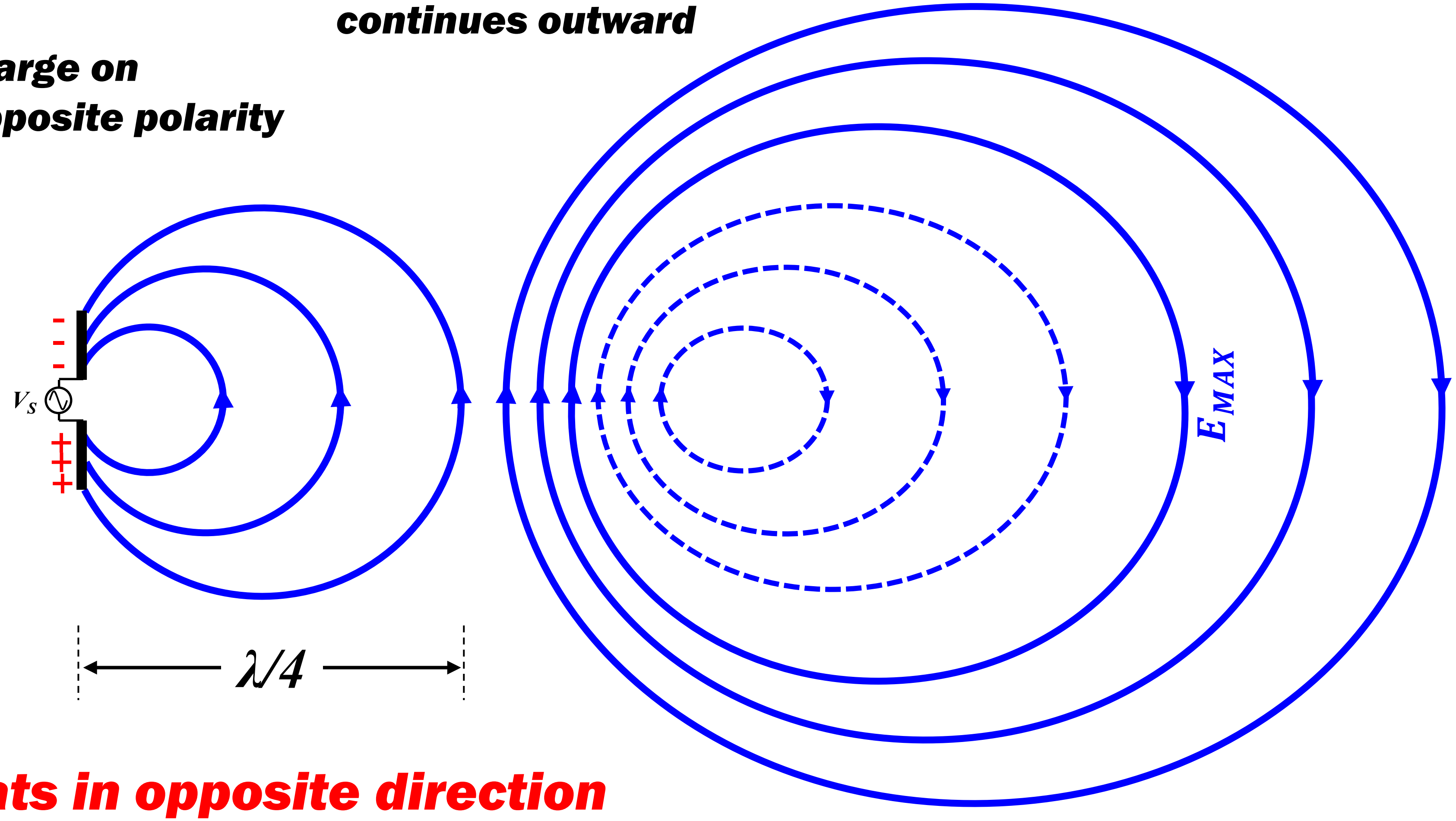
In next quarter period:

First group of field lines continues outward

Potential and charge on conductors of opposite polarity



$T/2 \leq t \leq 3T/4$

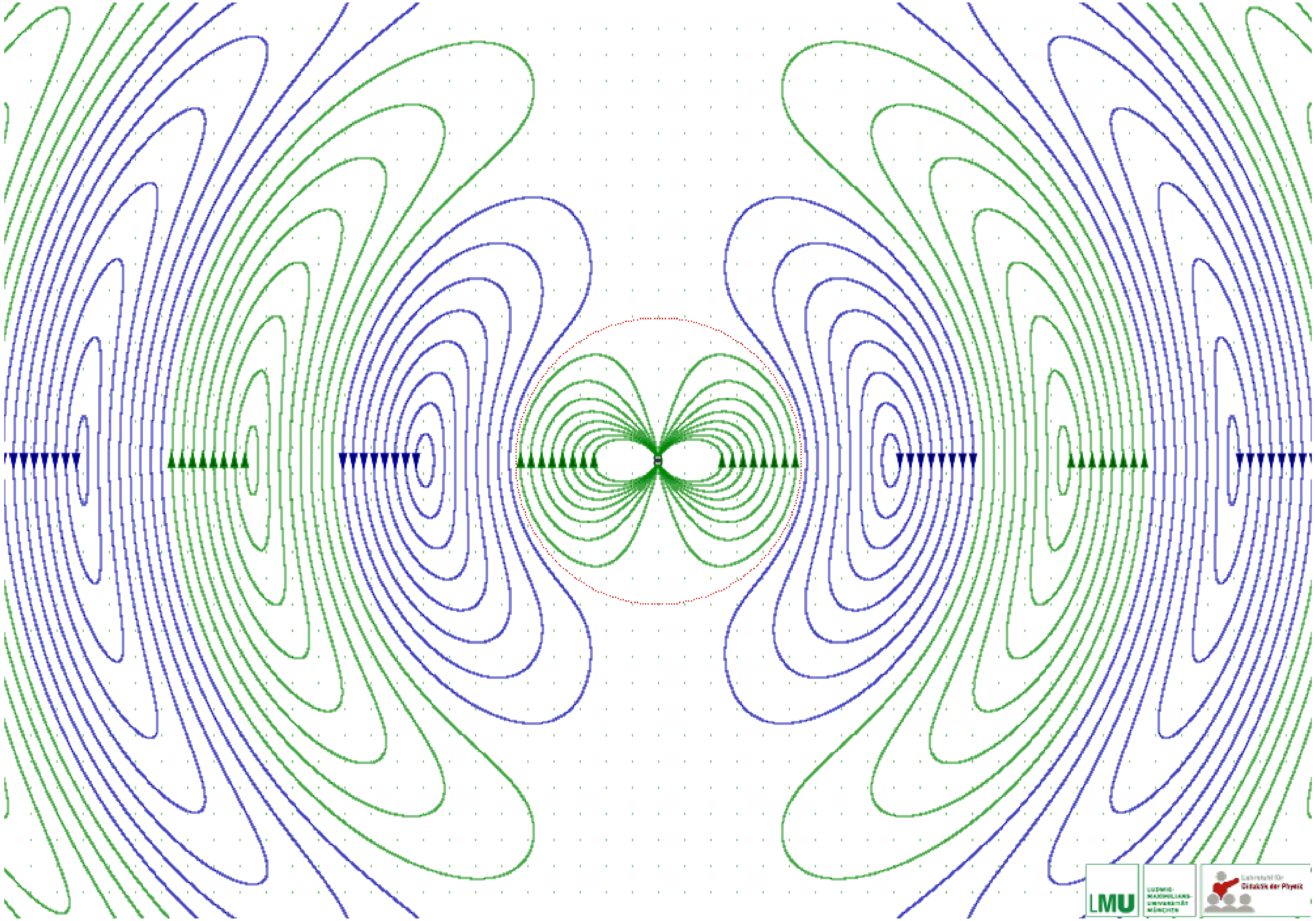


Process repeats in opposite direction

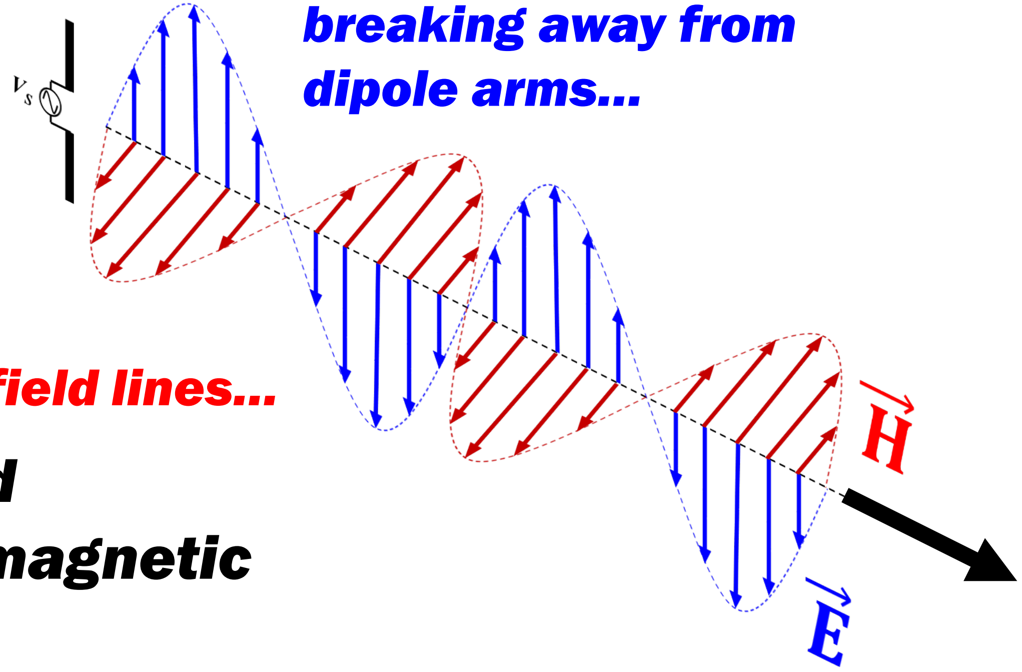
Simplified Radiation Mechanism – Short Dipole Animation

https://www.didaktik.physik.uni-muenchen.de/_assets/bilder/Multimedia/bilder_dipol/web_bilder_orig/dip_1h___o.gif

Courtesy Ludwig Maximilians-Universität Mũnchen



Simplified Radiation Mechanism – Short Dipole Example (cont.)



Electric field lines breaking away from dipole arms…

+ magnetic field lines…

= radiated electromagnetic waves

Poynting vector

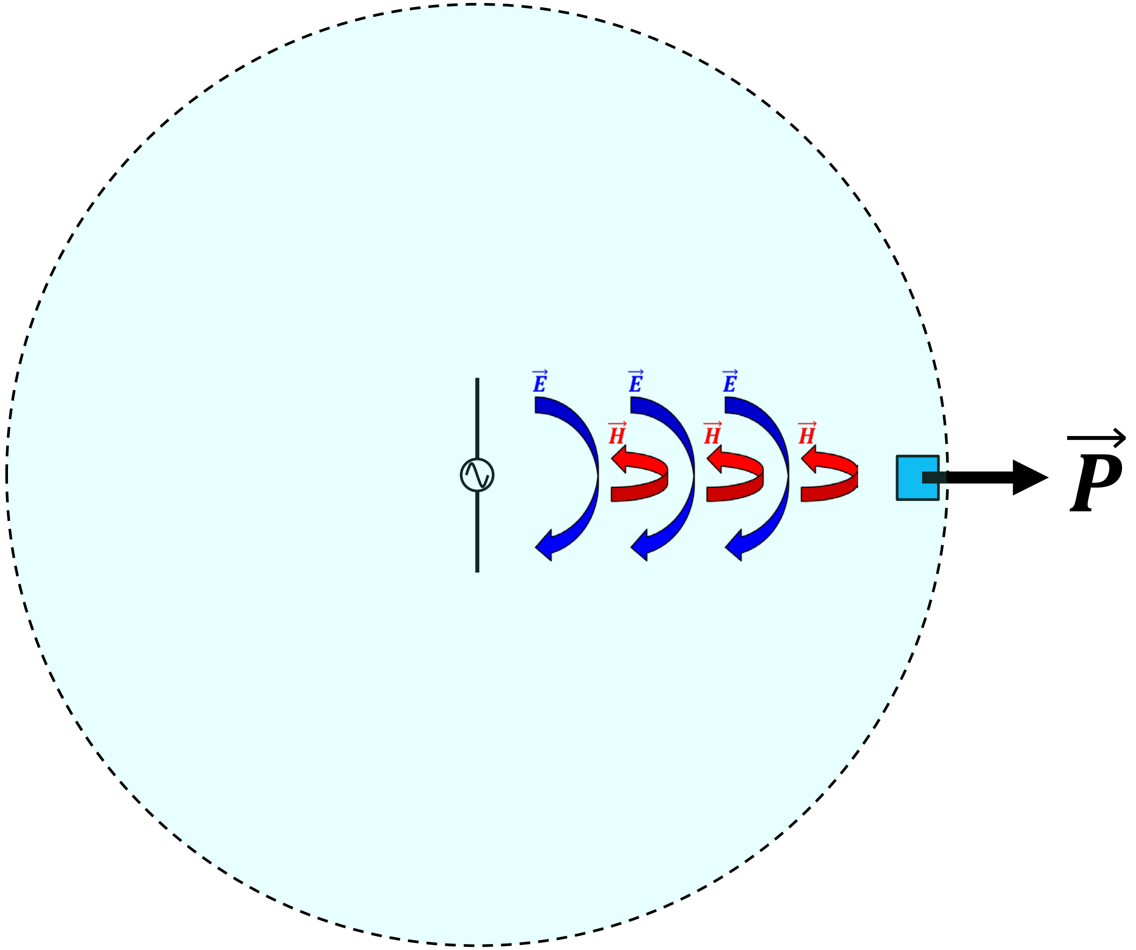
$$\vec{P} = \vec{E} \times \vec{H}$$

$$|\vec{P}| = |E||H|$$ ***Power density***

Propagating electromagnetic waves transmit power from antenna

$$\frac{V}{m} \cdot \frac{A}{m} = \frac{W}{m^2}$$

Transmitted Power



Total transmitted power

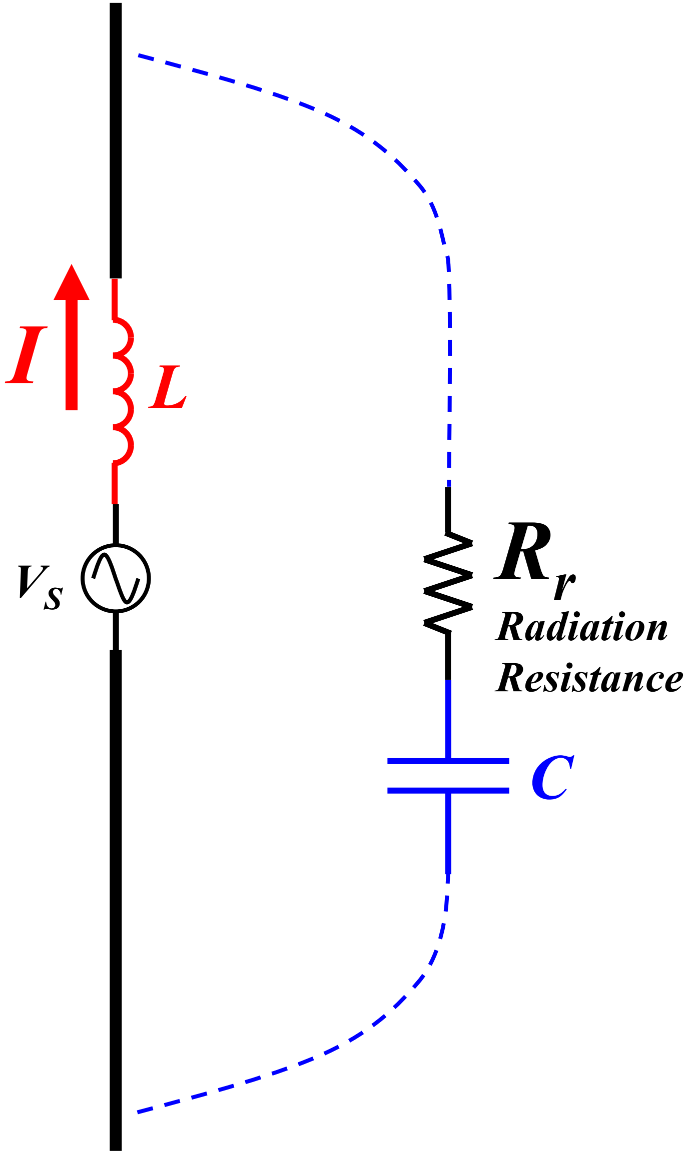
$$P_T = \iint \vec{\boldsymbol{P}} \cdot \boldsymbol{d}\vec{\boldsymbol{s}}$$

Integrated over
4π steradians

Radiation Resistance

Transmitted power proportional to square of dipole current:

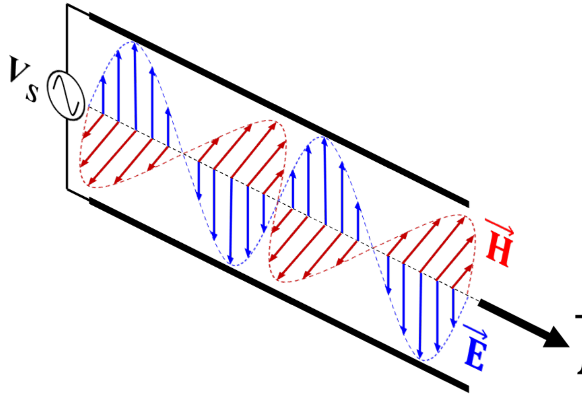
$$P_T \propto I^2$$



$$P_{av} = \frac{1}{2} I^2 R_r$$

Effective load resistance to relate dipole current to radiated power (NOT physical resistor)

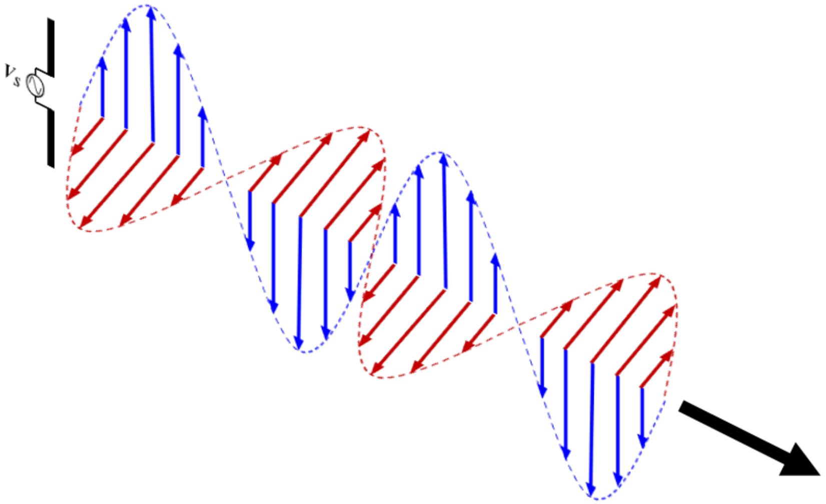
From Transmission Line to Dipole (revisited)



$$\vec{P} = \vec{E} \times \vec{H}$$

Transmission line:

- ***Spacing between conductors generally much less than wavelength***
- ***Fields and power contained in space between conductors***

