Fundamentals of Electromagnetics

Test and Measurement Overview

John McCloskey
Chief EMC Engineer
NASA/Goddard Space Flight Center
Greenbelt, MD
John.C.McCloskey@nasa.gov

Topics

- Measurements: Direct and Indirect
- Accuracy and Precision
- Why do EMC folks speak in dB (decibels)?
- 50 Ω System
- Transducers
- Linearity and Dynamic Range
- Thermal Noise

Measurement: Direct and Indirect

Direct measurement

Indirect measurement

Whether direct or indirect, any measurement setup must be <u>engineered</u> to provide a valid result...

Accuracy and Precision

- **Accuracy**
  - How closely a measurement agrees with its "true value" traced to some absolute reference (e.g. NIST)
- **Precision**
  - How closely a given set of measurements agree with each other
  - Repeatability

Why do EMC folks speak in dB (decibels)?

- It’s all about dynamic range; measurements can span many orders of magnitude
- Our brains have trouble comparing numbers that are very, very large or very, very small

1,000,000	*0.000001*
1,000,000,000	*0.000000001*
1,000,000,000,000	*0.000000000001*
1,000,000,000,000,000	*0.000000000000001*
1,000,000,000,000,000,000	*0.000000000000000001*
etc.	*etc.*

A whole bunch of 0s…

Expressing numbers on a logarithmic or dB scale can show a span of many orders of magnitude in a form that is more manageable for our brains to process

Why do EMC folks speak in dB (cont.)?

- **A number in dB always expresses a ratio of two power quantities:**

$$dB = 10\log_{10}\left(\frac{P_1}{P_2}\right)$$

$$P \propto V^2 \longrightarrow \left(\frac{V_1}{V_2}\right)_{dB} = 10\log_{10}\left(\frac{V_1}{V_2}\right)^2 = 20\log_{10}\left(\frac{V_1}{V_2}\right)$$

$$P \propto I^2 \longrightarrow \left(\frac{I_1}{I_2}\right)_{dB} = 10\log_{10}\left(\frac{I_1}{I_2}\right)^2 = 20\log_{10}\left(\frac{I_1}{I_2}\right)$$

Referenced to 1 µV: $dB\mu V = 20\log_{10}\left(\frac{V}{1\ \mu V}\right)$

0 dBµV = 1 µV
60 dBµV = 1 mV
120 dBµV = 1 V

Referenced to 1 µA: $dB\mu A = 20\log_{10}\left(\frac{I}{1\ \mu A}\right)$

0 dBµA = 1 µA
60 dBµA = 1 mA
120 dBµA = 1 A

Referenced to 1 mW: $dBm = 10\log_{10}\left(\frac{P}{1\ mW}\right)$

-30 dBm = 1 µW
0 dBm = 1 mW
30 dBm = 1 W

Properties of Logarithms

$$\log(AB) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

$$\log(A^B) = B \log A$$

$$\frac{1}{N}\sum_{n=1}^{N} \log A_n = \log(A_1 A_2 \dots A_N)^{\frac{1}{N}}$$

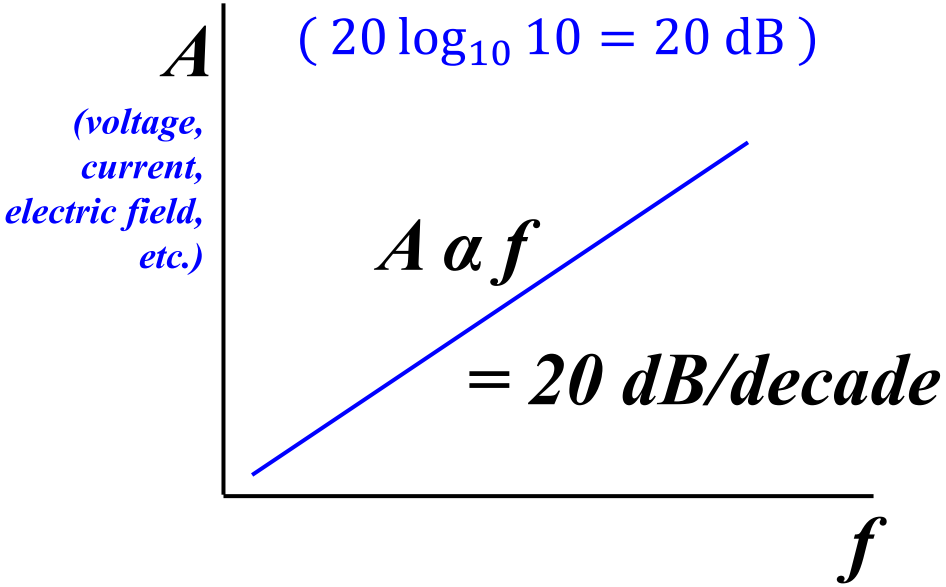
$$= \log\underbrace{\left(\sqrt[N]{A_1 A_2 \dots A_N}\right)}_{\text{Geometric mean of numeric values}}$$

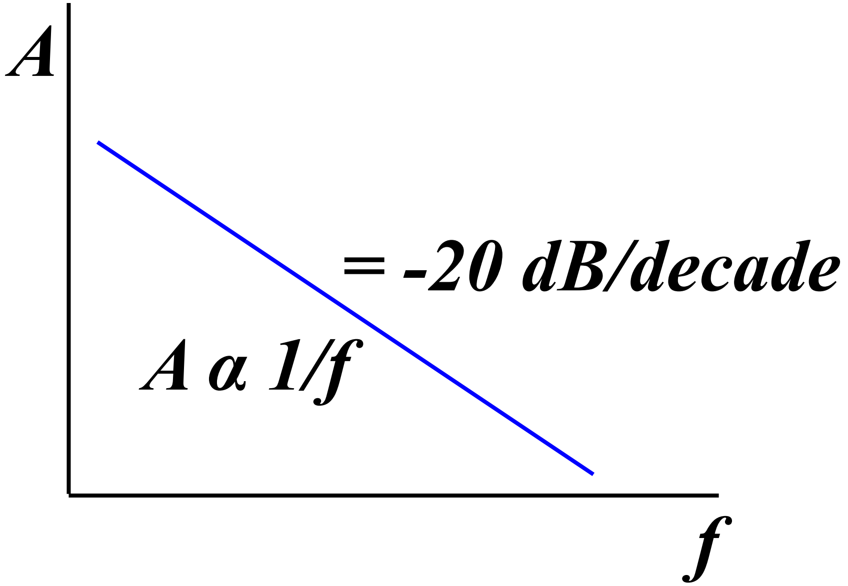
Arithmetic mean of logarithmic values

Some Handy dB Conversions

Power	**Voltage and Current**
$10\log_{10}10 = 10\ dB$	$20\log_{10}10 = 20\ dB$
$10\log_{10}2 \approx 3\ dB$	$20\log_{10}2 \approx 6\ dB$
$10\log_{10}3 \approx 5\ dB$	$20\log_{10}3 \approx 10\ dB$
$10\log_{10}5 = ?$	$20\log_{10}5 = ?$
$10\log_{10}\left(\frac{10}{2}\right) \approx 7\ dB$	$20\log_{10}\left(\frac{10}{2}\right) \approx 14\ dB$
$10\log_{10}10^n = 10n\ dB$	$20\log_{10}10^n = 20n\ dB$

Some Handy dB Conversions (cont.)





dBm to dBμV Conversion for 50 Ω System

$$P = \frac{V^2}{R}$$

For 50 Ω system: $P = \frac{V^2}{50\ \Omega}$

$$10\log_{10} P = 10\log_{10} V^2 - 10\log_{10}(50\ \Omega)$$

$$10\log_{10} P = 20\log_{10} V - \underline{10}\log_{10}(50\ \Omega)$$

(10·log in this context)

$1\ W = 1000\ mW$

$0\ dBW = 30\ dBm$

$$\begin{array}{ccc} dBW & = & dBV - 17 \\ \downarrow & & \downarrow \\ dBm - 30 & = & (dB\mu V - 120) - 17 \end{array}$$

$1\ V = 10^6\ \mu V$

$0\ dBV = 120\ dB\mu V$

$$dB\mu V = dBm + 107$$

For 50 Ω system only!!!