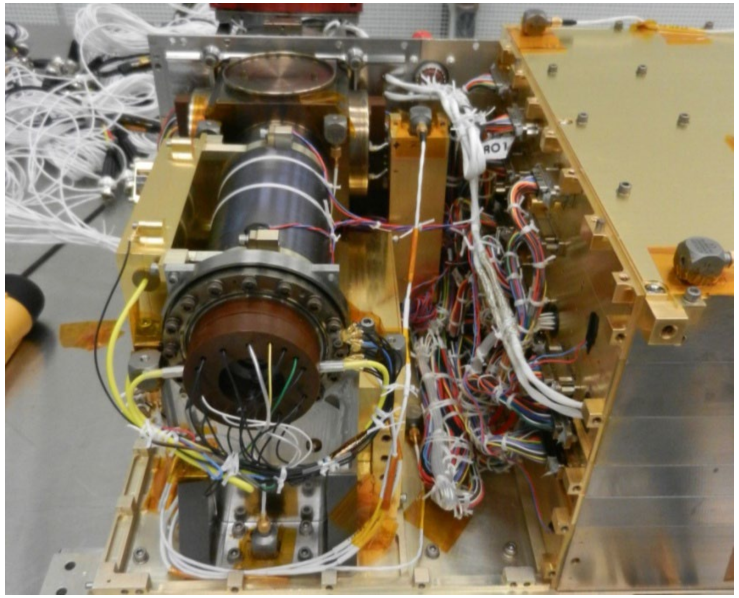


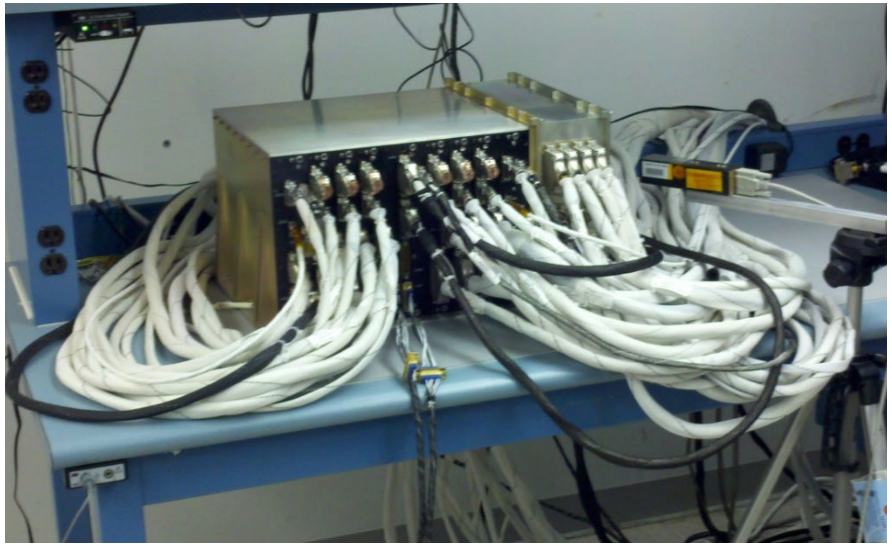
$$\vec{P} = \vec{E} \times \vec{H}$$

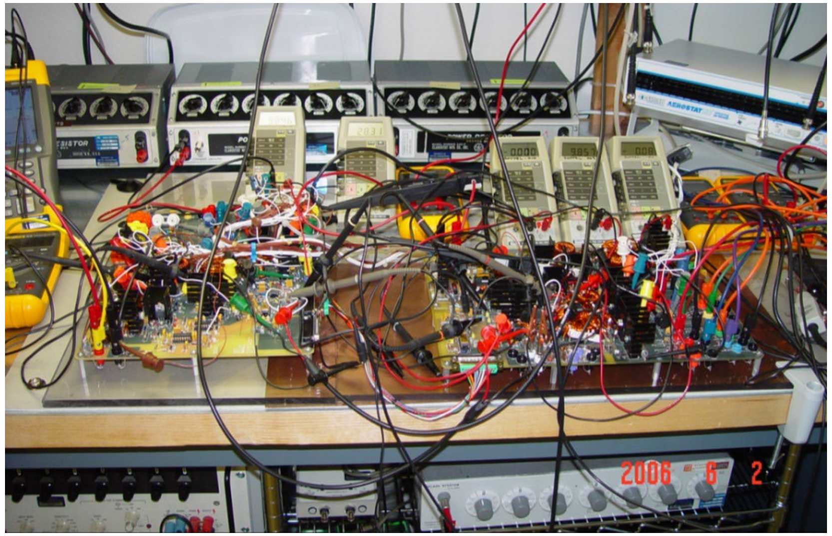
Dipole (and antennas in general):

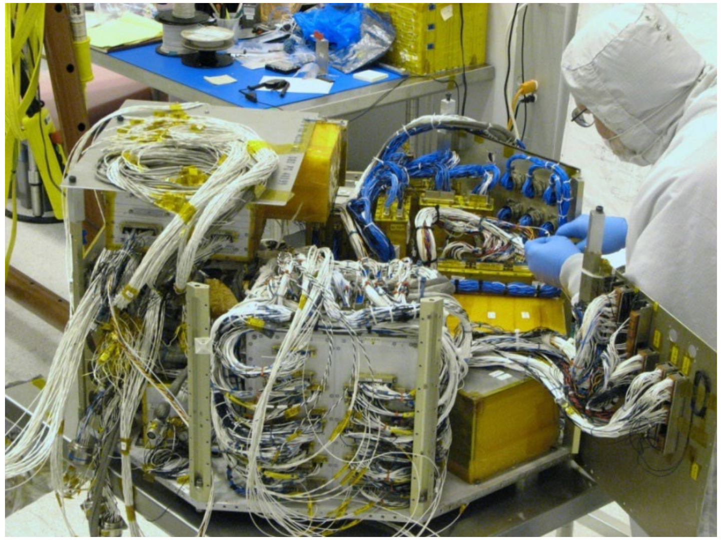
- ***Spacing between conductors is significant fraction of wavelength***
- ***Fields and power transmitted away from conductors***

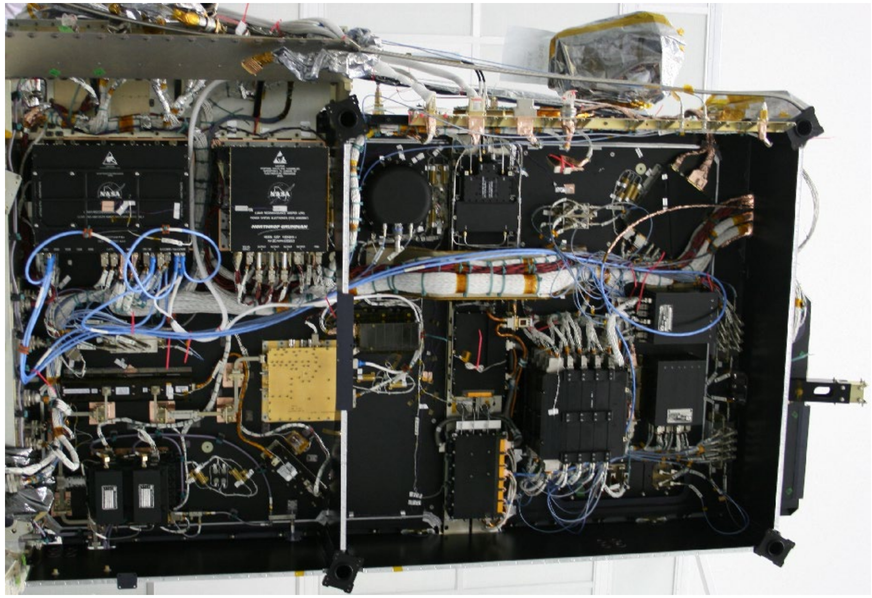
Can You Find the Dipoles in These Pictures?

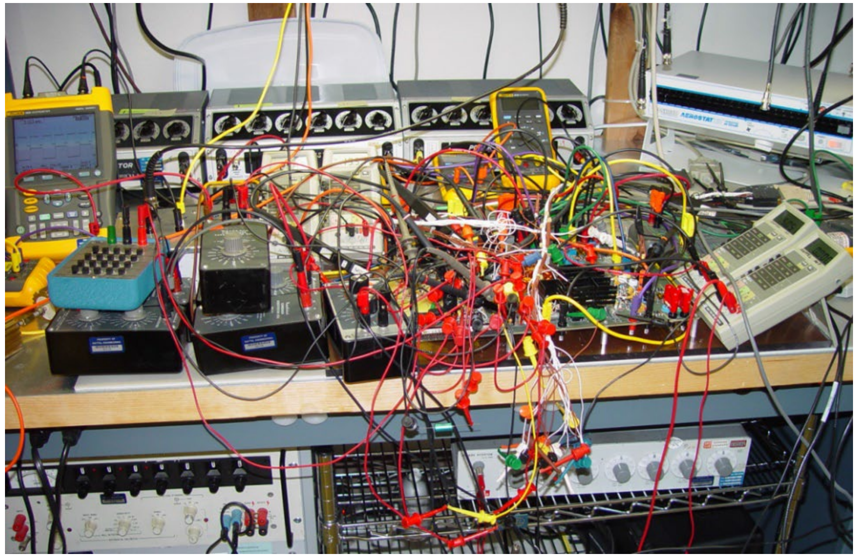








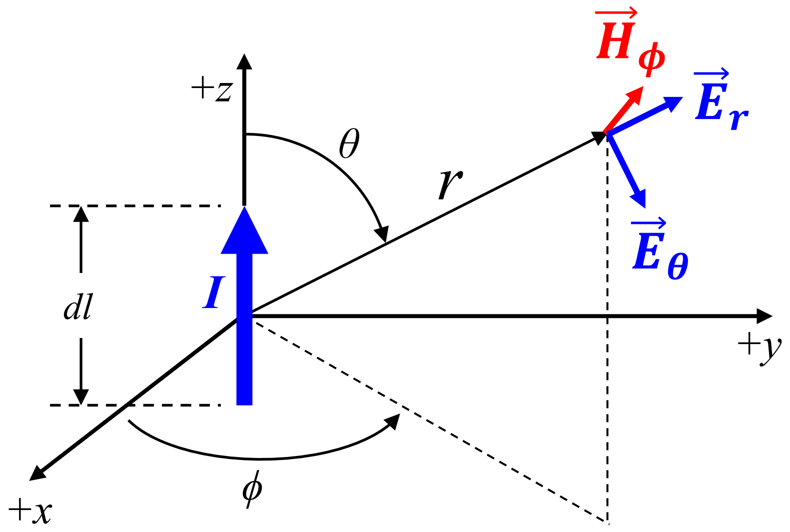




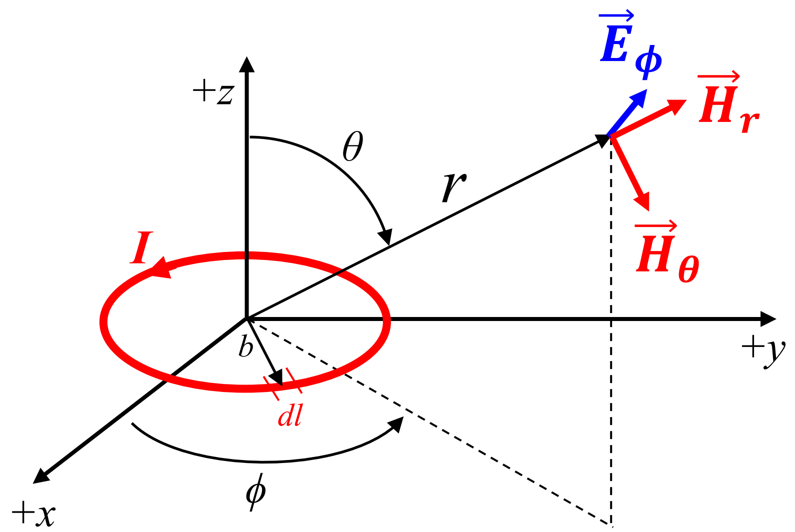
Elemental Dipoles

- Following discussion will address elemental dipoles, electric (Hertzian) and magnetic (loop)
- Covered in every electromagnetics and antennas textbook; you'll need to recognize them
- Very useful when used properly, but a few cautions are in order (sound familiar?)

Electric (Hertzian) Dipole



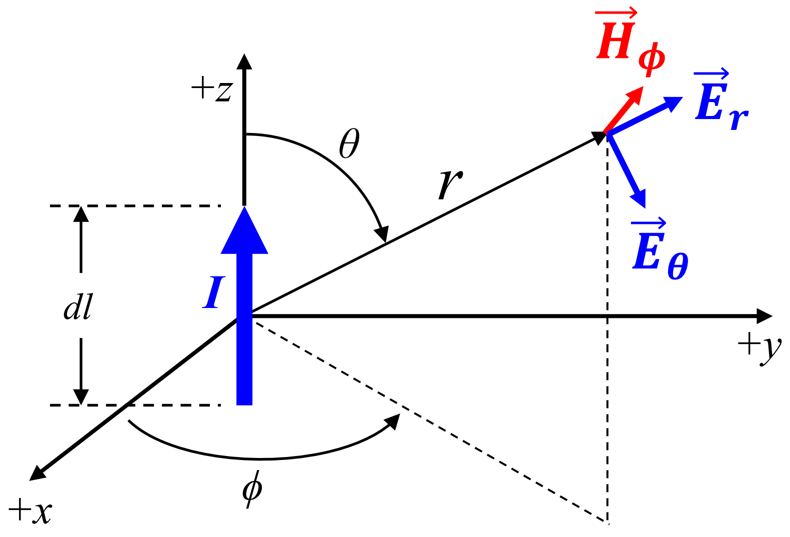
Magnetic (Loop) Dipole



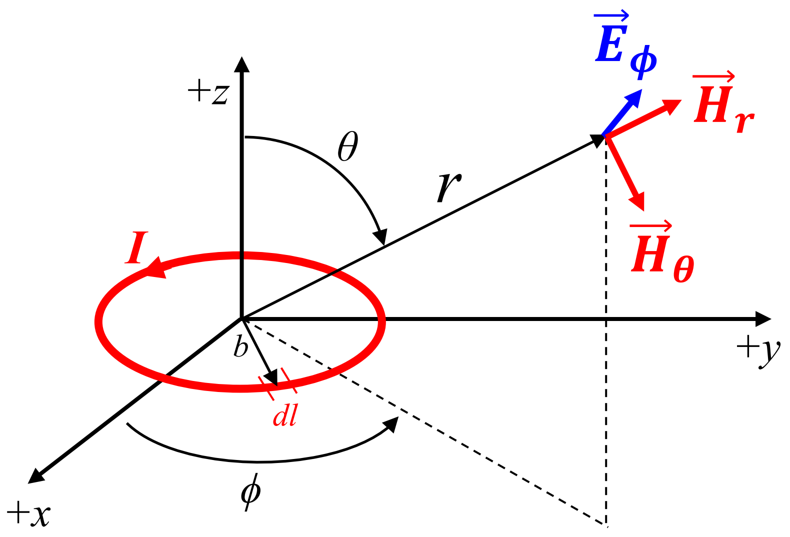
Elemental Dipoles (cont.)

- Each dipole model is treated as a <u>POINT SOURCE</u>
- All observation distances r taken to be sufficient to make it appear to be a <u>POINT SOURCE</u>
- NOT MEANINGFUL OR APPLICABLE AS $r \rightarrow dl$ or $r \rightarrow b$!!!
- Math gives infinite fields as $r \rightarrow 0$; fields are limited by available potential and current

Electric (Hertzian) Dipole



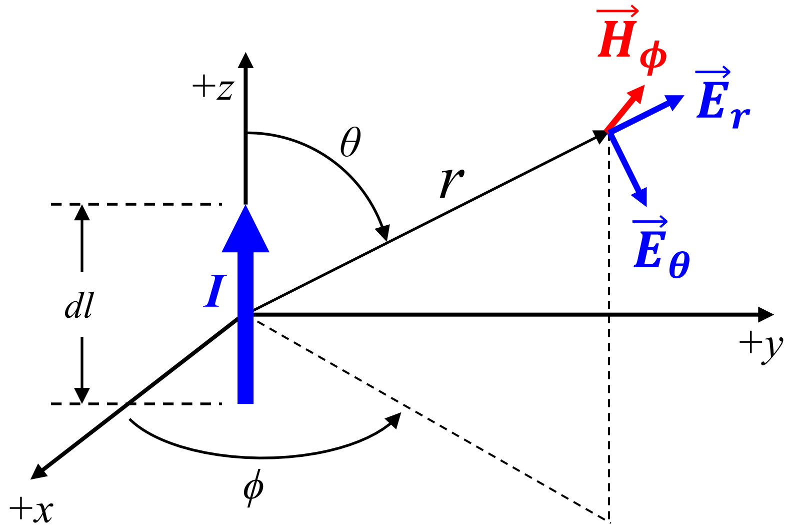
Magnetic (Loop) Dipole



Field Regions

- **From Balanis, section 2.2.4:**
  - Near-field (reactive region):
    - That region of the field immediately surrounding the antenna wherein the reactive field predominates
  - Intermediate field (Fresnel region):
    - That region of the field of an antenna between the reactive near-field region and the far-field region wherein radiation fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna
  - Far-field (Fraunhofer region):
    - That region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna

Elemental Electric (Hertzian) Dipole (cont.)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity:

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = -\frac{Idl}{4\pi}\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right] e^{-j\beta r}$$

Electric field intensity:

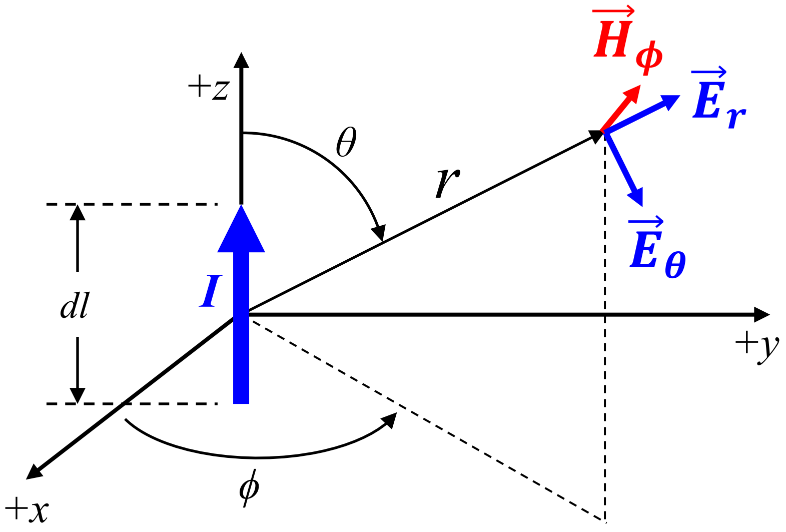
$$E_r = -2\frac{Idl}{4\pi}\eta\beta^2 \cos\theta \left[\frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\theta = -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\phi = 0$$

Derivations in backup slides

Elemental Electric Dipole: Near Field (Reactive Region)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = -\frac{Idl}{4\pi}\beta^2 \sin\theta \left[\underbrace{\cancel{\frac{1}{j\beta r}}}_{\rightarrow 0} + \frac{1}{(j\beta r)^2}\right] e^{-j\beta r}$$

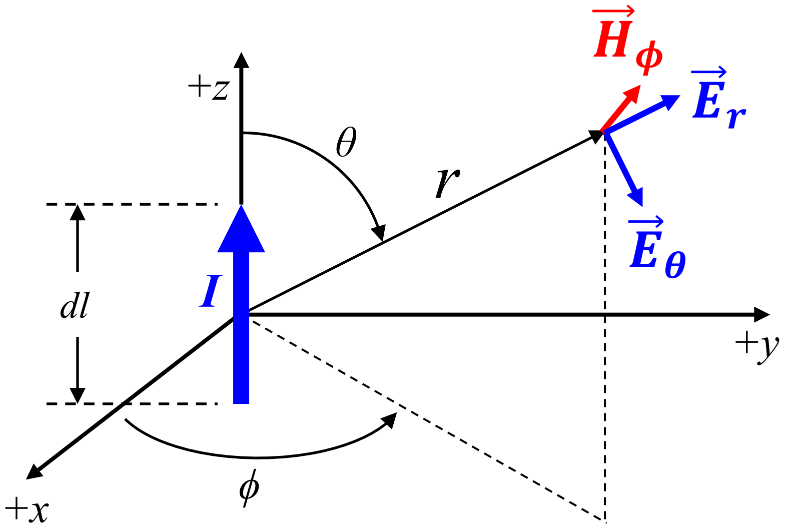
Electric field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$E_r = -2\frac{Idl}{4\pi}\eta\beta^2 \cos\theta \left[\cancel{\frac{1}{(j\beta r)^2}} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\theta = -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[\cancel{\frac{1}{j\beta r}} + \underbrace{\cancel{\frac{1}{(j\beta r)^2}}}_{\rightarrow 0} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\phi = 0$$

Elemental Electric Dipole: Near Field (cont.)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi \approx -\frac{Idl}{4\pi}\beta^2 \sin\theta \left[\frac{1}{(j\beta r)^2}\right] e^{-j\beta r}$$

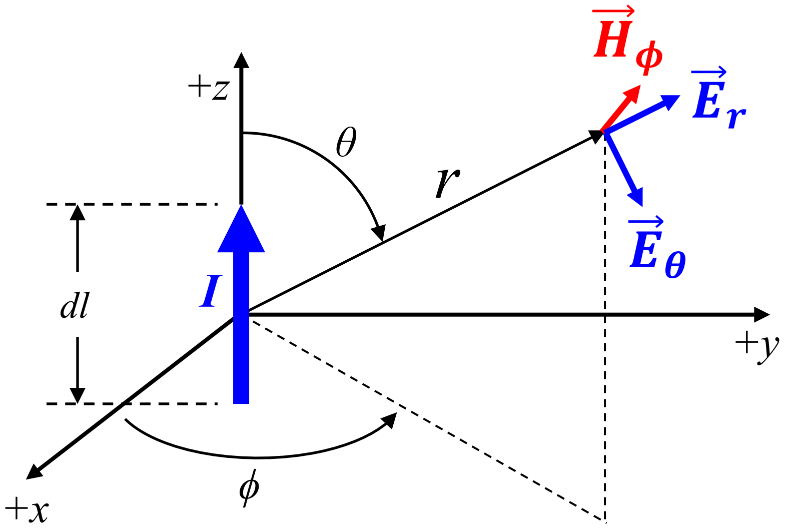
Electric field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$E_r \approx -2\frac{Idl}{4\pi}\eta\beta^2 \cos\theta \left[\frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\theta \approx -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[\frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\phi = 0$$

Elemental Electric Dipole: Near Field (cont.)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi \approx -j\frac{Idl}{4\pi\beta r^2}\sin\theta\, e^{-j\beta r}$$

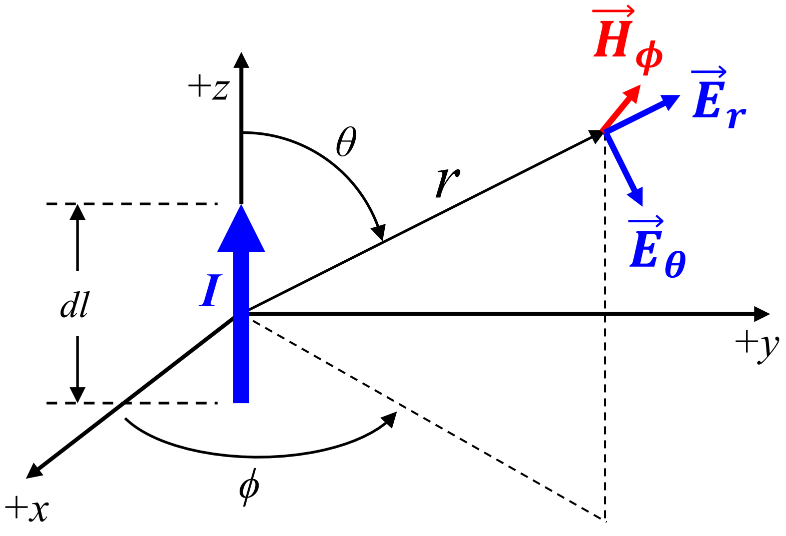
Electric field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$E_r \approx -j\frac{Idl}{2\pi\beta r^3}\eta\cos\theta\, e^{-j\beta r}$$

$$E_\theta \approx -j\frac{Idl}{4\pi\beta r^3}\eta\sin\theta\, e^{-j\beta r}$$

$$E_\phi = 0$$

Elemental Electric Dipole: Intermediate Field (Fresnel Region)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi \approx -\frac{Idl}{4\pi}\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \cancelto{0}{\frac{1}{(j\beta r)^2}}\right] e^{-j\beta r}$$

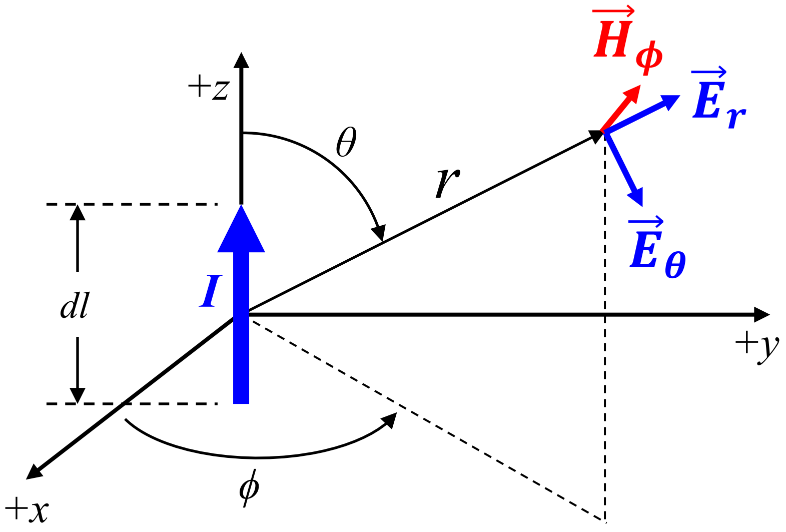
Electric field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$E_r \approx -2\frac{Idl}{4\pi}\eta\beta^2 \cos\theta \left[\frac{1}{(j\beta r)^2} + \cancel{\frac{1}{(j\beta r)^3}}\right] e^{-j\beta r}$$

$$E_\theta \approx -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \cancel{\frac{1}{(j\beta r)^2}} + \cancel{\frac{1}{(j\beta r)^3}}\right] e^{-j\beta r} \quad \rightarrow 0$$

$$E_\phi = 0$$

Elemental Electric Dipole: Intermediate Field (cont.)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi \approx -\frac{Idl}{4\pi}\beta^2 \sin\theta \left[\frac{1}{j\beta r}\right] e^{-j\beta r}$$

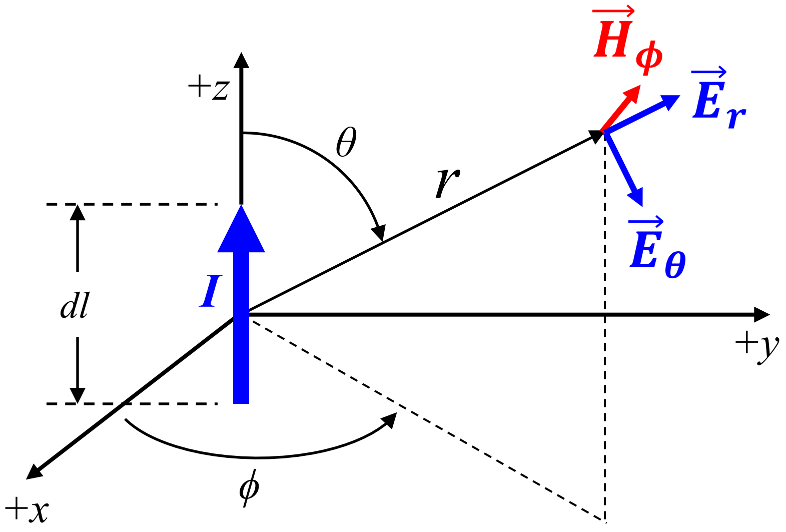
Electric field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$E_r \approx -2\frac{Idl}{4\pi}\eta\beta^2 \cos\theta \left[\frac{1}{(j\beta r)^2}\right] e^{-j\beta r}$$

$$E_\theta \approx -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[\frac{1}{j\beta r}\right] e^{-j\beta r}$$

$$E_\phi = 0$$

Elemental Electric Dipole: Intermediate Field (cont.)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi \approx j\frac{Idl}{4\pi r}\beta \sin\theta\, e^{-j\beta r}$$

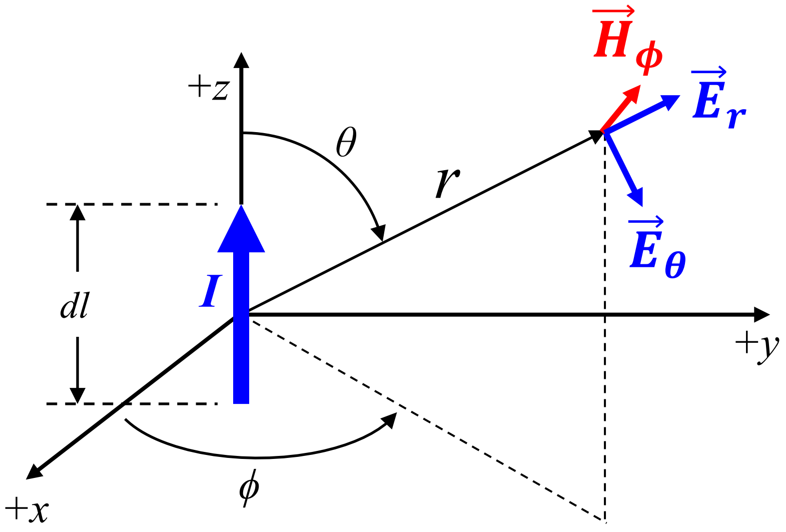
Electric field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$E_r \approx \frac{Idl}{2\pi r^2}\eta \cos\theta\, e^{-j\beta r}$$

$$E_\theta \approx j\frac{Idl}{4\pi r}\eta\beta \sin\theta\, e^{-j\beta r}$$

$$E_\phi = 0$$

Elemental Electric Dipole: Far Field (Fraunhofer Region)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi \approx j\frac{Idl}{4\pi r}\beta \sin\theta \, e^{-j\beta r}$$

Electric field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

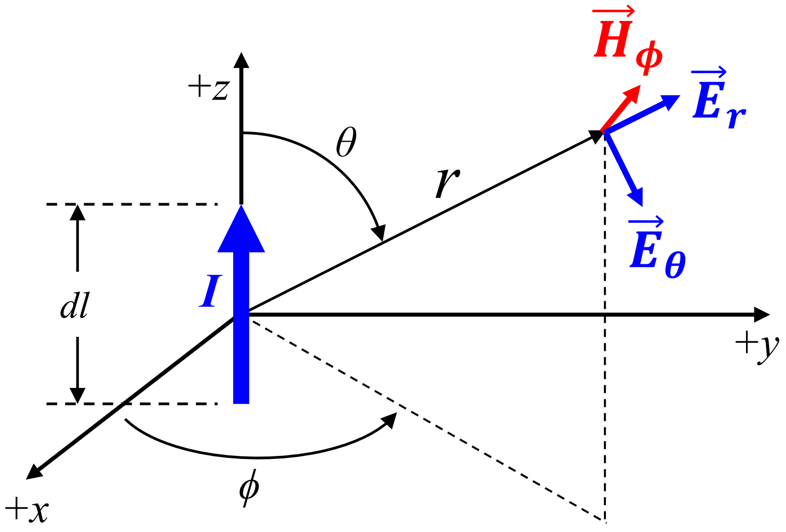
$$E_r \approx \cancel{\frac{Idl}{2\pi r^2}}_{\rightarrow 0}\eta \cos\theta \, e^{-j\beta r}$$

$E_r = 0$ in "far field"

$$E_\theta \approx j\frac{Idl}{4\pi r}\eta\beta \sin\theta \, e^{-j\beta r}$$

$$E_\phi = 0$$

Elemental Electric Dipole: Far Field (cont.)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi \approx j\frac{Idl}{4\pi r}\beta \sin\theta \, e^{-j\beta r}$$

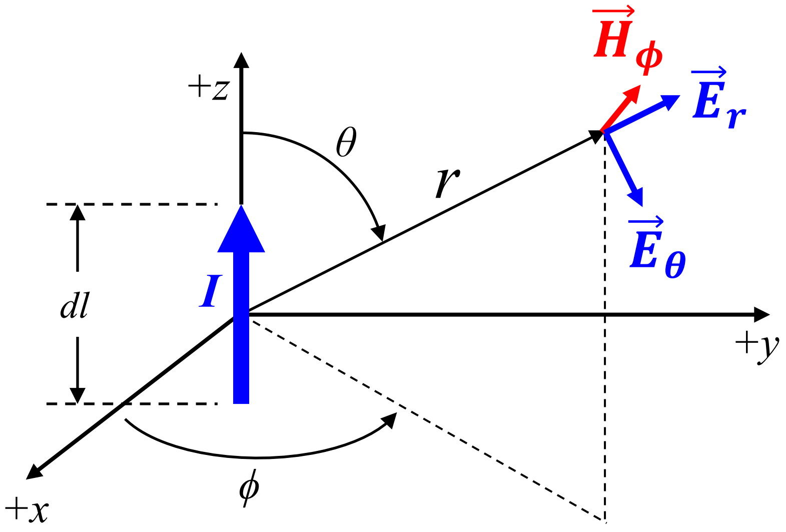
Electric field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$E_r = 0$$

$$E_\theta \approx j\frac{Idl}{4\pi r}\eta\beta \sin\theta \, e^{-j\beta r}$$

$$E_\phi = 0$$

Elemental Electric Dipole: Far Field (cont.)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$H_\phi = j\frac{Idl}{4\pi r}\beta \sin\theta e^{-j\beta r}$$

Electric field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

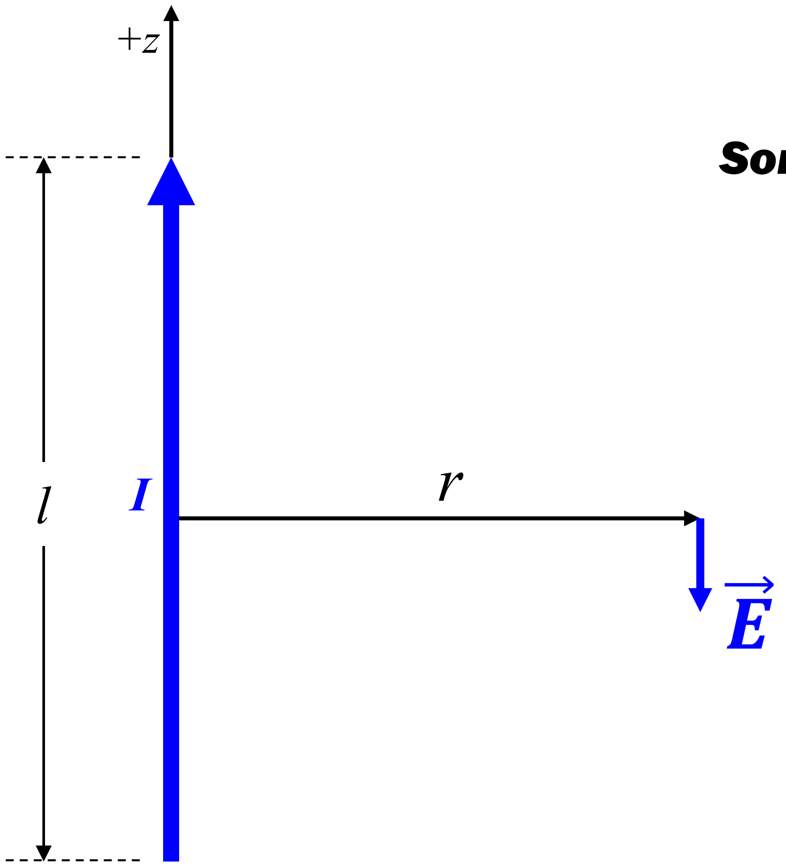
$$E_\theta = j\frac{Idl}{4\pi r}\eta\beta \sin\theta e^{-j\beta r}$$

In far field:

$$\frac{E_\theta}{H_\phi} = \eta$$

Characteristic impedance of medium (377 Ω in air/vacuum)

Electric Field from Current-Carrying Wire



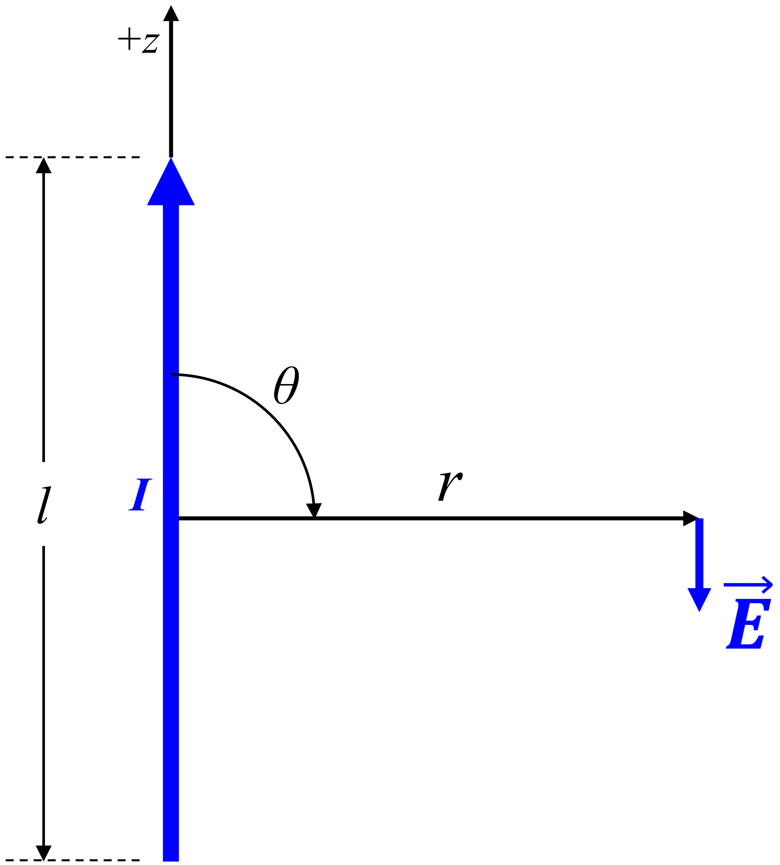
Sometimes you will see this equation…

$$E = \frac{\mu I l f}{2r}$$

WARNING!!!
WARNING!!!
WARNING!!!

Use only if you understand the underlying assumptions…

Electric Field from Current-Carrying Wire (cont.)