Remember this!

0 dBm = 1 mW = 107 dBμV = 224 mVrms

dBμA to dBμV Conversion for 50 Ω System

$V = IR$ ***For 50 Ω system:*** $V = I(50\ \Omega)$

$$20\log_{10} V = 20\log_{10} I + \underline{20}\log_{10}(50\ \Omega)$$

(20·log in this context)

$$dBV = dBA + 34\ dB\Omega$$

$$dB\mu V - 120 = (dB\mu A - 120) + 34\ dB\Omega$$

$$\left.\begin{array}{r} dB\mu V = dB\mu A + 34\ dB\Omega \\ \textit{For 50 Ω system only!!!} \end{array}\right\}$$

Remember this!

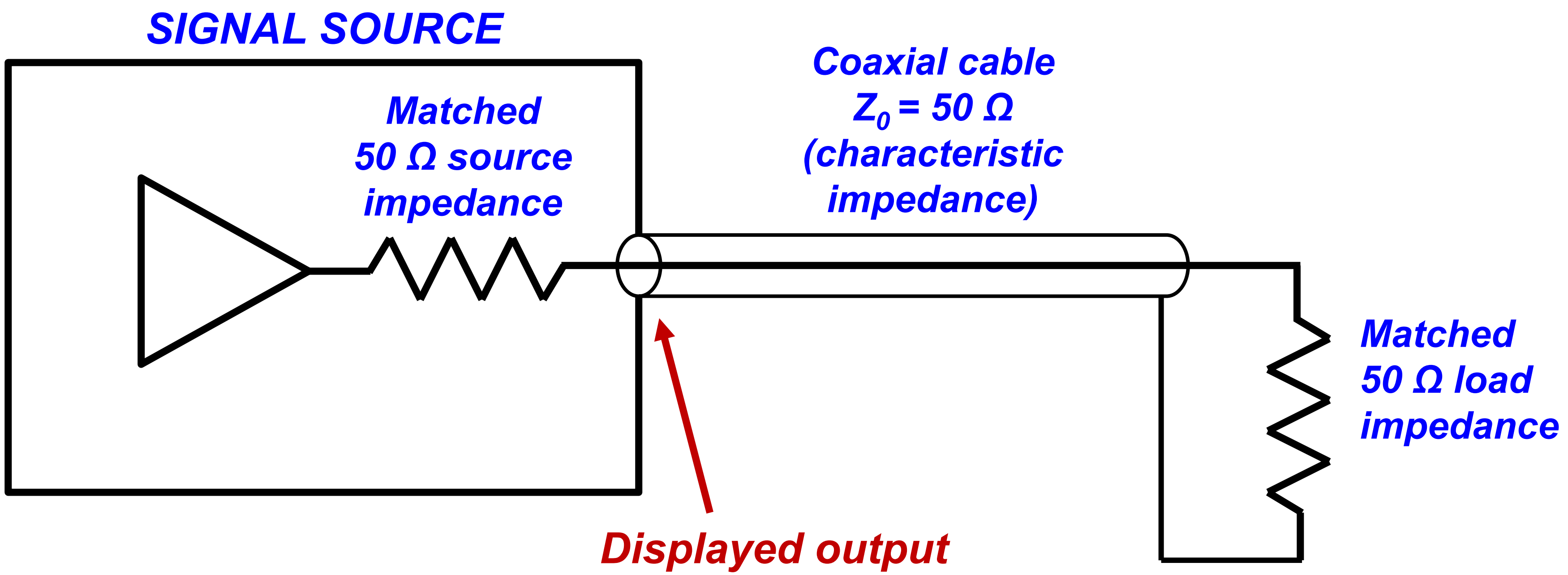
"dB Literacy"

- **Remember that a number in dB is ALWAYS a ratio of 2 power numbers**
- **Denominator provides reference value, e.g.:**
  - dBµV referenced to 1 µV
  - dBµA referenced to 1 µA
  - dBm referenced to 1 mW
- **When discussing a level above a test limit, the limit is the reference**
  - **CORRECT: "X dB above the limit"**
  - **INCORRECT: "X dBµV/m above the limit"**

- ***Numeric counterexample:***
  - *Exceeded speed limit by a factor of 2, or...*
  - *Exceeded limit by 2 miles per hour*

VERY different meanings

50 Ω System



SIGNAL SOURCE

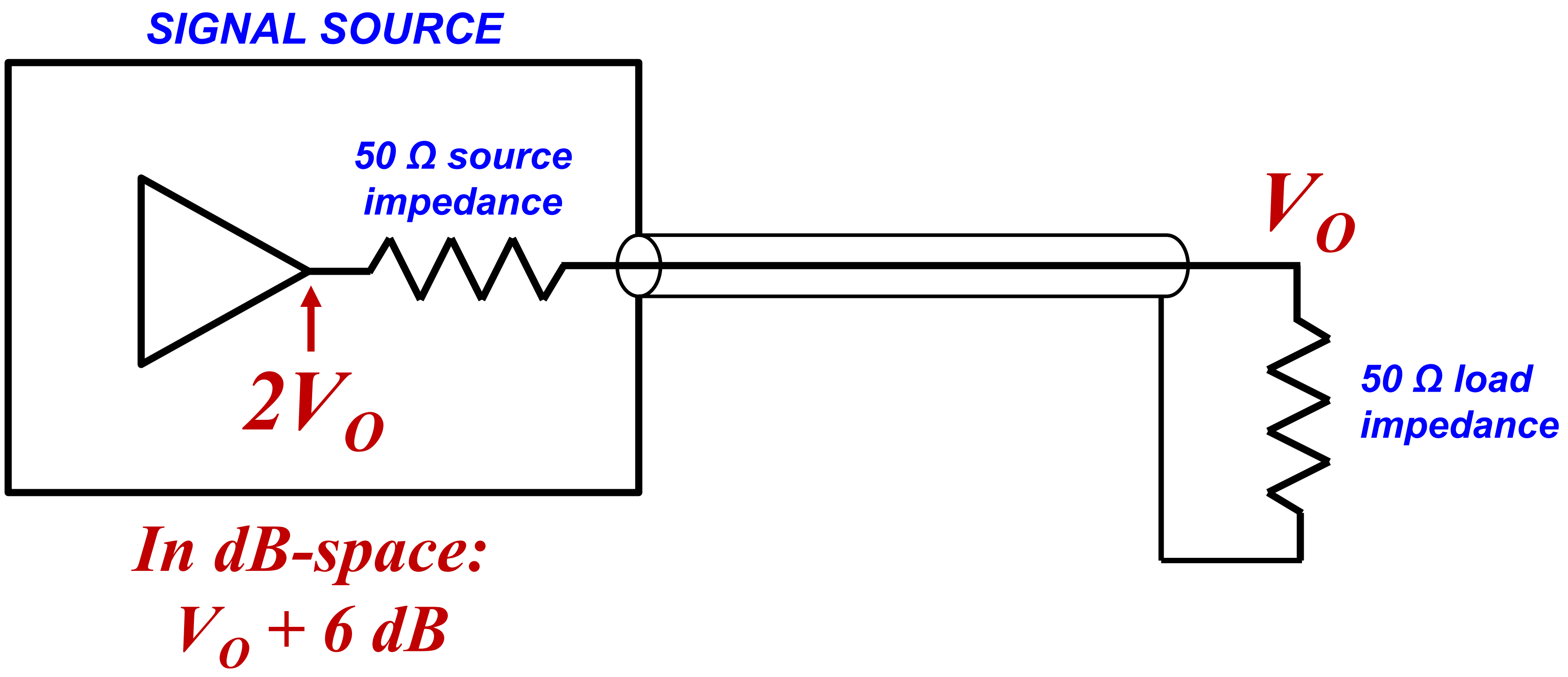
Matched
50 Ω source
impedance

Coaxial cable
$Z_0 = 50\ \Omega$
(characteristic
impedance)

Matched
50 Ω load
impedance

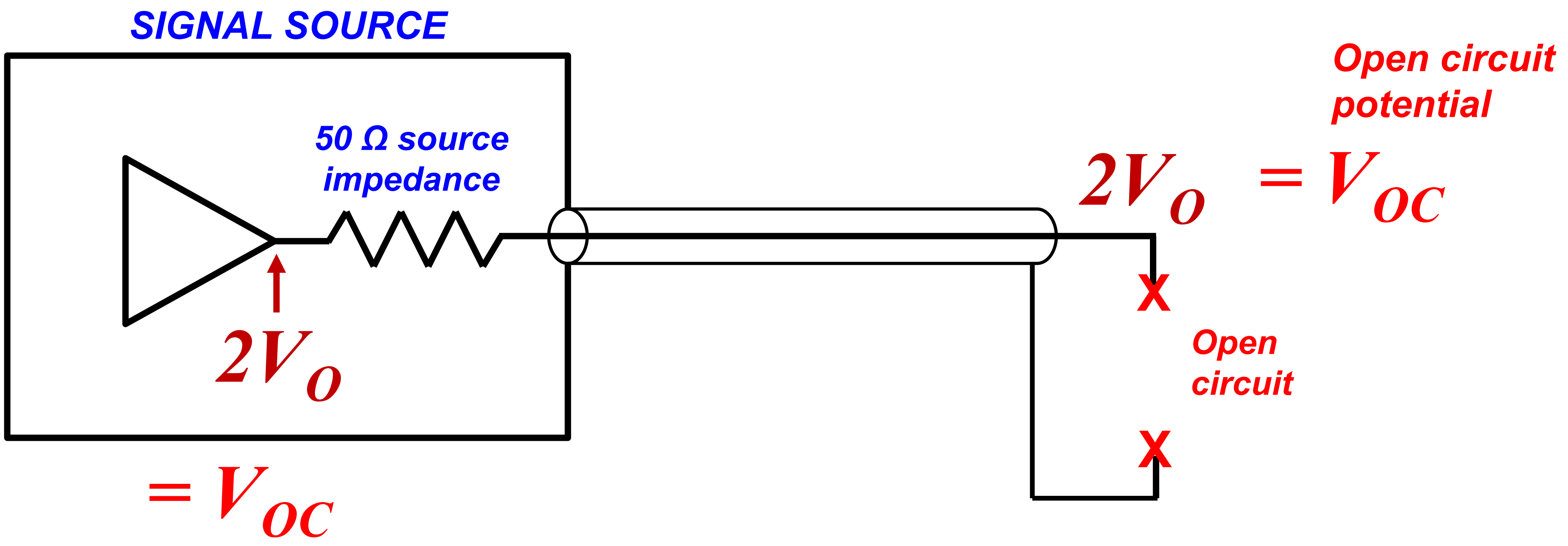
Displayed output
level assumes
50 Ω load

50 Ω System (cont.)



In dB-space:
$V_O + 6\ dB$

50 Ω System (cont.)