Contribution from element dz:

$$dE = -\frac{I(z)dz}{4\pi}\eta\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

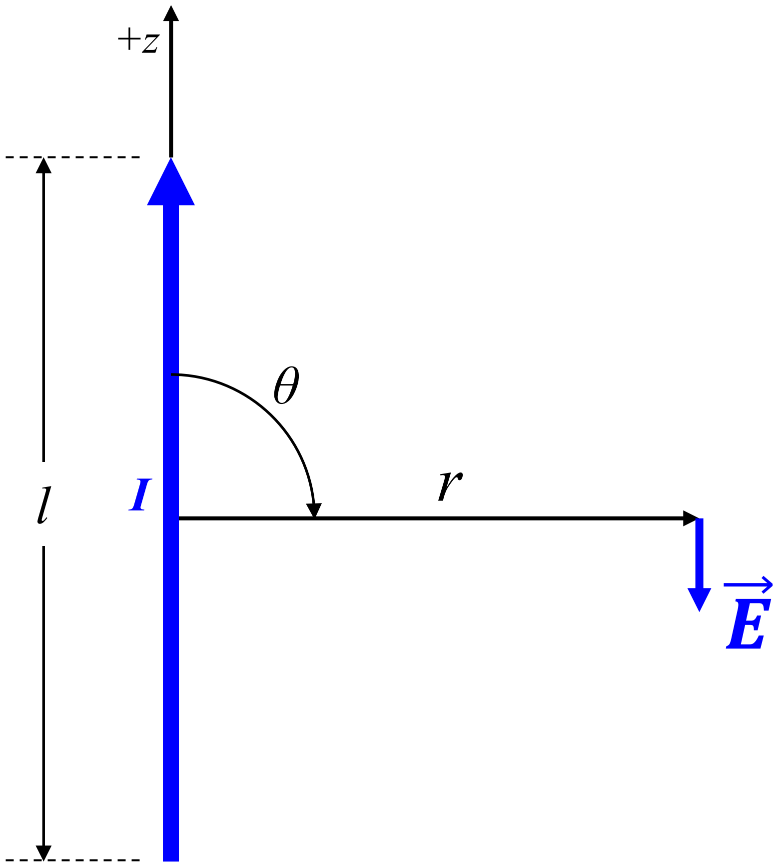
Total electric field at distance r from midpoint of wire:

$$E = -\frac{\eta\beta^2}{4\pi} \underbrace{\int_{-l/2}^{l/2} \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r} I(z)dz}_{\text{a VERY sick integral}}$$

I, r, θ
all vary with z

a VERY sick integral

Electric Field from Current-Carrying Wire (cont.)



$$E = -\frac{\eta\beta^2}{4\pi}\int_{-l/2}^{l/2} \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r} I(z)\,dz$$

$$E \approx -\frac{\eta\beta^2}{4\pi}\int_{-l/2}^{l/2} \sin\theta \left[\frac{1}{j\beta r}\right] e^{-j\beta r} I(z)\,dz$$

Assume far field
1/(jβr) term dominates

$$\approx j\frac{\eta\beta}{4\pi r}\int_{-l/2}^{l/2} \sin\theta\, I(z)\,dz$$

Assume all points on wire are equidistant from observation point (constant r) and are in phase (lose exponential term)

$$\approx j\frac{\eta\beta I l}{4\pi r}$$

Assume constant I along full length ($l << \lambda$)
Set $\theta = \pi/2$

$$|E|_{MAX} \approx \frac{\eta\beta I l}{4\pi r}$$

Take magnitude only
Set as maximum upper bound
(given above assumptions)

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \qquad \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \beta = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu\varepsilon}$$

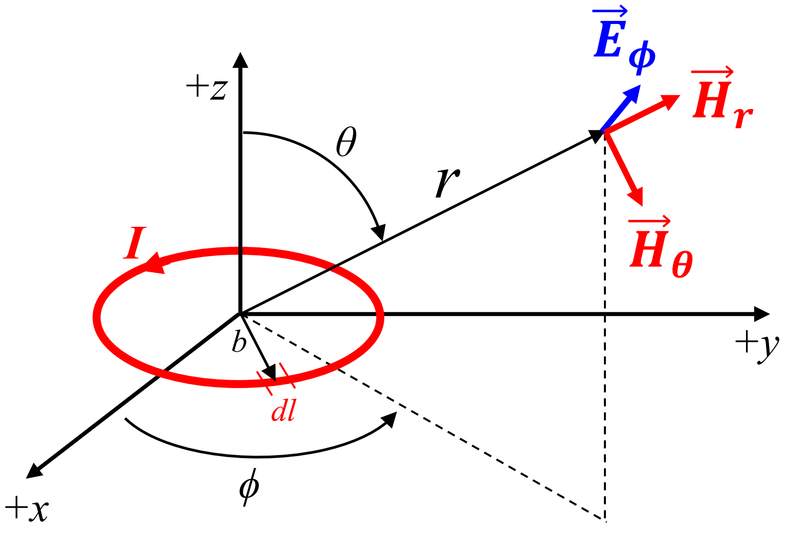
$$|E|_{MAX} \approx \frac{Il}{4\pi r}\cdot\sqrt{\frac{\mu}{\varepsilon}}\cdot 2\pi f\sqrt{\mu\varepsilon}$$

Assumptions a-plenty

$$|E|_{MAX} \approx \frac{\mu I l f}{2r}$$

Upper bound estimate ONLY
NOT TO BE USED AS PRECISE VALUE

Elemental Magnetic Dipole



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I \pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity:

$$H_r = -j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\theta = -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\phi = 0$$

Electric field intensity:

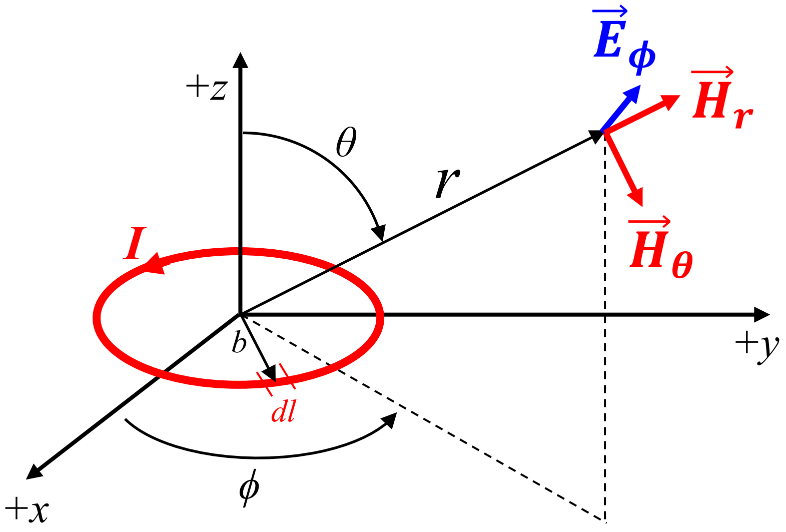
$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi = j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

Derivations in backup slides

Elemental Magnetic Dipole: Near Field (Reactive Region)



Vector magnetic moment:

$$\vec{\boldsymbol{m}} = \vec{\boldsymbol{a}}_{\boldsymbol{z}} I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$H_r = -j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\cancel{\frac{1}{(j\beta r)^2}} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\theta = -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\cancel{\frac{1}{j\beta r}} + \cancel{\frac{1}{(j\beta r)^2}} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$\rightarrow 0$

$$H_\phi = 0$$

Electric field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

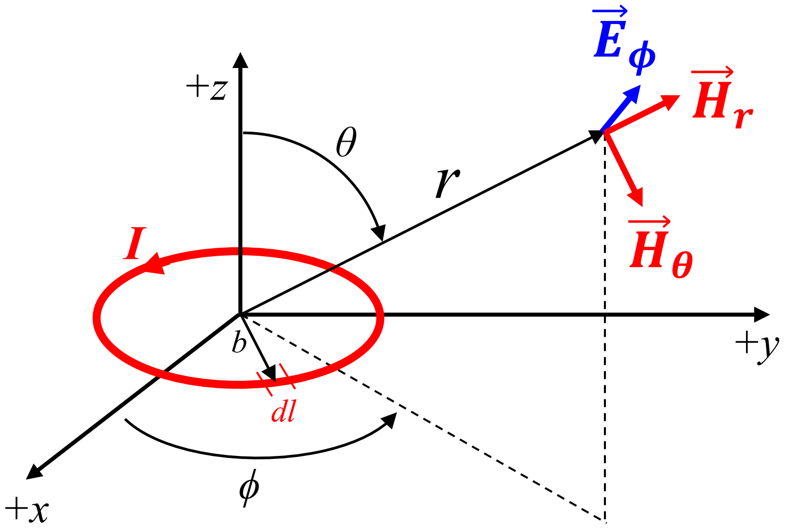
$$E_r = 0$$

$$E_\theta = 0$$

$\rightarrow 0$

$$E_\phi = j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[\cancel{\frac{1}{j\beta r}} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

Elemental Magnetic Dipole: Near Field (cont.)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I \pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$H_r \approx -j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\theta \approx -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\phi = 0$$

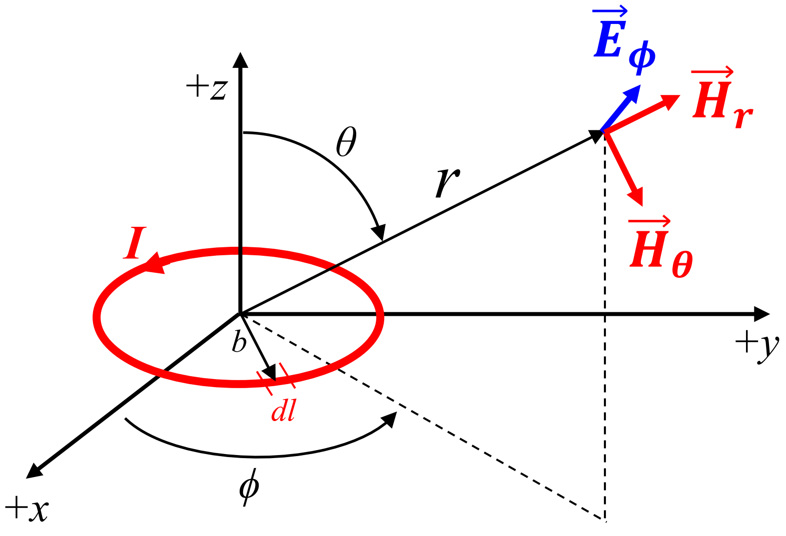
Electric field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi \approx j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[\frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

Elemental Magnetic Dipole: Near Field (cont.)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$\left.\begin{aligned} H_r &\approx \frac{\omega\mu m}{2\pi\eta\beta r^3}\cos\theta\, e^{-j\beta r} \\ H_\theta &\approx \frac{\omega\mu m}{4\pi\eta\beta r^3}\sin\theta\, e^{-j\beta r} \end{aligned}\right\} \frac{\omega\mu}{\eta\beta} = ?$$

$$H_\phi = 0$$

Electric field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

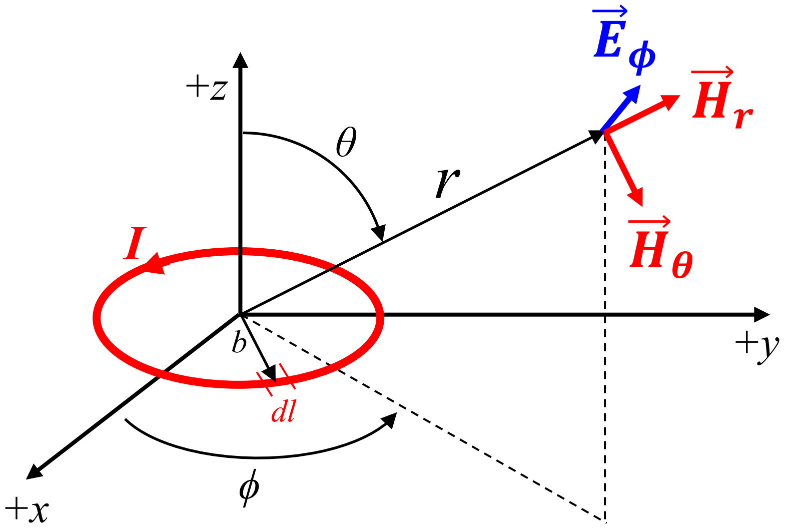
$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi \approx -j\frac{\omega\mu m}{4\pi r^2}\sin\theta\, e^{-j\beta r}$$

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}} \qquad \frac{\omega\mu}{\eta\beta} = \mu\cdot\frac{1}{\sqrt{\mu\varepsilon}}\cdot\frac{1}{\sqrt{\mu/\varepsilon}} = \mu\frac{1}{\mu} = 1$$

Elemental Magnetic Dipole: Near Field (cont.)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$H_r \approx \frac{m}{2\pi r^3}\cos\theta\, e^{-j\beta r}$$

$$H_\theta \approx \frac{m}{4\pi r^3}\sin\theta\, e^{-j\beta r}$$

$$H_\phi = 0$$

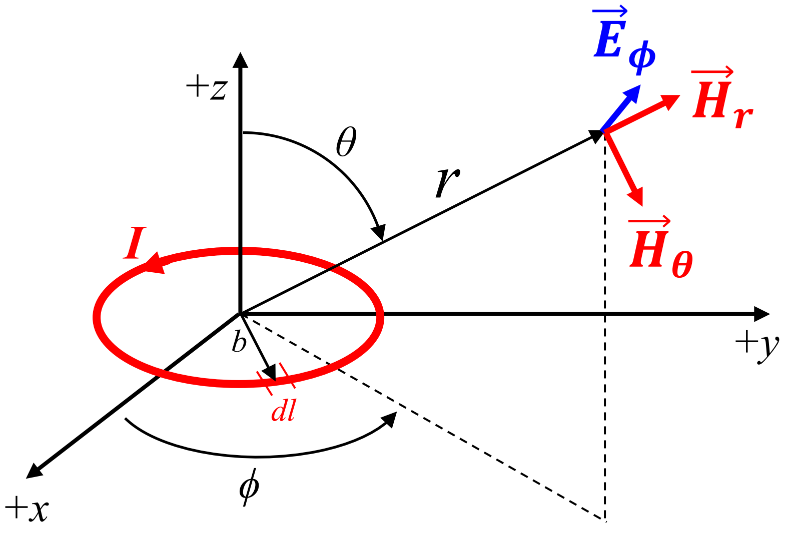
Electric field intensity, "near field" ($\beta r << 1 \rightarrow r << \lambda/2\pi$):

$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi \approx -j\frac{m}{4\pi r^2}\eta\beta\sin\theta\, e^{-j\beta r}$$

Elemental Magnetic Dipole: Intermediate Field (Fresnel Region)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$H_r = -j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\frac{1}{(j\beta r)^2} + \cancel{\frac{1}{(j\beta r)^3}}\right]e^{-j\beta r}$$

$\rightarrow 0$

$$H_\theta = -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{j\beta r} + \cancel{\frac{1}{(j\beta r)^2}} + \cancel{\frac{1}{(j\beta r)^3}}\right]e^{-j\beta r}$$

$$H_\phi = 0$$

Electric field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

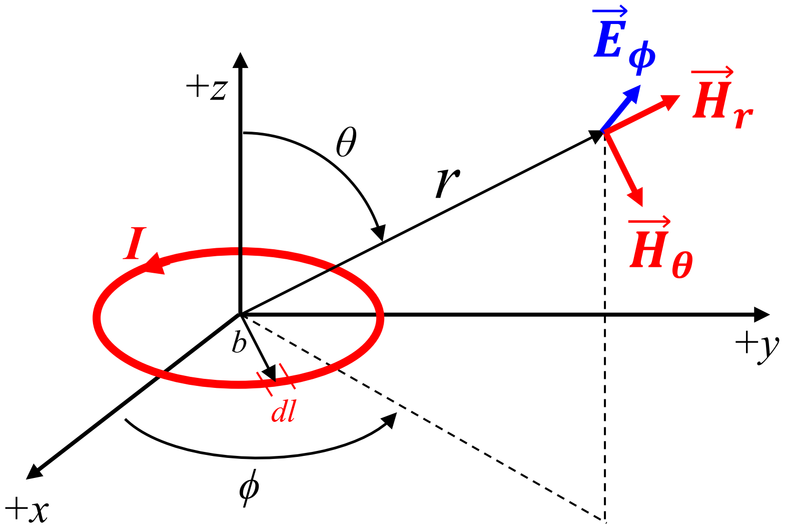
$$E_r = 0$$

$$E_\theta = 0$$

$\rightarrow 0$

$$E_\phi = j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta r} + \cancel{\frac{1}{(j\beta r)^2}}\right]e^{-j\beta r}$$

Elemental Magnetic Dipole: Intermediate Field (cont.)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$H_r \approx -j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

$$H_\theta \approx -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{j\beta r}\right]e^{-j\beta r}$$

$$H_\phi = 0$$

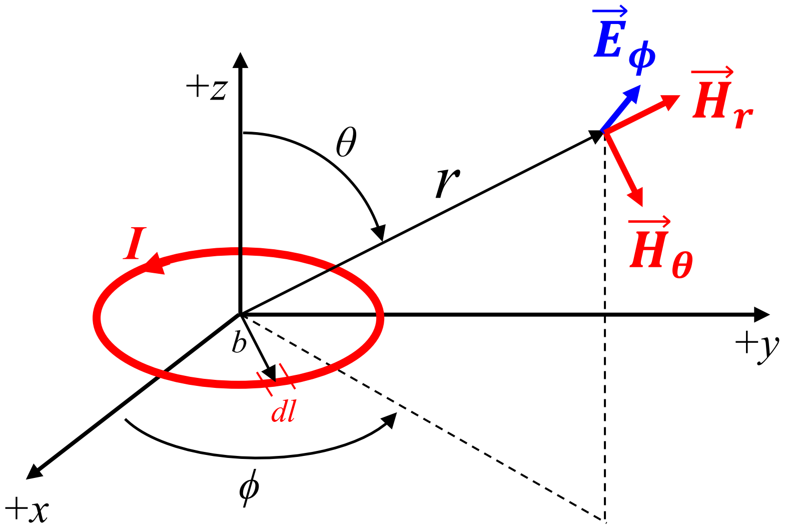
Electric field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi \approx j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta r}\right]e^{-j\beta r}$$

Elemental Magnetic Dipole: Intermediate Field (cont.)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$H_r \approx j\frac{\omega\mu m}{2\pi\eta r^2}\cos\theta \, e^{-j\beta r}$$

$$H_\theta \approx -\frac{\omega\mu m}{4\pi\eta r}\beta \sin\theta \, e^{-j\beta r}$$

$$H_\phi = 0$$

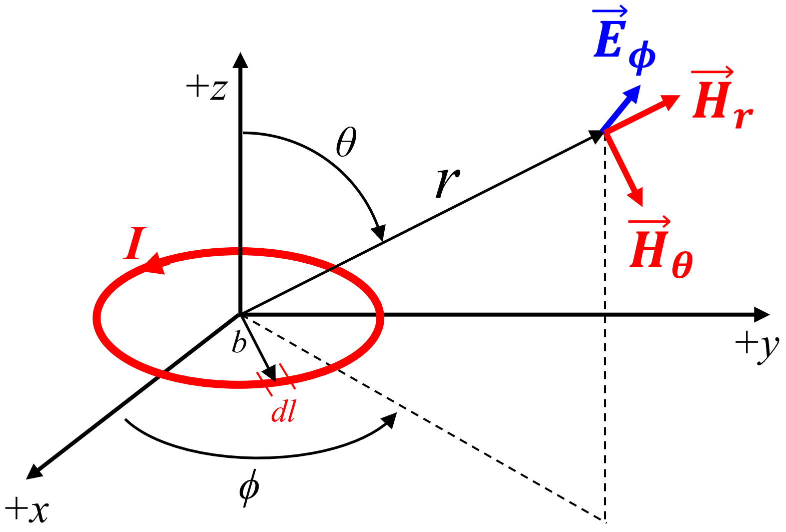
Electric field intensity, "intermediate field" ($\beta r \approx 1 \rightarrow r \approx \lambda/2\pi$):

$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi \approx \frac{\omega\mu m}{4\pi r}\beta \sin\theta \, e^{-j\beta r}$$

Elemental Magnetic Dipole: Far Field (Fraunhofer Region)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I \pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$H_r \approx j \frac{\omega \mu m}{\cancel{2\pi \eta r^2}} \cos\theta \, e^{-j\beta r} \rightarrow 0$$

$H_r = 0$ in "far field"

$$H_\theta \approx -\frac{\omega \mu m}{4\pi \eta r} \beta \sin\theta \, e^{-j\beta r}$$

$$H_\phi = 0$$

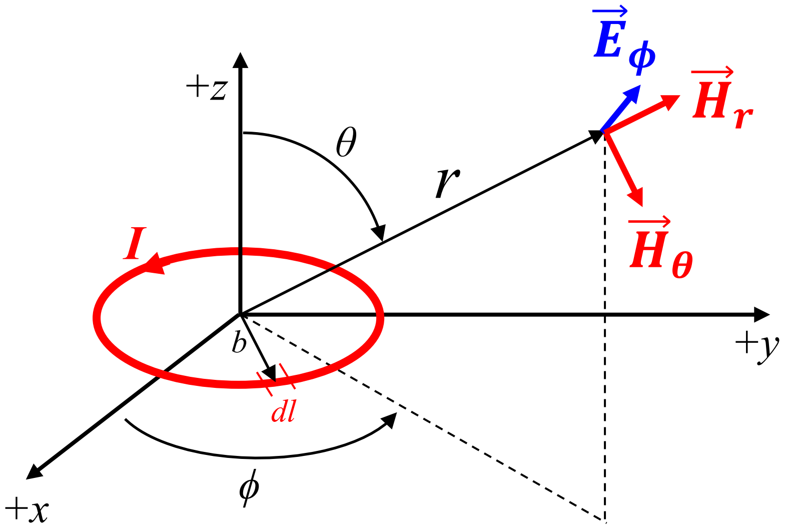
Electric field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi \approx \frac{\omega \mu m}{4\pi r} \beta \sin\theta \, e^{-j\beta r}$$

Elemental Magnetic Dipole: Far Field (cont.)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$H_r = 0$$

$$H_\theta \approx -\frac{\omega\mu m}{4\pi\eta r}\beta \sin\theta \, e^{-j\beta r}$$

$$H_\phi = 0$$

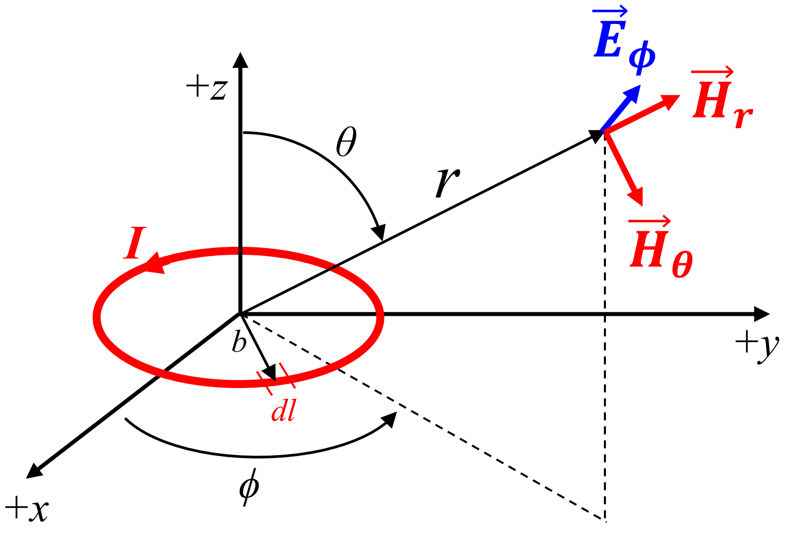
Electric field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi \approx \frac{\omega\mu m}{4\pi r}\beta \sin\theta \, e^{-j\beta r}$$

Elemental Magnetic Dipole: Far Field (cont.)