SIGNAL SOURCE

50 Ω source impedance

$2V_O$

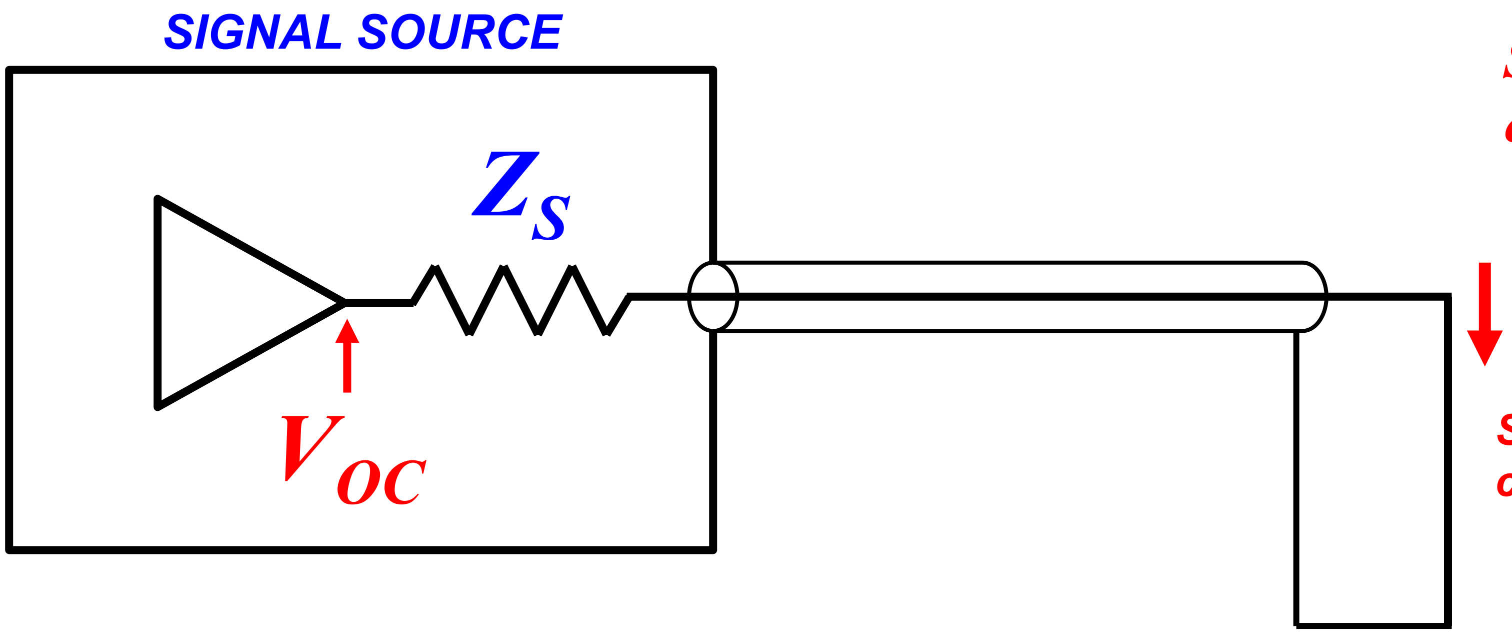
$= V_{OC}$

Open circuit potential

$2V_O = V_{OC}$

Open circuit

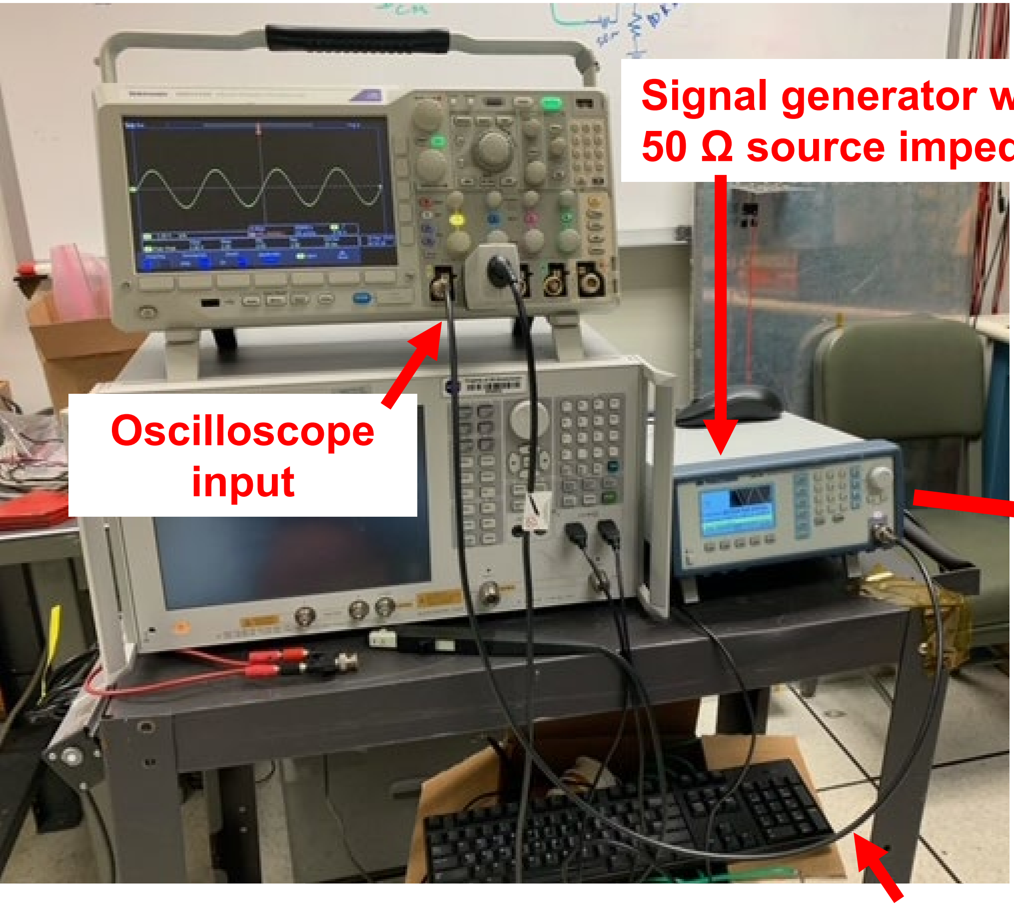
50 Ω System (cont.)



Short-circuit current:

$$I_{SC} = \frac{V_{OC}}{Z_S}$$

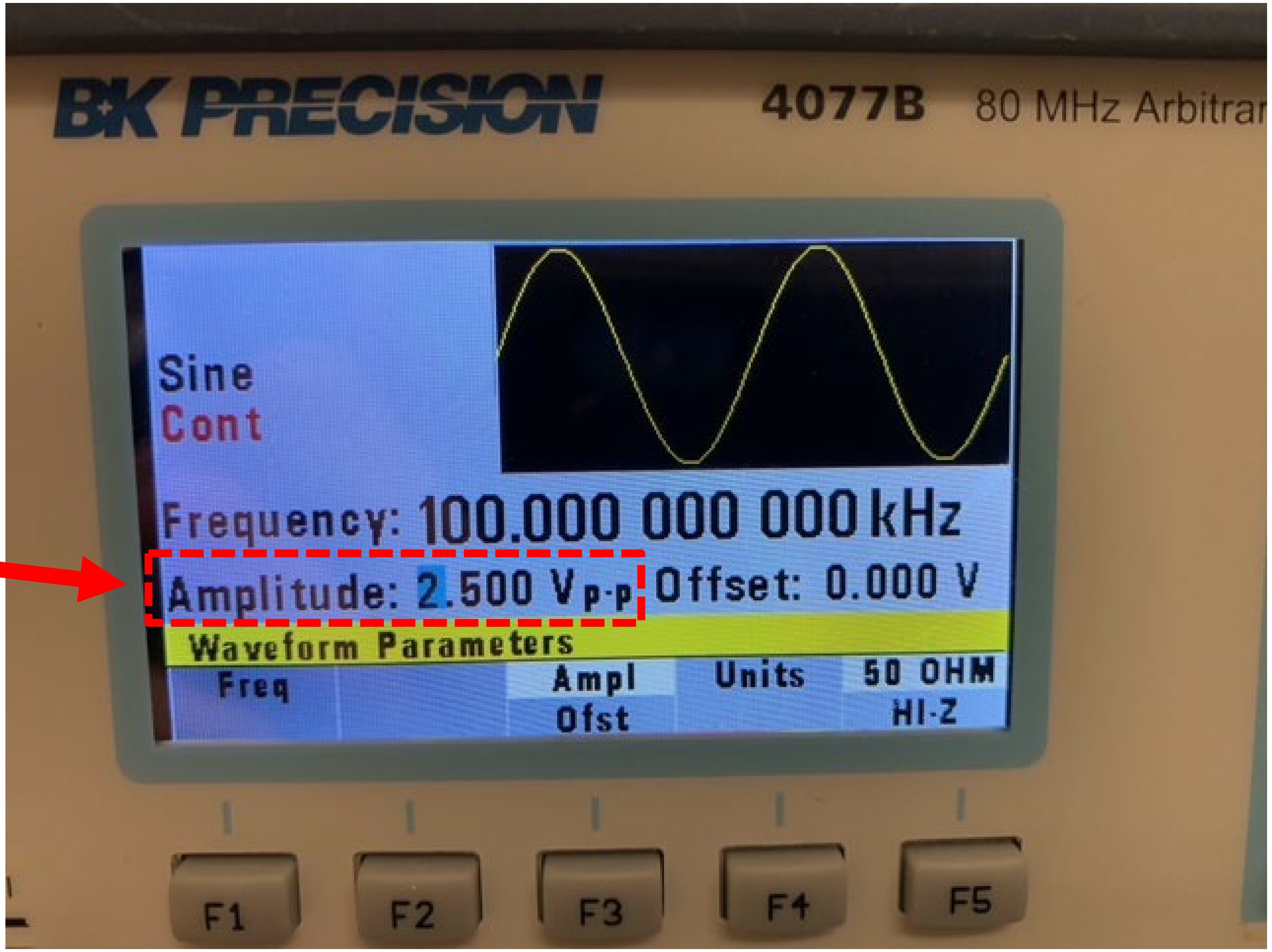
50 Ω System (cont.)



Signal generator with 50 Ω source impedance

Oscilloscope input

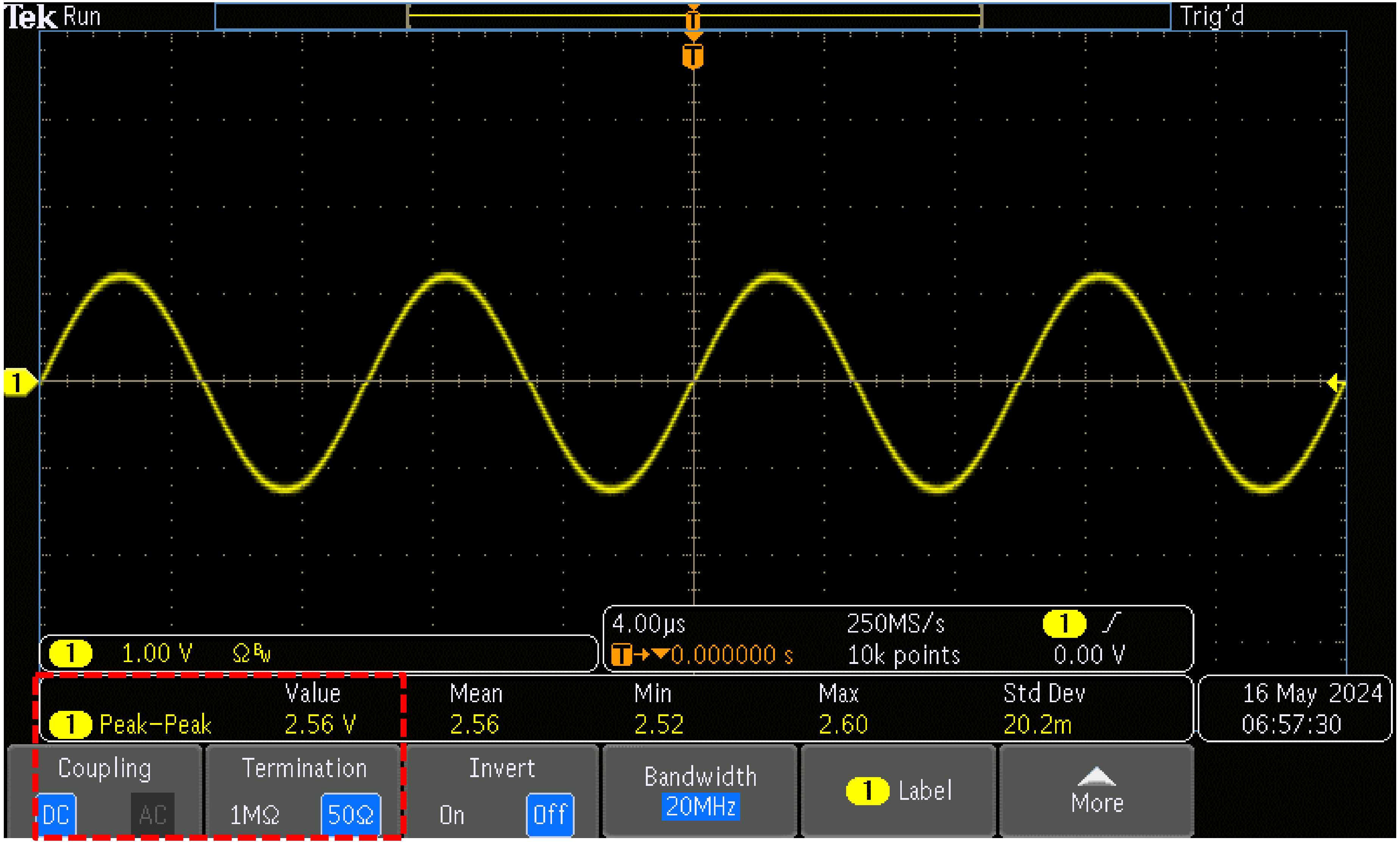
Coaxial cable $Z_0 = 50\ \Omega$



Frequency: 100.000 000 000 kHz

Amplitude: 2.500 $V_{p\text{-}p}$ Offset: 0.000 V

50 Ω System (cont.)



Tek Run

Trig’d

1 1.00 V Ω B_W

4.00µs 250MS/s 1 ⁄

T→▼0.000000 s 10k points 0.00 V

	Value	Mean	Min	Max	Std Dev
1 Peak−Peak	2.56 V	2.56	2.52	2.60	20.2m

16 May 2024
06:57:30

Coupling: DC

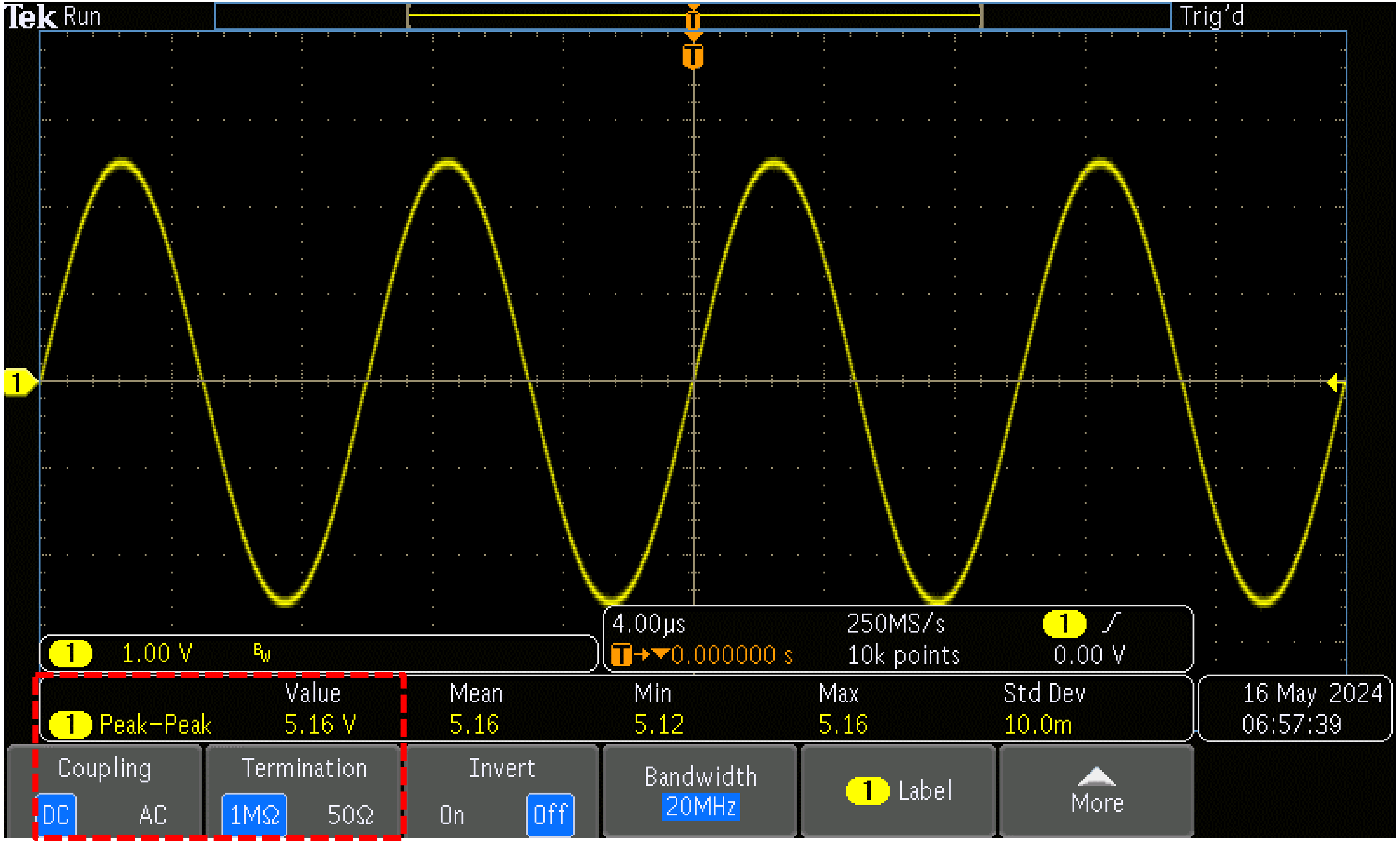
Termination: 50Ω

Invert: Off

Bandwidth: 20MHz

Displayed potential (V_O) into 50 Ω

50 Ω System (cont.)