Vector magnetic moment:

$$\vec{\boldsymbol{m}} = \vec{\boldsymbol{a}}_{\boldsymbol{z}} I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$H_\theta \approx -\frac{\omega\mu m}{4\pi\eta r}\beta \sin\theta e^{-j\beta r}$$

Electric field intensity, "far field" ($\beta r >> 1 \rightarrow r >> \lambda/2\pi$):

$$E_\phi \approx \frac{\omega\mu m}{4\pi r}\beta \sin\theta e^{-j\beta r}$$

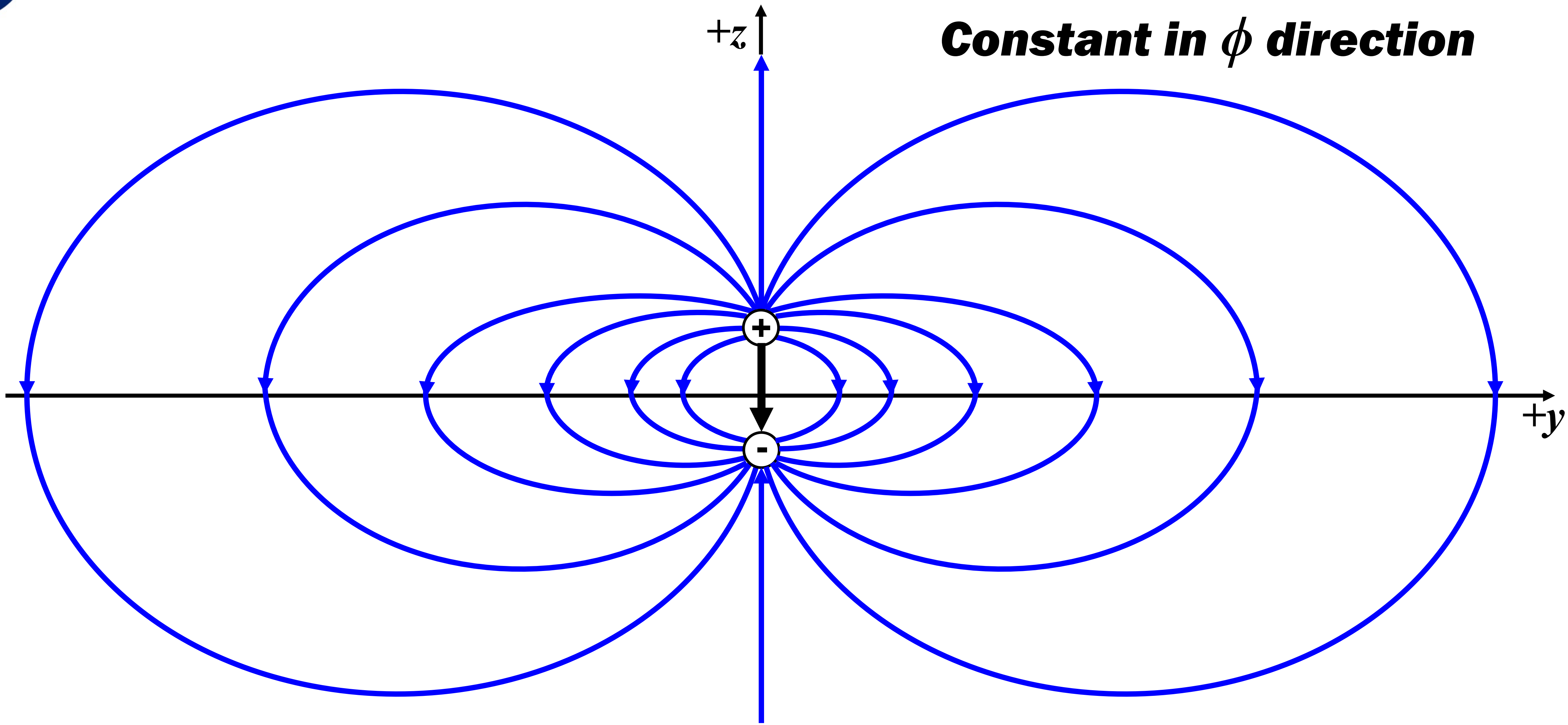
In far field:

$$\left|\frac{E_\theta}{H_\phi}\right| = \eta$$

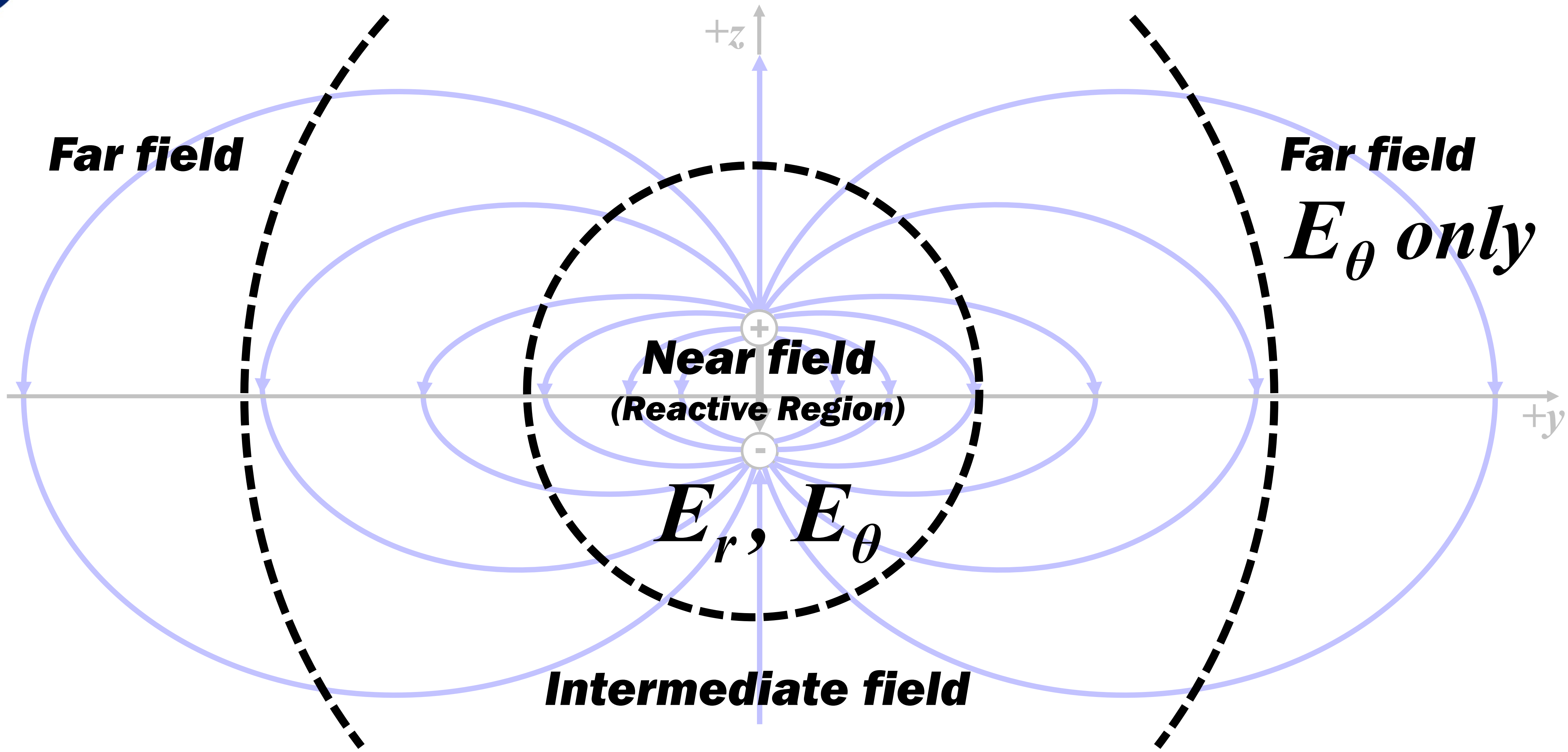
Characteristic impedance of medium (377 Ω in air/vacuum)

Electric Dipole: Electric Field Lines

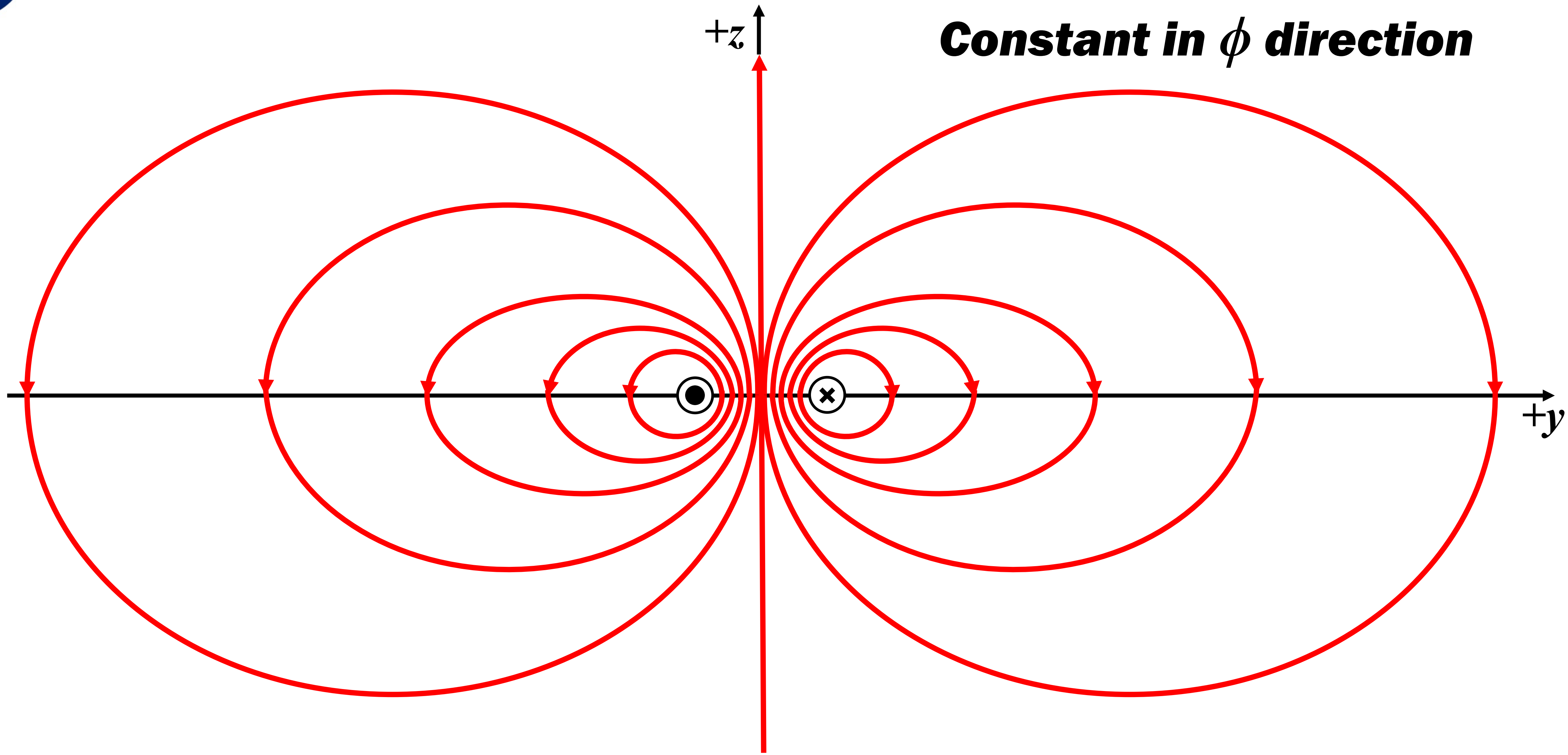
Constant in ϕ direction



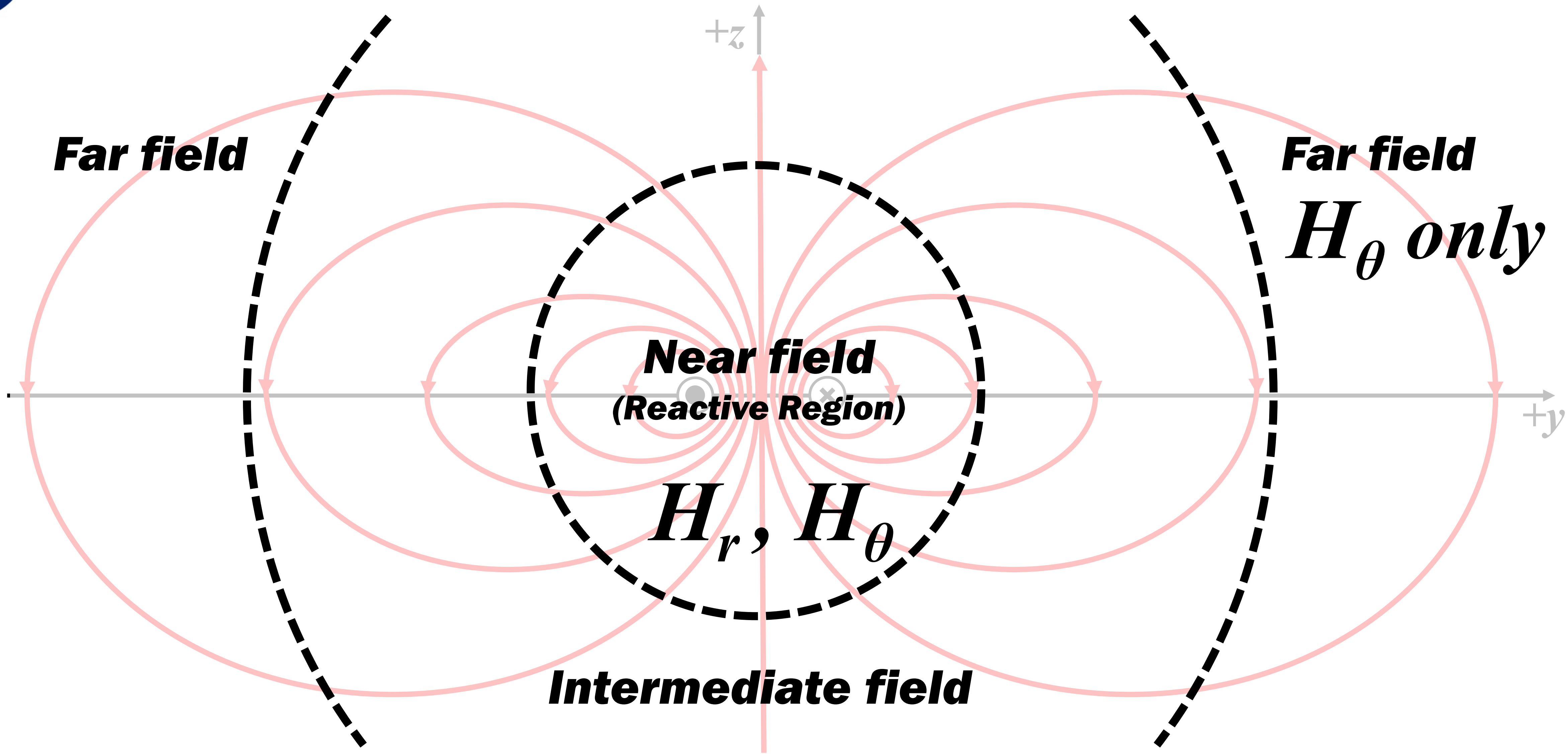
Electric Dipole: Electric Field Lines (cont.)



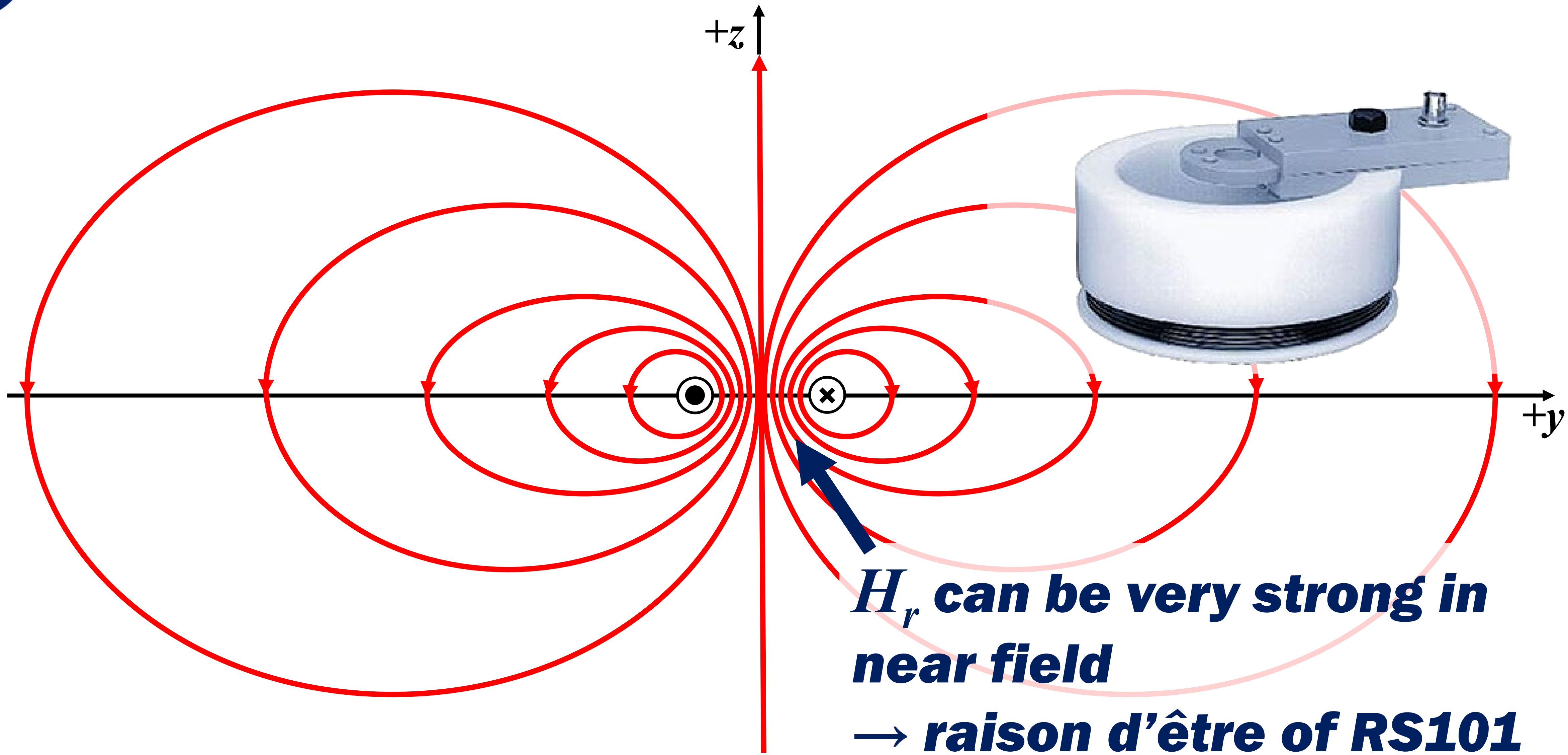
Magnetic Dipole: Magnetic Field Lines



Magnetic Dipole: Magnetic Field Lines (cont.)



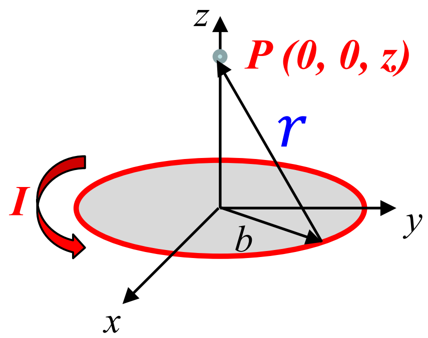
Magnetic Dipole: Magnetic Field Lines (cont.)