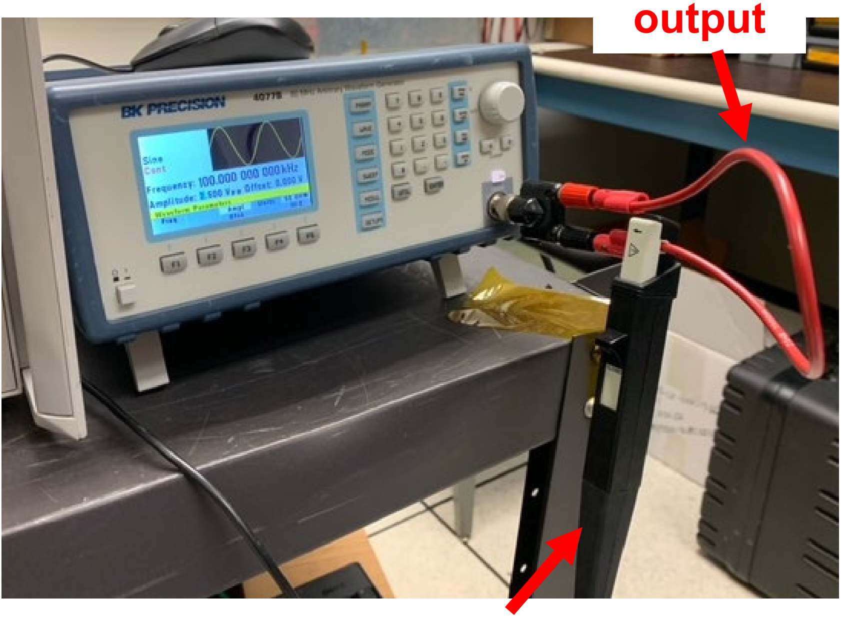
	Value	Mean	Min	Max	Std Dev
1 Peak−Peak	5.16 V	5.16	5.12	5.16	10.0m

Coupling: DC [✓] AC

Termination: 1MΩ [✓] 50Ω

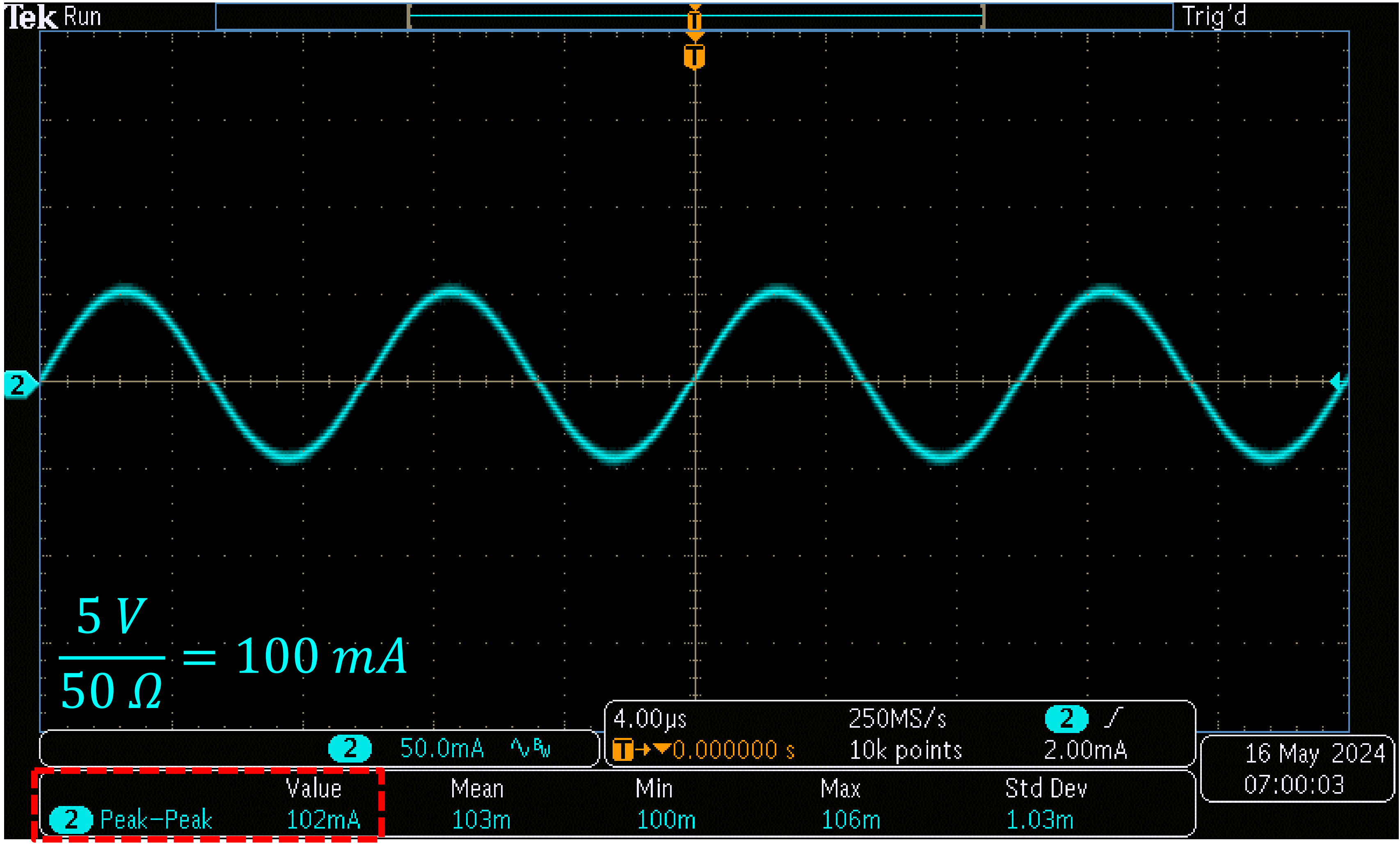
2x displayed potential ($2V_O$) into 1 MΩ (V_{OC})

50 Ω System (cont.)



Shorted output

Current probe



$$\frac{5\ V}{50\ \Omega} = 100\ mA$$

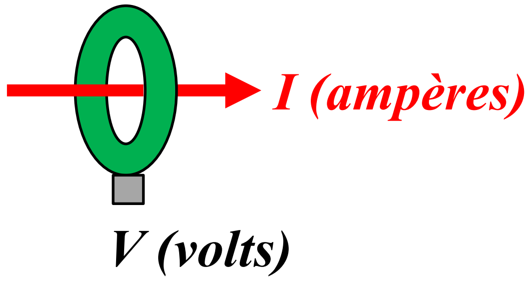
```
                 Value    Mean    Min     Max     Std Dev
2 Peak-Peak      102mA    103m    100m    106m    1.03m
```

Short circuit current = V_{OC} / Z_S

Where Measurements Start: Transducers

- **transducer /tranzˈdo͞osər/, noun**
  1. a device that converts energy from one form to another
  2. a device that converts variations in a physical quantity (e.g. current or electric field), into an electrical signal (e.g. potential), or vice versa

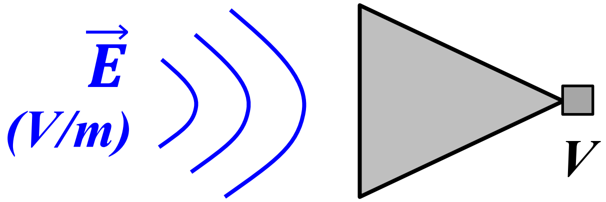
Current probe



$$\textit{Transfer function} = \frac{volts}{ampères} = \Omega \quad \textit{Transfer impedance, } Z_t$$

$$20\log_{10} Z_t = Z_t(\mathrm{dB}\Omega)$$

Antenna



$$\textit{Transfer function} = \frac{volts}{volts/meter} = \mathrm{meters} \quad \textit{Effective height/ length, } h_e$$

$$20\log_{10} h_e = h_e(\mathrm{dB}\cdot\mathrm{meter})$$

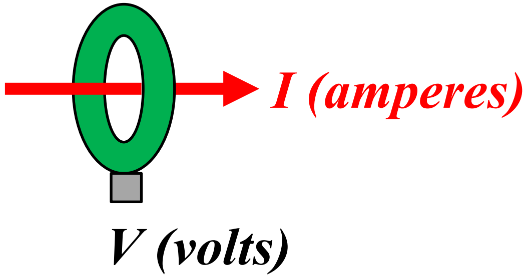
Transducers: Transfer Function and Transducer/Probe Factor