H_r can be very strong in near field

→ raison d'être of RS101

Magnetic Field on Axis Near Large Loop



***N* = number of turns**
(RS101 coil)

$$\vec{H} = \vec{a}_z \frac{Ib^2N}{2(z^2+b^2)^{3/2}} \qquad \vec{m} = \vec{a}_z I\pi b^2$$

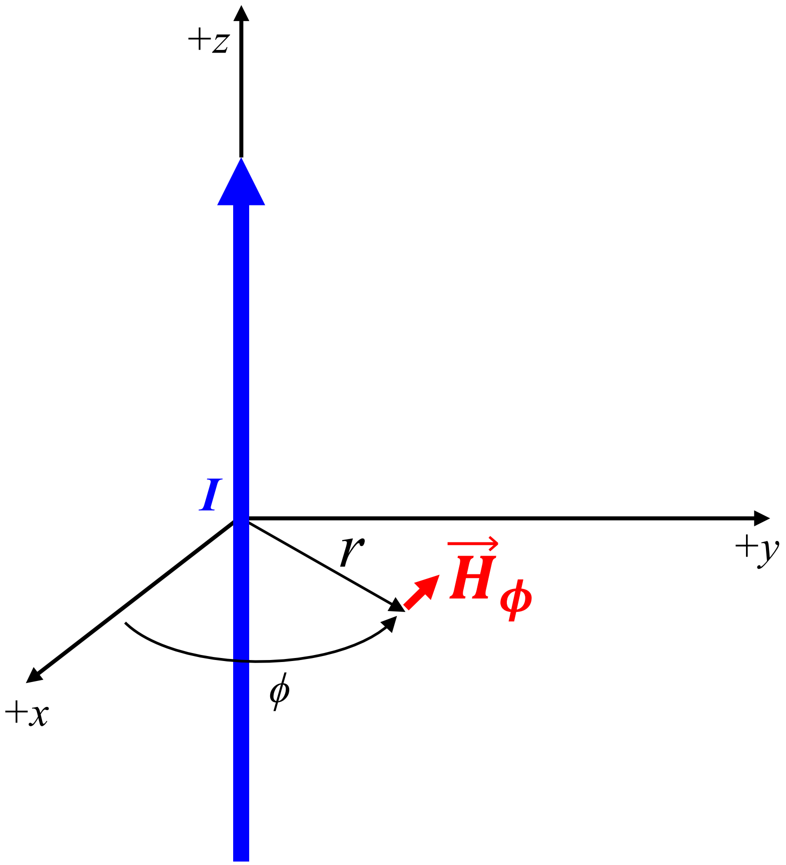
$$= \vec{a}_z \frac{mN}{2\pi(z^2+b^2)^{3/2}} \qquad r = \sqrt{z^2+b^2}$$

$$= \vec{a}_z \frac{mN}{2\pi r^3} \qquad z \gg b \ \rightarrow r \approx z$$

$$H_r \approx \frac{m}{2\pi r^3} \underline{\cos\theta}\, e^{-j\beta r}$$

Looks a bit like near-field radial term when $\theta = 0$

Magnetic Field Around Long Current-Carrying Wire



From Ampère's Law:

$$\oint \vec{H} \cdot d\vec{l} = I$$

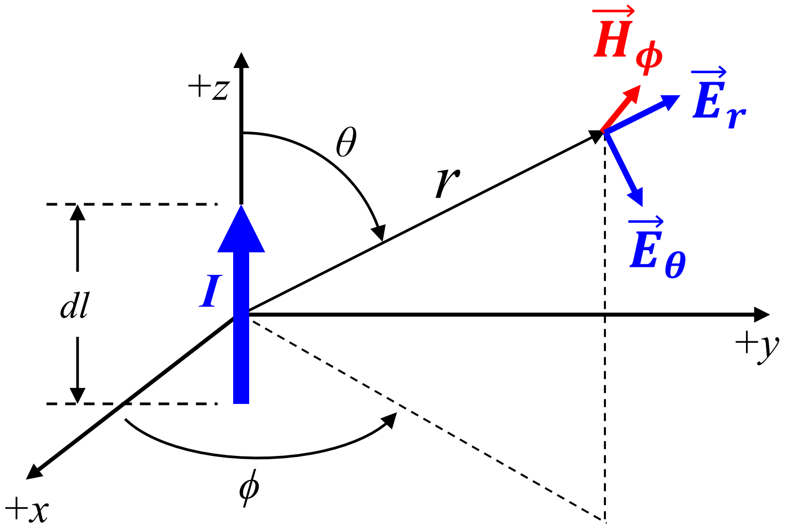
$$H_\phi = \frac{I}{2\pi r}$$

VERY accurate

Current probes employ this very principle

Handy for inductive crosstalk analysis
(much more so than attempts at near-field analysis)

Elemental Electric (Hertzian) Dipole: Wave Impedance



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity:

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = -\frac{Idl}{4\pi}\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right] e^{-j\beta r}$$

Electric field intensity:

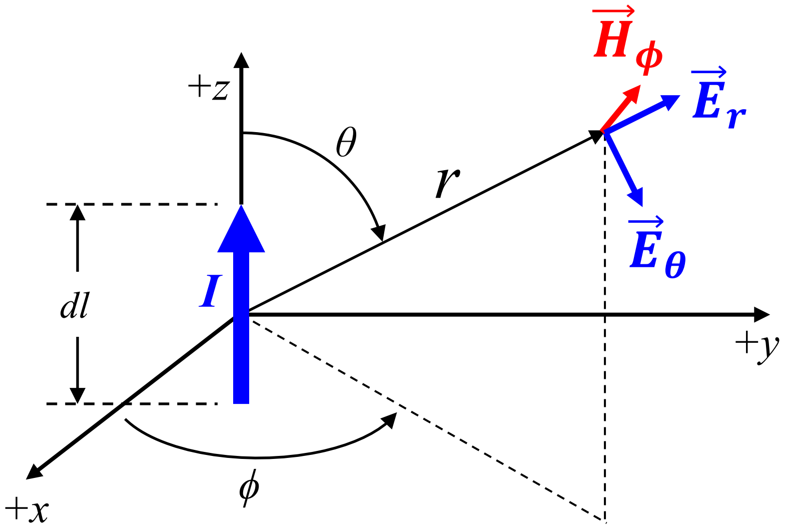
$$E_r = -2\frac{Idl}{4\pi}\eta\beta^2 \cos\theta \left[\frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\theta = -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\phi = 0$$

H_ϕ and E_θ dominate for $\theta = \pi/2$

Elemental Electric (Hertzian) Dipole: Wave Impedance (cont.)



$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity:

$$H_\phi = -\frac{Idl}{4\pi}\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right] e^{-j\beta r}$$

$$H_\phi = -\frac{Idl}{4\pi}\beta^2 \sin\theta \left[-\frac{1}{(\beta r)^2} - j\frac{1}{\beta r}\right] e^{-j\beta r}$$

Electric field intensity:

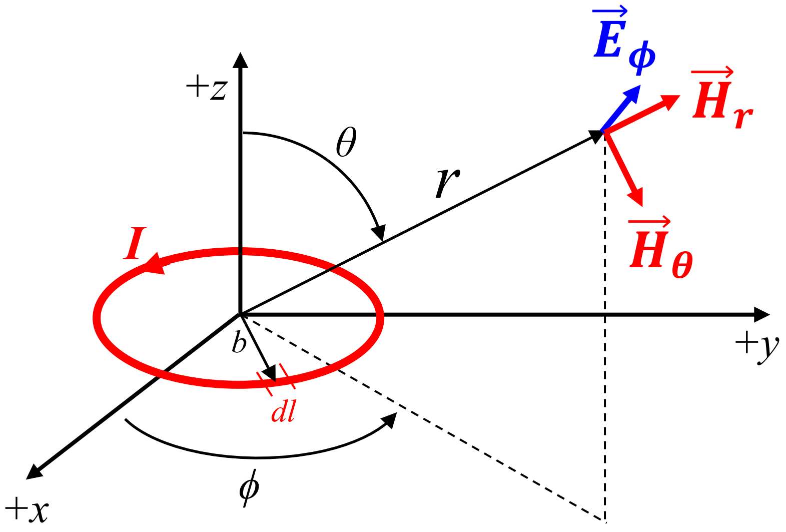
$$E_\theta = -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

$$E_\theta = -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[-\frac{1}{(\beta r)^2} + j\left(\frac{1}{(\beta r)^3} - \frac{1}{\beta r}\right)\right] e^{-j\beta r}$$

$$|Z_E| = \left|\frac{E_\theta}{H_\phi}\right| = \eta \frac{\sqrt{\left(\frac{1}{\beta r}\right)^4 + \left[\frac{1}{(\beta r)^3} - \frac{1}{\beta r}\right]^2}}{\sqrt{\left(\frac{1}{\beta r}\right)^4 + \left(\frac{1}{\beta r}\right)^2}}$$

$\approx \eta$ for $\beta r >> 1$

Elemental Magnetic Dipole: Wave Impedance



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I \pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity:

$$H_r = -j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\theta = -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\phi = 0$$

Electric field intensity:

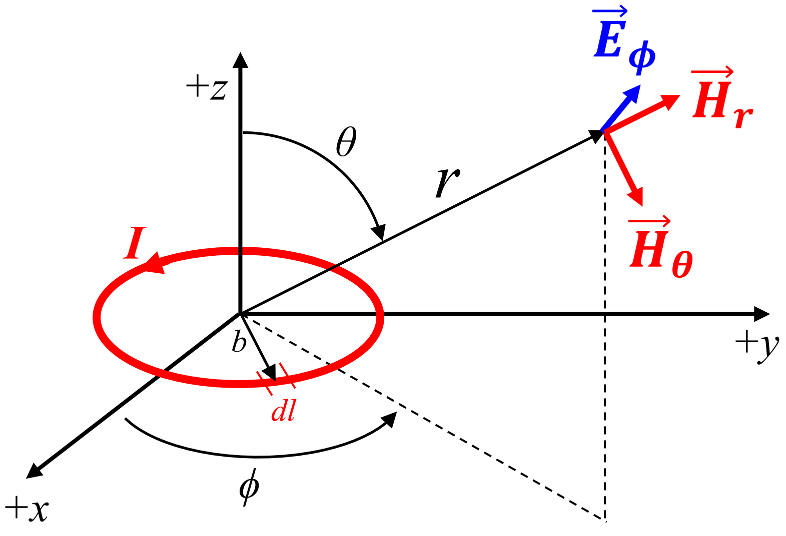
$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi = j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

H_θ and E_ϕ dominate for $\theta = \pi/2$

Elemental Magnetic Dipole: Wave Impedance (cont.)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I\pi b^2$$

$$\beta = \frac{2\pi}{\lambda}$$

Magnetic field intensity:

$$H_\theta = -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\theta = -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[-\frac{1}{(\beta r)^2}+j\left(\frac{1}{(\beta r)^3}-\frac{1}{\beta r}\right)\right]e^{-j\beta r}$$

Electric field intensity:

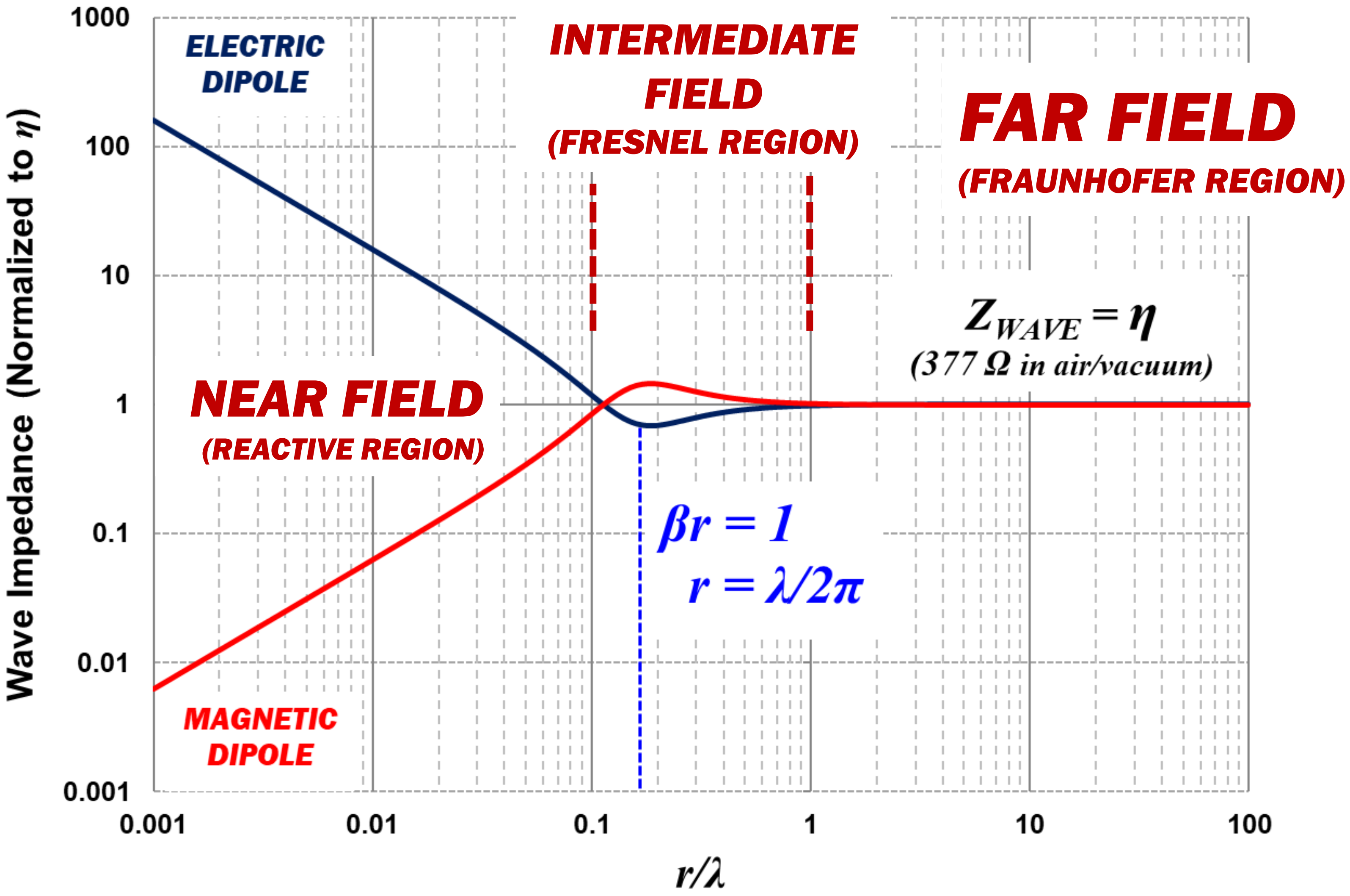
$$E_\phi = j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

$$E_\phi = j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[-\frac{1}{(\beta r)^2}-j\frac{1}{\beta r}\right]e^{-j\beta r}$$

$$|Z_M| = \left|\frac{E_\phi}{H_\theta}\right| = \eta\frac{\sqrt{\left(\frac{1}{\beta r}\right)^4+\left(\frac{1}{\beta r}\right)^2}}{\sqrt{\left(\frac{1}{\beta r}\right)^4+\left[\frac{1}{(\beta r)^3}-\frac{1}{\beta r}\right]^2}}$$

$\approx \eta$ for $\beta r >> 1$

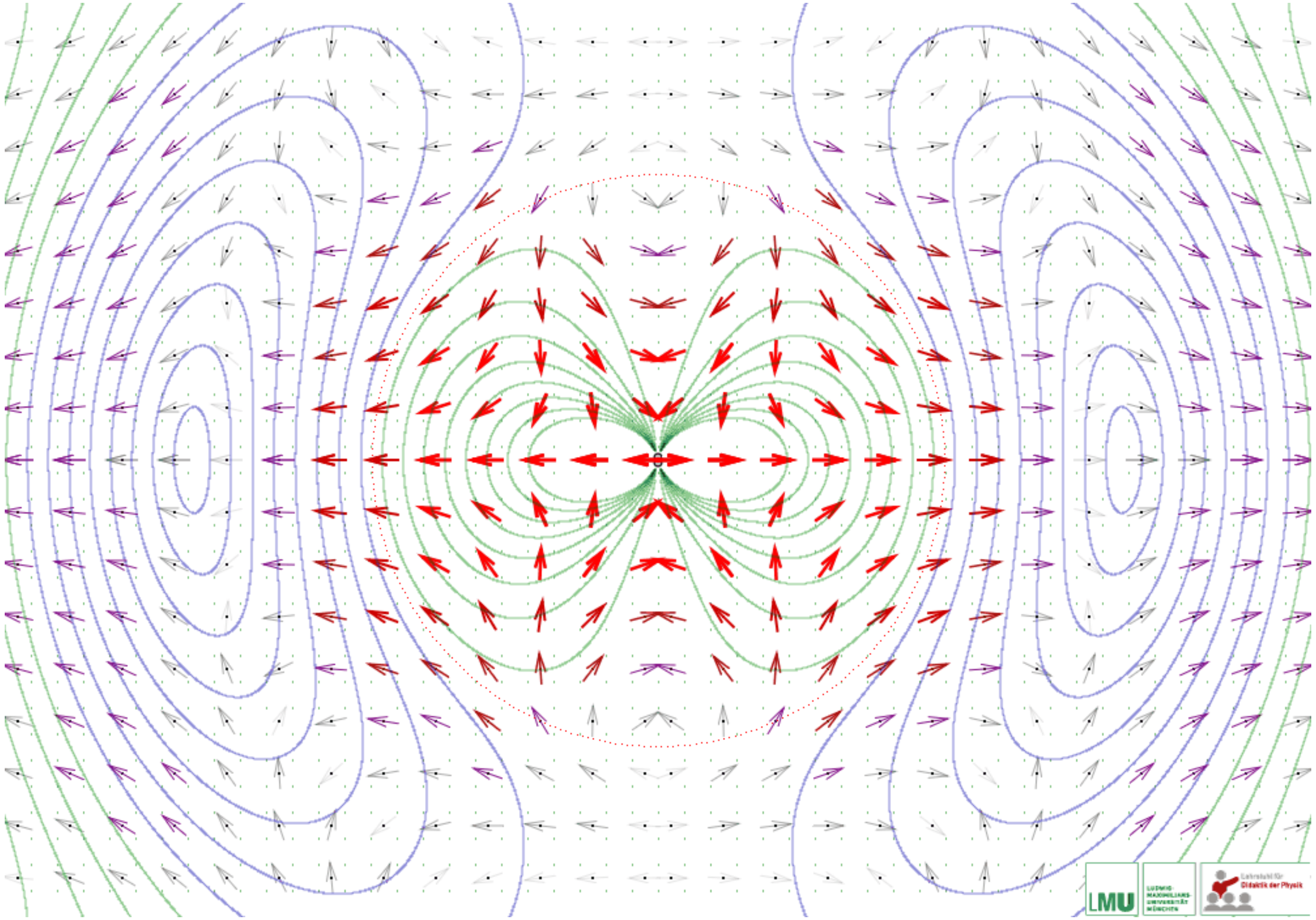
Wave Impedances of Elemental Dipoles (cont.)