Working forward: $IZ_t = V$ $\quad I(dB\mu A) + Z_t(dB\Omega) = V(dB\mu V)$ $\qquad Eh_e = V$ $\quad E(dB\mu V/m) + h_e(dB \cdot meter) = V(dB\mu V)$

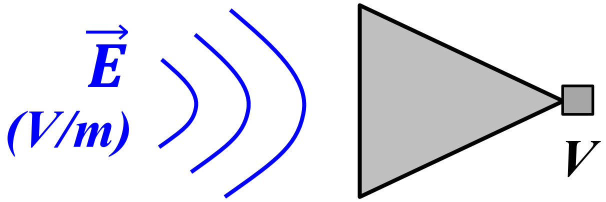
Working backwards: $\dfrac{V}{Z_t} = I$ $\quad V(dB\mu V) \underline{- Z_t(dB\Omega)} = I(dB\mu A)$ $\qquad \dfrac{V}{h_e} = E$ $\quad V(dB\mu V) \underline{- h_e(dB \cdot meter)} = E(dB\mu V/m)$

Measurement software generally expects "Transducer Factor" as number in dB to be <u>ADDED</u> to measured value

Current probe



Antenna



$$\textit{Transfer function} = \frac{volts}{ampères} = \Omega$$ ***Transfer impedance, Z_t***

$$20\log_{10} Z_t = Z_t(\mathrm{dB\Omega})$$

$$\textit{Transfer function} = \frac{volts}{volts/meter} = \text{meters}$$ ***Effective height/ length, h_e***

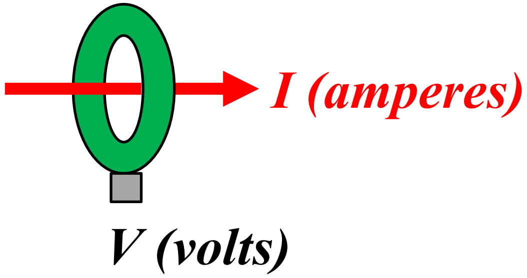
$$20\log_{10} h_e = h_e(\mathrm{dB \cdot meter})$$

Transducers: Transfer Function and Transducer/Probe Factor

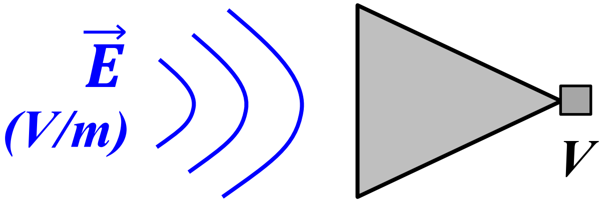
Working forward: $IZ_t = V \qquad I(dB\mu A) + Z_t(dB\Omega) = V(dB\mu V)$ $\qquad Eh_e = V \qquad E(dB\mu V/m) + h_e(dB \cdot meter) = V(dB\mu V)$

Working backwards: $\dfrac{V}{Z_t} = I \qquad V(dB\mu V) - Z_t(dB\Omega) = I(dB\mu A)$ $\qquad \dfrac{V}{h_e} = E \qquad V(dB\mu V) - h_e(dB \cdot meter) = E(dB\mu V/m)$

Current probe



Antenna



Probe factor: $PF = \dfrac{1}{Z_t} \qquad PF(dB\Omega^{-1}) = -Z_t(dB\Omega)$

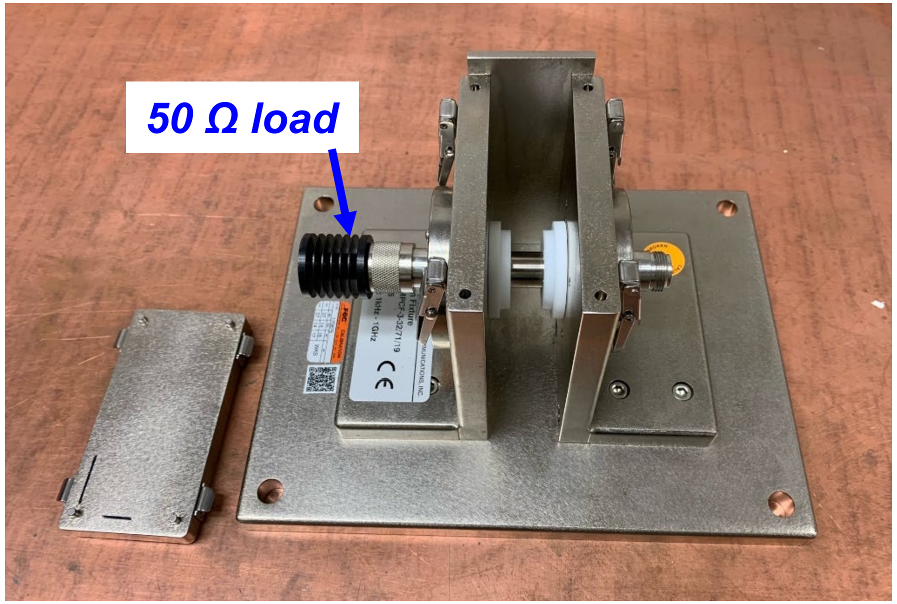
$$V(dB\mu V) + PF(dB\Omega^{-1}) = I(dB\mu A)$$

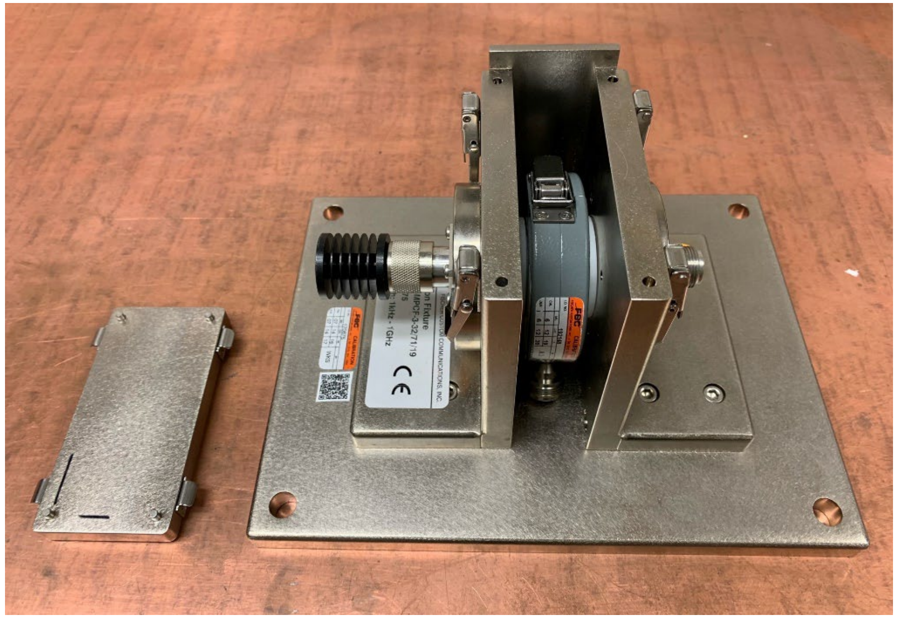
Antenna factor: $AF = \dfrac{1}{h_e} \qquad AF(dB \cdot meter^{-1}) = -h_e(dB \cdot meter)$