Hertzian Dipole: Energy Flow Animation

https://www.didaktik.physik.uni-muenchen.de/_assets/bilder/Multimedia/bilder_dipol/web_bilder_orig/dip_1h___o.gif

Courtesy Ludwig Maximilians-Universität Mũnchen

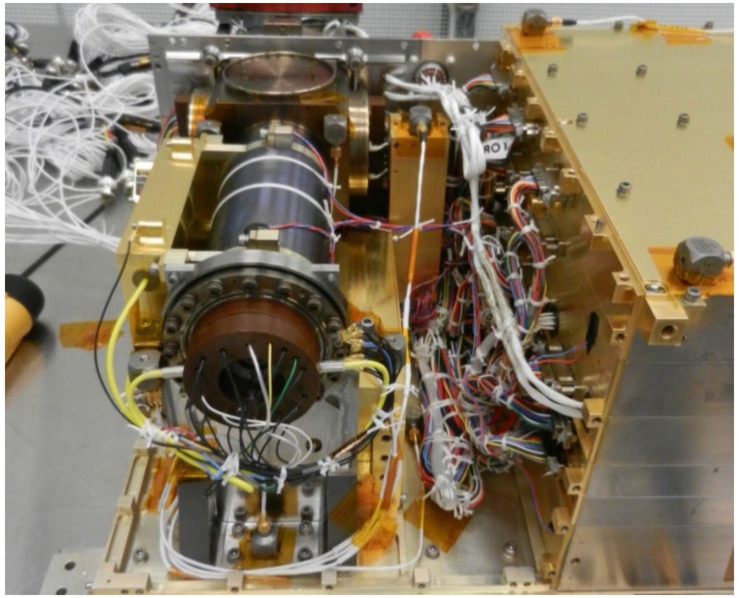


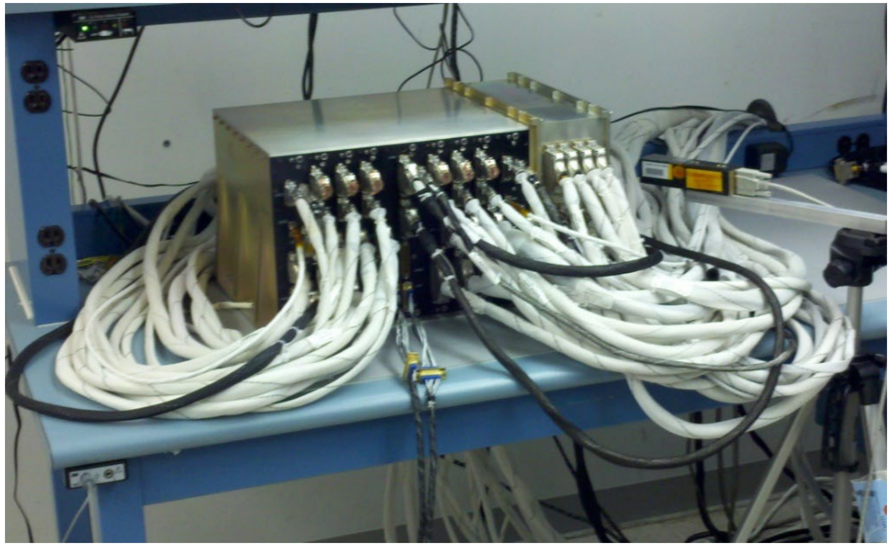
Far Field
Energy propagates away from dipole (propagating wave)

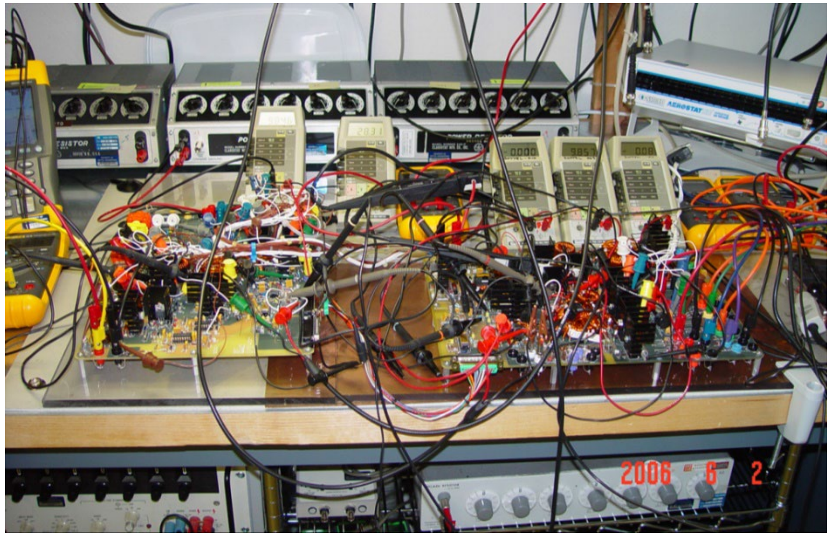
Fields no longer react immediately to stimulus (reflect earlier state of stimulus)

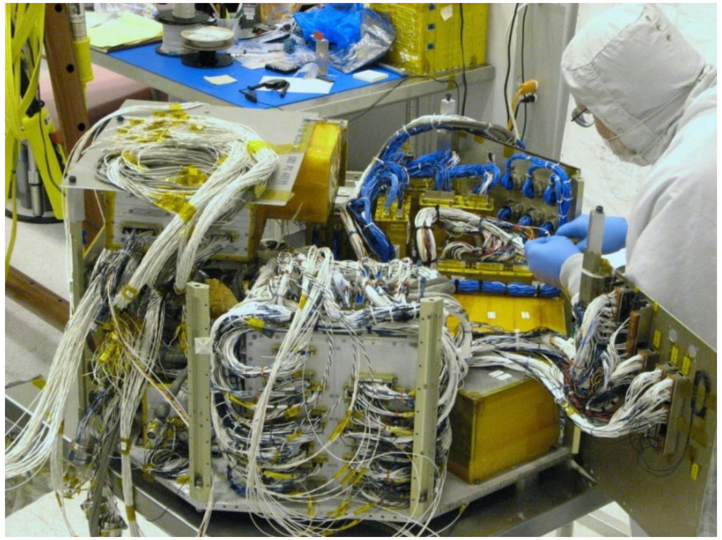
Near Field (Reactive Region)
Fields react immediately to stimulus signal
Energy circulates near dipole (standing waves)

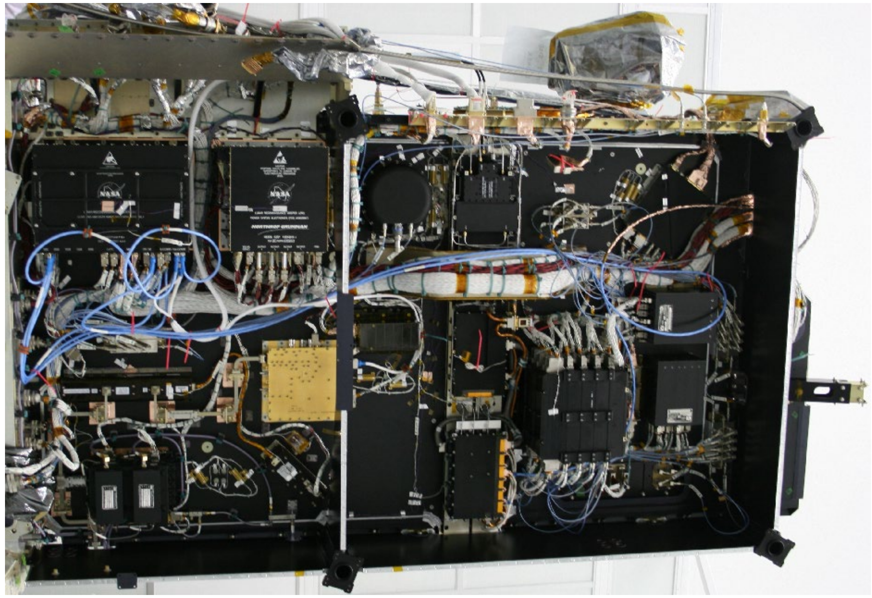
Can You Find the Dipoles in These Pictures?

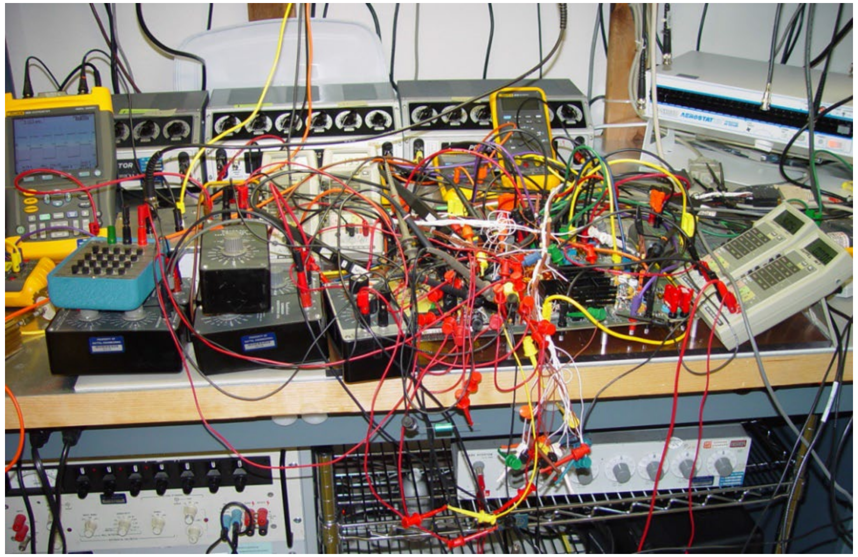






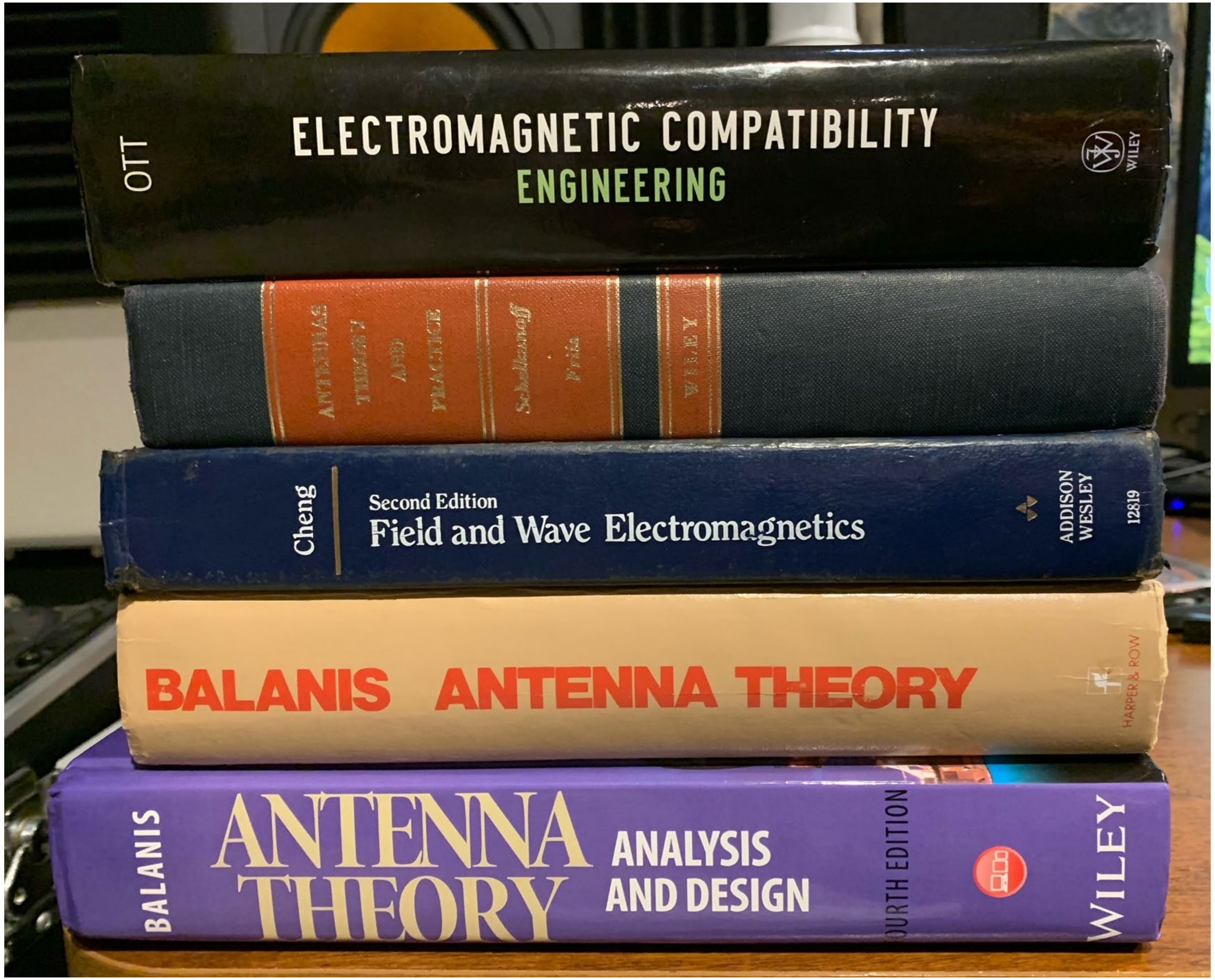






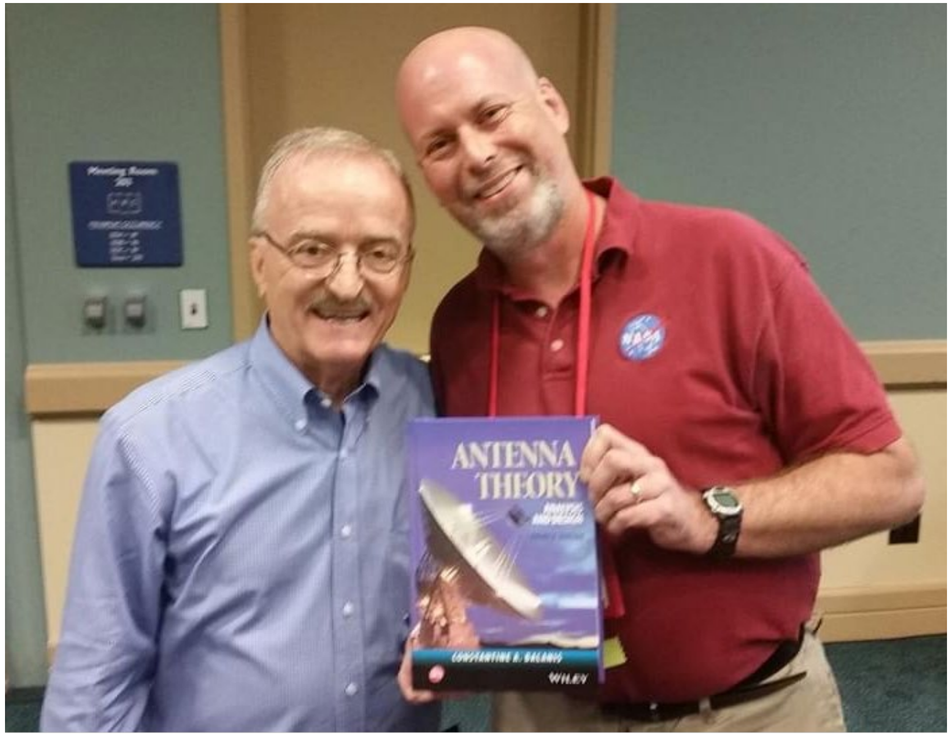
Summary

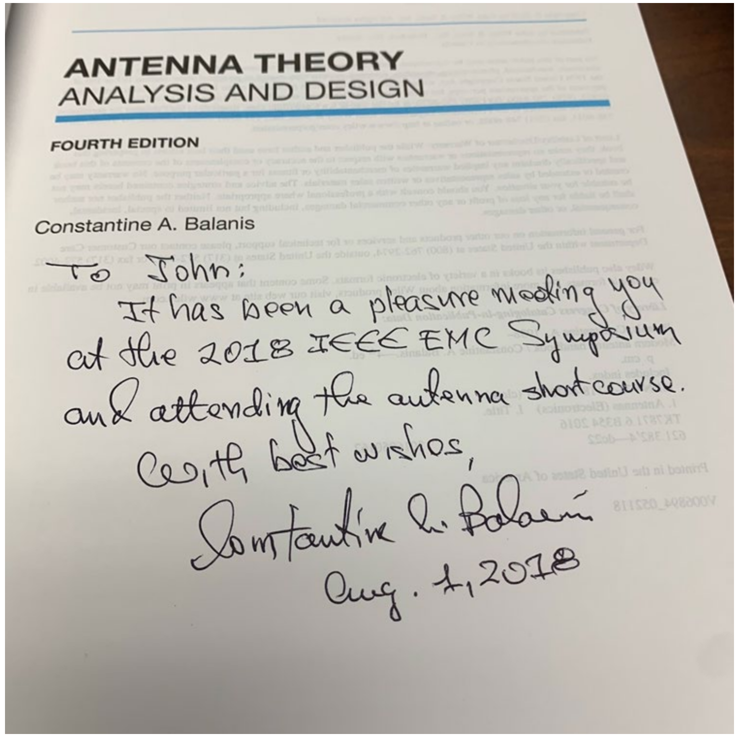
- **That's it for Part 1**
- **In Part 2, we will delve into:**
  - Linear Dipole (finite length)
  - General antenna parameters of interest
- **Recommendations for further reading provided on following slide**



References

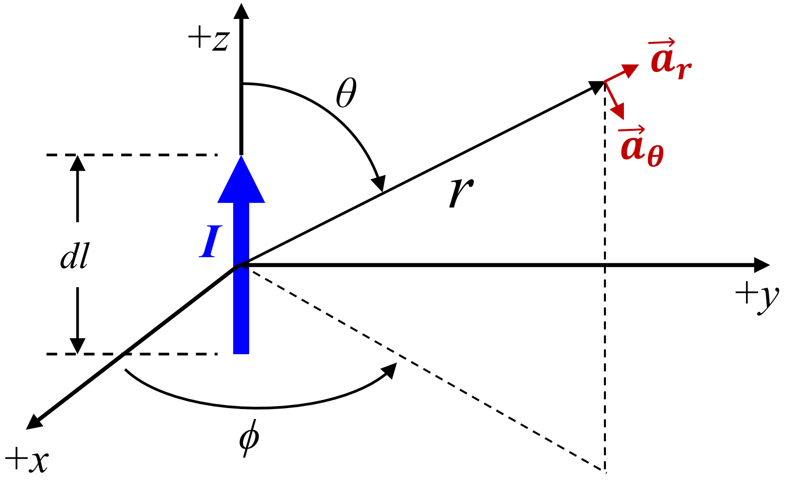
- Balanis, Constantine: Antenna Theory - Analysis and Design
- Cheng, David: Field and Wave Electromagnetics
- Kraus: Antennas
- Ott, Henry: Electromagnetic Compatibility Engineering (Appendix D: “Dipoles for Dummies”)
- Schelkunoff, Sergei, and Friis, Harald: Antennas, Theory and Practice





Dipole Antenna Basics
BACKUP

Elemental Electric (Hertzian) Dipole



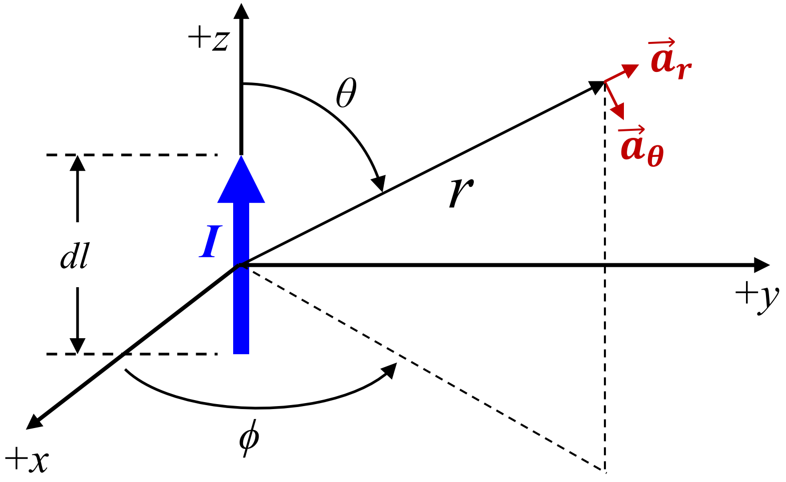
Vector magnetic ("retarded") potential:

$$\vec{A} = \frac{\mu}{4\pi}\int_{V'}\frac{\vec{J}e^{-j\beta r}}{r}dv'$$

$$\vec{H} = \frac{1}{\mu}(\vec{\nabla} \times \vec{A})$$

$$\vec{E} = \frac{1}{j\omega\varepsilon}(\vec{\nabla} \times \vec{H})$$

Elemental Electric (Hertzian) Dipole (cont.)



Vector magnetic ("retarded") potential:

$$\vec{A} = \frac{\mu}{4\pi}\int_{V'} \frac{\vec{J}e^{-j\beta r}}{r}\,dv'$$

$$\vec{A} = \vec{a}_z \frac{\mu I dl}{4\pi}\left(\frac{e^{-j\beta r}}{r}\right) \qquad r >> dl$$

$$\vec{a}_z = \vec{a}_R \cos\theta - \vec{a}_\theta \sin\theta$$

$$A_r = A_z \cos\theta = \frac{\mu I dl}{4\pi r}\cos\theta\, e^{-j\beta r}$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I dl}{4\pi r}\sin\theta\, e^{-j\beta r}$$

$$A_\phi = 0$$

Elemental Electric (Hertzian) Dipole (cont.)

$$\vec{\boldsymbol{H}} = \frac{1}{\mu}(\vec{\nabla} \times \vec{\boldsymbol{A}}) \longrightarrow \vec{\nabla} \times \vec{\boldsymbol{A}} = \frac{1}{r^2 \sin\theta} \begin{bmatrix} \vec{\boldsymbol{a}}_r & \vec{\boldsymbol{a}}_\theta r & \vec{\boldsymbol{a}}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ r & rA_\theta & r\sin\theta\, A_\phi \end{bmatrix}$$

Curl in spherical coordinates

$$= \frac{1}{r^2 \sin\theta}\left[\vec{\boldsymbol{a}}_r \underbrace{\left(\frac{\partial}{\partial \theta} r\sin\theta\, A_\phi - \frac{\partial}{\partial \phi} rA_\theta\right)}_{= 0} + \vec{\boldsymbol{a}}_\theta r \underbrace{\left(\frac{\partial}{\partial \phi} A_r - \frac{\partial}{\partial r} r\sin\theta\, A_\phi\right)}_{= 0} + \vec{\boldsymbol{a}}_\phi r\sin\theta\left(\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial \theta} A_r\right)\right]$$

$$A_r = A_z \cos\theta = \frac{\mu I dl}{4\pi r}\cos\theta\, e^{-j\beta r}$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I dl}{4\pi r}\sin\theta\, e^{-j\beta r}$$

$$A_\phi = 0$$

$$\vec{\nabla} \times \vec{\boldsymbol{A}} = \frac{1}{r^2 \sin\theta}\left[\vec{\boldsymbol{a}}_\phi r\sin\theta\left(\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial \theta} A_r\right)\right]$$

$$= \vec{\boldsymbol{a}}_\phi \frac{1}{r}\left(\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial \theta} A_r\right)$$

$$= \vec{\boldsymbol{a}}_\phi \frac{1}{r}\left[-\frac{\mu I dl}{4\pi}\sin\theta\,(-j\beta)e^{-j\beta r} + \frac{\mu I dl}{4\pi r}\sin\theta\, e^{-j\beta r}\right]$$

$$= \vec{\boldsymbol{a}}_\phi \mu\beta^2 \sin\theta\left[-\frac{I dl}{4\pi}\left(\frac{1}{j\beta r}\right)e^{-j\beta r} - \frac{I dl}{4\pi}\left(\frac{1}{(j\beta r)^2}\right)e^{-j\beta r}\right]$$

$$\vec{\nabla} \times \vec{\boldsymbol{A}} = -\vec{\boldsymbol{a}}_\phi \mu \frac{I dl}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

$$\underline{\vec{\boldsymbol{H}} = \frac{1}{\mu}(\vec{\nabla} \times \vec{\boldsymbol{A}}) = -\vec{\boldsymbol{a}}_\phi \frac{I dl}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}}$$

Elemental Electric (Hertzian) Dipole (cont.)

$$\vec{E} = \frac{1}{j\omega\varepsilon}(\vec{\nabla} \times \vec{H}) \longrightarrow \vec{\nabla} \times \vec{H} = \frac{1}{r^2 \sin\theta}\begin{bmatrix} \vec{a}_r & \vec{a}_\theta r & \vec{a}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\, H_\phi \end{bmatrix}$$

Curl in spherical coordinates

$$= \frac{1}{r^2\sin\theta}\left[\vec{a}_r\left(\frac{\partial}{\partial\theta} r\sin\theta\, H_\phi - \underbrace{\frac{\partial}{\partial\phi} rH_\theta}_{=0}\right) + \vec{a}_\theta r\left(\underbrace{\frac{\partial}{\partial\phi} H_r}_{=0} - \frac{\partial}{\partial r} r\sin\theta\, H_\phi\right) + \vec{a}_\phi r\sin\theta\left(\underbrace{\frac{\partial}{\partial r} rH_\theta - \frac{\partial}{\partial\theta} H_r}_{=0}\right)\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\vec{a}_r\left(\frac{\partial}{\partial\theta} r\sin\theta\, H_\phi\right) + \vec{a}_\theta r\left(-\frac{\partial}{\partial r} r\sin\theta\, H_\phi\right)\right]$$

$$\vec{\nabla} \times \vec{H} = \vec{a}_r \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(H_\phi \sin\theta\right) - \vec{a}_\theta \frac{1}{r}\frac{\partial}{\partial r}\left(rH_\phi\right)$$

$$\vec{H} = -\vec{a}_\phi \frac{Idl}{4\pi}\beta^2 \sin\theta\left(\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right)e^{-j\beta r}$$

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \omega = \frac{\beta}{\sqrt{\mu\varepsilon}} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\omega\varepsilon = \frac{\beta\varepsilon}{\sqrt{\mu\varepsilon}} = \frac{\beta}{\eta}$$

$$H_\phi \sin\theta = \frac{Idl}{4\pi}\beta^2 \sin^2\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

$$\frac{\partial}{\partial\theta}\left(H_\phi \sin\theta\right) = \frac{Idl}{4\pi}\beta^2(2\sin\theta\cos\theta)\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

$$E_r = -\frac{1}{j\omega\varepsilon}\cdot\frac{1}{r\sin\theta}\cdot\frac{Idl}{4\pi}\beta^2(2\sin\theta\cos\theta)\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

$$E_r = -\frac{\eta}{j\beta}\frac{Idl}{4\pi r}\beta^2(2\cos\theta)\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

$$E_r = -2\frac{Idl}{4\pi}\eta\beta^2\cos\theta\left[\frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

Elemental Electric (Hertzian) Dipole (cont.)

$$\vec{\boldsymbol{E}} = \frac{1}{j\omega\varepsilon}(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{H}}) \qquad \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{H}} = \vec{\boldsymbol{a}}_{\boldsymbol{r}} \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(H_\phi \sin\theta) - \vec{\boldsymbol{a}}_{\boldsymbol{\theta}} \frac{1}{r}\frac{\partial}{\partial r}(rH_\phi)$$

$$\vec{\boldsymbol{H}} = -\vec{\boldsymbol{a}}_{\boldsymbol{\phi}} \frac{Idl}{4\pi}\beta^2 \sin\theta \left(\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right) e^{-j\beta r}$$

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \omega = \frac{\beta}{\sqrt{\mu\varepsilon}} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\omega\varepsilon = \frac{\beta\varepsilon}{\sqrt{\mu\varepsilon}} = \frac{\beta}{\eta}$$

$$E_\theta = \frac{1}{j\omega\varepsilon}\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{Idl}{4\pi} r\beta^2 \sin\theta \left(\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right) e^{-j\beta r}\right]$$

$$= \frac{\eta}{j\beta}\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{Idl}{4\pi} r\beta^2 \sin\theta \left(\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right) e^{-j\beta r}\right]$$

$$= \frac{Idl}{4\pi}\eta\beta^2 \sin\theta \frac{1}{j\beta r}\frac{\partial}{\partial r}\left[r\left(\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right) e^{-j\beta r}\right]$$

$$= \frac{Idl}{4\pi}\eta\beta^2 \sin\theta \frac{1}{j\beta r}\frac{\partial}{\partial r}\left[\left(\frac{1}{j\beta} + \frac{1}{(j\beta)^2 r}\right) e^{-j\beta r}\right]$$

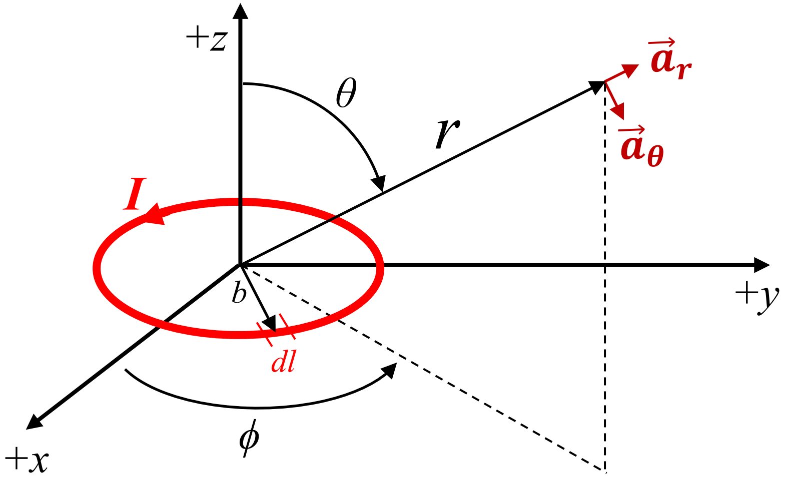
$$= \frac{Idl}{4\pi}\eta\beta^2 \sin\theta \frac{1}{j\beta r}\frac{\partial}{\partial r}\left[\left(\frac{1}{j\beta} + \frac{1}{(j\beta)^2 r}\right) e^{-j\beta r}\right]$$

$$= \frac{Idl}{4\pi}\eta\beta^2 \sin\theta \frac{1}{j\beta r}\frac{\partial}{\partial r}\left[\left(\frac{1}{j\beta} e^{-j\beta r} + \frac{1}{(j\beta)^2 r} e^{-j\beta r}\right)\right]$$

$$= \frac{Idl}{4\pi}\eta\beta^2 \sin\theta \frac{1}{j\beta r}\left[-e^{-j\beta r} - \frac{1}{(j\beta r)^2} e^{-j\beta r} - \frac{1}{j\beta r} e^{-j\beta r}\right]$$

$$E_\theta = -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right] e^{-j\beta r}$$

Elemental Magnetic Dipole



Vector magnetic moment:

$$\vec{\boldsymbol{m}} = \vec{\boldsymbol{a}}_z I\pi b^2$$

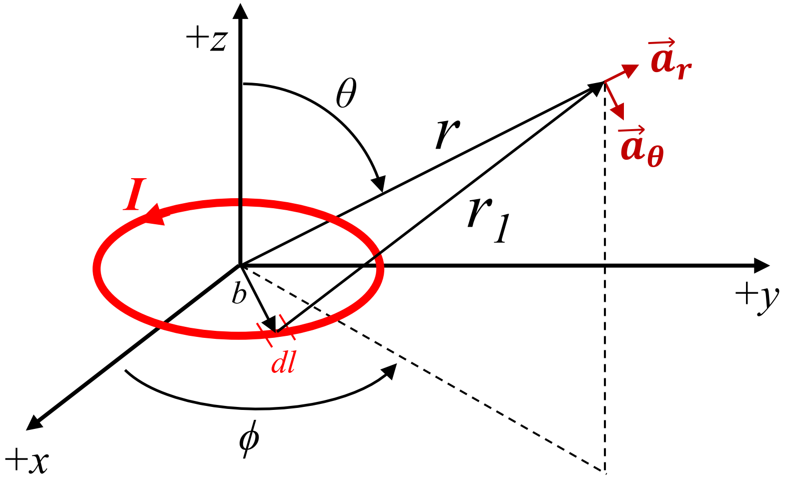
Vector magnetic ("retarded") potential:

$$\vec{\boldsymbol{A}} = \frac{\mu}{4\pi}\int_{V'}\frac{\vec{\boldsymbol{J}}e^{-j\beta r}}{r}dv'$$

$$\vec{\boldsymbol{H}} = \frac{1}{\mu}(\vec{\boldsymbol{\nabla}}\times\vec{\boldsymbol{A}})$$

$$\vec{\boldsymbol{E}} = \frac{1}{j\omega\varepsilon}(\vec{\boldsymbol{\nabla}}\times\vec{\boldsymbol{H}})$$

Elemental Magnetic Dipole (cont.)



Vector magnetic moment:

$$\vec{m} = \vec{a}_z I\pi b^2$$

Vector magnetic ("retarded") potential:

$$\vec{A} = \frac{\mu}{4\pi}\int_{V'}\frac{\vec{J}e^{-j\beta r}}{r}dv'$$

$$\vec{A} = \frac{\mu I}{4\pi}\oint\frac{e^{-j\beta r_1}}{r_1}dl$$

$r_1 >> b$

$$e^{-j\beta r_1} = e^{-j\beta r}e^{-j\beta(r_1 - r)}$$

$$e^{-j\beta r_1} \approx e^{-j\beta r}[1 - j\beta(r_1 - r)]$$

From Taylor series expansion

$$\vec{A} = \frac{\mu I}{4\pi}e^{-j\beta r}\left[(1 + j\beta r)\oint\frac{dl}{r_1} - j\beta\oint dl\right]$$

$$\vec{A} = \vec{a}_\phi\frac{\mu m}{4\pi r^2}\sin\theta$$

Vector magnetic potential of static dipole

$$\vec{A} = \vec{a}_\phi\frac{\mu m}{4\pi r^2}(1 + j\beta r)e^{-j\beta r}\sin\theta$$

Elemental Magnetic Dipole (cont.)

$$\vec{\boldsymbol{H}} = \frac{1}{\mu}(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{A}}) \longrightarrow \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{A}} = \frac{1}{r^2 \sin\theta}\begin{bmatrix} \vec{\boldsymbol{a}}_{\boldsymbol{r}} & \vec{\boldsymbol{a}}_{\boldsymbol{\theta}} r & \vec{\boldsymbol{a}}_{\boldsymbol{\phi}} r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ r & rA_\theta & r\sin\theta\, A_\phi \end{bmatrix}$$

Curl in spherical coordinates

$$= \frac{1}{r^2\sin\theta}\left[\vec{\boldsymbol{a}}_{\boldsymbol{r}}\left(\frac{\partial}{\partial\theta} r\sin\theta\, A_\phi - \underbrace{\frac{\partial}{\partial\phi} rA_\theta}_{=0}\right) + \vec{\boldsymbol{a}}_{\boldsymbol{\theta}} r\left(\underbrace{\frac{\partial}{\partial\phi}A_r}_{=0} - \frac{\partial}{\partial r} r\sin\theta\, A_\phi\right) + \vec{\boldsymbol{a}}_{\boldsymbol{\phi}} r\sin\theta\left(\underbrace{\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial\theta}A_r}_{=0}\right)\right]$$

$$A_r = 0$$

$$A_\theta = 0$$

$$A_\phi = \frac{\mu m}{4\pi r^2}(1 + j\beta r)e^{-j\beta r}\sin\theta$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{A}} = \frac{1}{r^2\sin\theta}\left[\vec{\boldsymbol{a}}_{\boldsymbol{r}}\left(\frac{\partial}{\partial\theta} r\sin\theta\, A_\phi\right) - \vec{\boldsymbol{a}}_{\boldsymbol{\theta}} r\left(\frac{\partial}{\partial r} r\sin\theta\, A_\phi\right)\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\vec{\boldsymbol{a}}_{\boldsymbol{r}}\frac{\partial}{\partial\theta}\left(\frac{\mu m}{4\pi r}(1 + j\beta r)e^{-j\beta r}\sin^2\theta\right) - \vec{\boldsymbol{a}}_{\boldsymbol{\theta}} r\frac{\partial}{\partial r}\left(\frac{\mu m}{4\pi r}(1 + j\beta r)e^{-j\beta r}\sin^2\theta\right)\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\vec{\boldsymbol{a}}_{\boldsymbol{r}}\left(\frac{\mu m}{4\pi r}(1 + j\beta r)e^{-j\beta r}(2\sin\theta\cos\theta)\right) - \vec{\boldsymbol{a}}_{\boldsymbol{\theta}} r\frac{\partial}{\partial r}\left(\frac{\mu m}{4\pi r}(1 + j\beta r)e^{-j\beta r}\sin^2\theta\right)\right]$$

$$H_r = \frac{1}{r^2\sin\theta}\cdot\frac{m}{4\pi r}(1 + j\beta r)e^{-j\beta r}(2\sin\theta\cos\theta)$$

$$= \frac{m}{4\pi r^3}(1 + j\beta r)e^{-j\beta r}(2\cos\theta)$$

$$= -j\frac{\beta^3 m}{4\pi}(2\cos\theta)\left[\frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_r = -j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$\beta = \omega\sqrt{\mu\varepsilon} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\cdot\frac{\sqrt{\mu/\varepsilon}}{\eta} = \frac{\omega\mu}{\eta}$$

$$H_\theta = \frac{1}{r^2\sin\theta}(-r\sin^2\theta)\frac{\mu m}{4\pi}\frac{\partial}{\partial r}\left[\frac{1}{r}e^{-j\beta r} + j\beta e^{-j\beta r}\right]$$

$$= -\frac{\mu m}{4\pi r}\sin\theta\left[-\frac{1}{r^2}e^{-j\beta r} - \frac{1}{r}j\beta e^{-j\beta r} - (j\beta)^2 e^{-j\beta r}\right]$$

$$= \frac{\mu m}{4\pi}\sin\theta\left[\frac{1}{r^3} + \frac{1}{r^2}(j\beta) + \frac{1}{r}(j\beta)^2\right]e^{-j\beta r}$$

$$= (j\beta)^3\frac{\mu m}{4\pi}\sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\theta = -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

Elemental Magnetic Dipole (cont.)

$$\vec{E} = \frac{1}{j\omega\varepsilon}(\vec{\nabla}\times\vec{H}) \longrightarrow \vec{\nabla}\times\vec{H} = \frac{1}{r^2\sin\theta}\begin{bmatrix} \vec{a}_r & \vec{a}_\theta r & \vec{a}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ H_r & rH_\theta & r\sin\theta\, H_\phi \end{bmatrix}$$

Curl in spherical coordinates

$$= \frac{1}{r^2\sin\theta}\left[\vec{a}_r\underbrace{\left(\frac{\partial}{\partial\theta}r\sin\theta\, H_\phi - \frac{\partial}{\partial\phi}rH_\theta\right)}_{=0} + \vec{a}_\theta r\left(\underbrace{\frac{\partial}{\partial\phi}H_r - \frac{\partial}{\partial r}r\sin\theta\, H_\phi}_{=0}\right) + \vec{a}_\phi r\sin\theta\left(\frac{\partial}{\partial r}rH_\theta - \frac{\partial}{\partial\theta}H_r\right)\right]$$

$$H_r = -j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\theta = -j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$H_\phi = 0$$

$$E_\phi = \frac{1}{j\omega\varepsilon}\frac{1}{r}\frac{\partial}{\partial r}\left\{-j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\left[\frac{1}{j\beta}+\frac{1}{(j\beta)^2 r}+\frac{1}{(j\beta)^3 r^2}\right]e^{-j\beta r}\right\} - \frac{1}{j\omega\varepsilon}\frac{1}{r}\frac{\partial}{\partial\theta}\left\{-j\frac{\omega\mu m}{4\pi\eta}\beta^2(2\cos\theta)\left[\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r}\right\}$$

$$= \frac{1}{j\omega\varepsilon}\frac{1}{r}\left(-j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\right)\left\{\left[-\frac{1}{(j\beta r)^2}-\frac{2}{(j\beta r)^3}\right]e^{-j\beta r} - \left[1+\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}\right]e^{-j\beta r}\right\} + \frac{1}{j\omega\varepsilon}\frac{1}{r}\left[j\frac{\omega\mu m}{4\pi\eta}\beta^2(-2\sin\theta)\left[\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r}\right]$$

$$= \frac{1}{j\omega\varepsilon}\frac{1}{r}\left(j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\, e^{-j\beta r}\right)\left[1+\frac{1}{j\beta r}+\frac{2}{(j\beta r)^2}+\frac{2}{(j\beta r)^3}\right] - \frac{1}{j\omega\varepsilon}\frac{1}{r}\left[j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\, e^{-j\beta r}\left[\frac{2}{(j\beta r)^2}+\frac{2}{(j\beta r)^3}\right]\right]$$

$$= \frac{1}{j\omega\varepsilon}\frac{1}{r}\left(j\frac{\omega\mu m}{4\pi\eta}\beta^2\sin\theta\, e^{-j\beta r}\right)\left[1+\frac{1}{j\beta r}+\frac{2}{(j\beta r)^2}+\frac{2}{(j\beta r)^3}-\frac{2}{(j\beta r)^2}-\frac{2}{(j\beta r)^3}\right]$$

$$\omega\varepsilon = \frac{\beta\varepsilon}{\sqrt{\mu\varepsilon}} = \frac{\beta}{\eta} \longrightarrow = \frac{1}{j\beta r}\left(j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\, e^{-j\beta r}\right)\left[1+\frac{1}{j\beta r}\right]$$

$$\underline{E_\phi = j\frac{\omega\mu m}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}\right]e^{-j\beta r}}$$