$$V(dB\mu V) + AF(dB \cdot meter^{-1}) = E(dB\mu V/m)$$

Current Probe Calibration

Example:
Fischer F-33-2





Coaxial calibration fixture

Center conductor feeds through

Shield connects to fixture housing

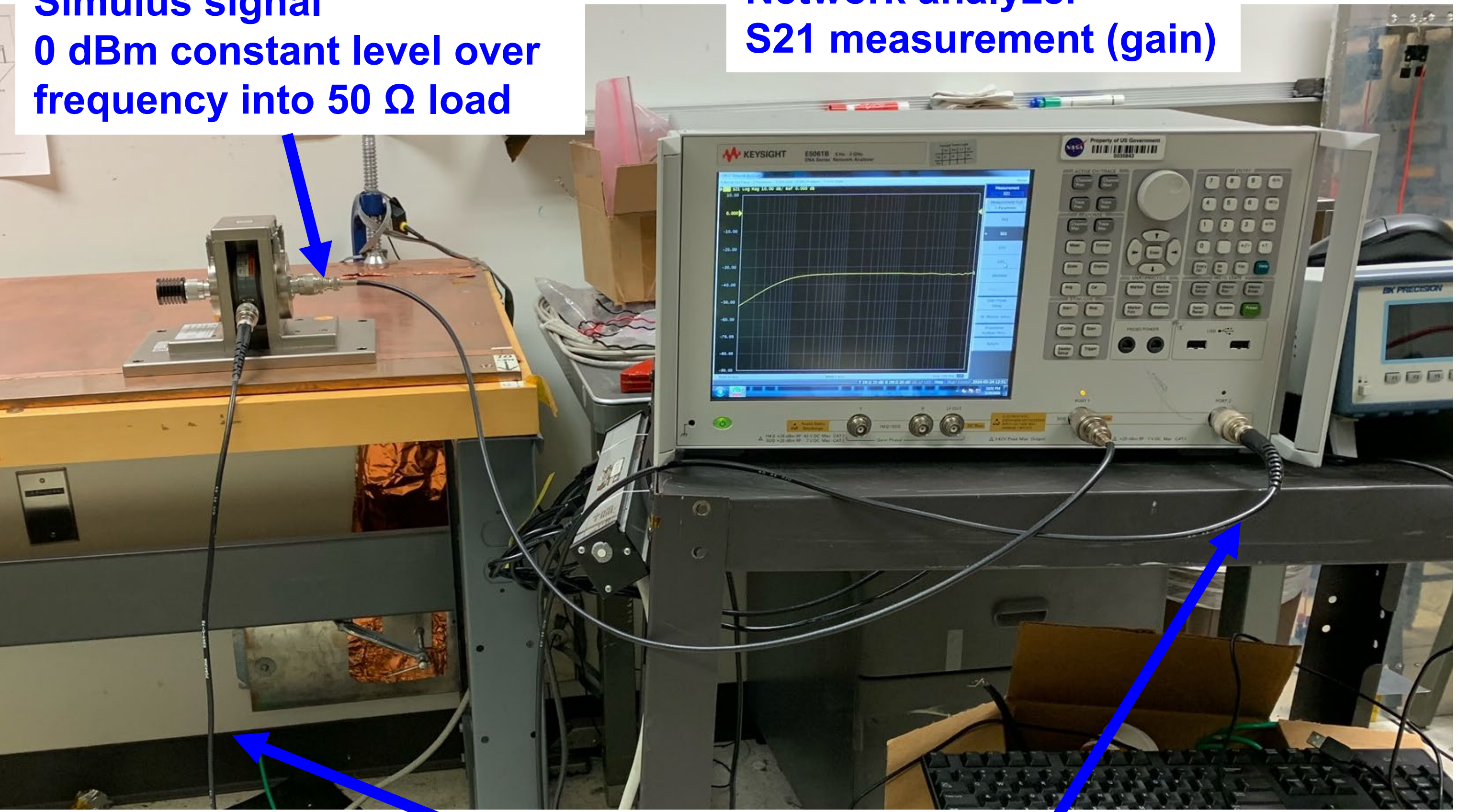
Put known current through probe

Measure output potential as function of frequency

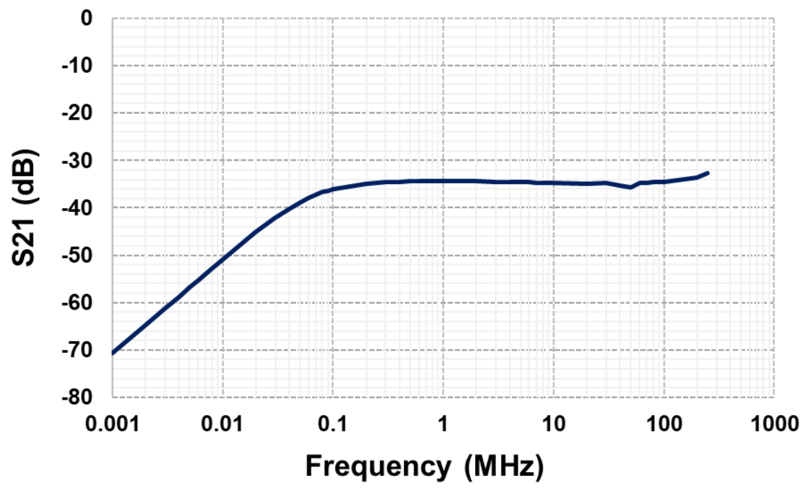
Current Probe Calibration (cont.)

Port 1:
Simulus signal
0 dBm constant level over frequency into 50 Ω load

Network analyzer
S21 measurement (gain)



Port 2:
Current probe output potential



$$S21(dB) = P2(dBm) - P1(dBm)$$

$$Z_t(dB\Omega) = V2(dB\mu V) - I(dB\mu A) \ \textbf{???}$$

$$P2(dBm) + 107 = V2(dB\mu V)$$
$$P1(dBm) + 107 = V1(dB\mu V)$$

$$S21(dB) = V2(dB\mu V) - V1(dB\mu V)$$

$$I = \frac{V1}{50\ \Omega} \qquad I(dB\mu A) = V1(dB\mu V) - 34\text{ dB}\Omega$$

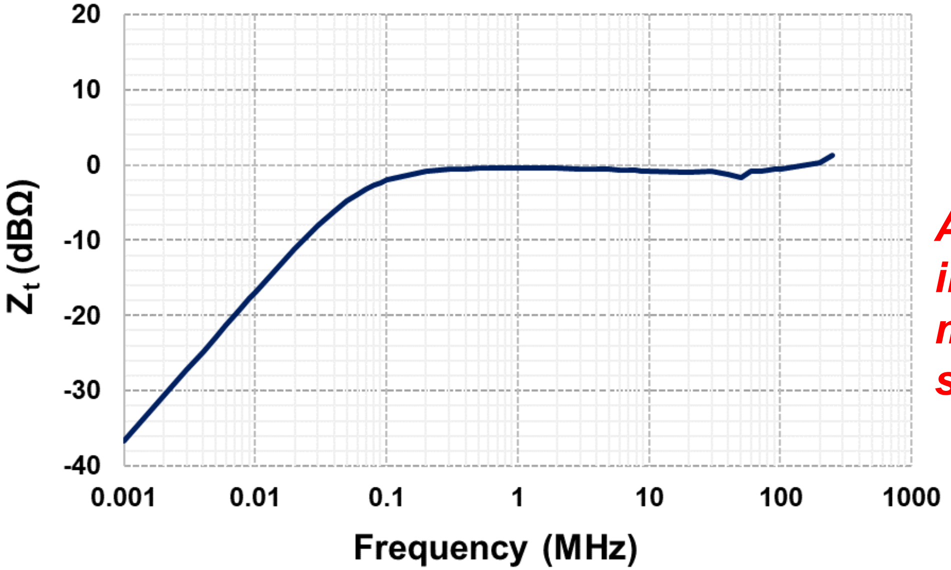
$$Z_t(dB\Omega) = V2(dB\mu V) - [V1(dB\mu V) - 34\text{ dB}\Omega]$$
$$= [V2(dB\mu V) - V1(dB\mu V)] + 34\text{ dB}\Omega$$

$$\underline{Z_t(dB\Omega) = S21(dB) + 34\text{ dB}\Omega}$$

Current Probe Calibration (cont.)

Transfer impedance:

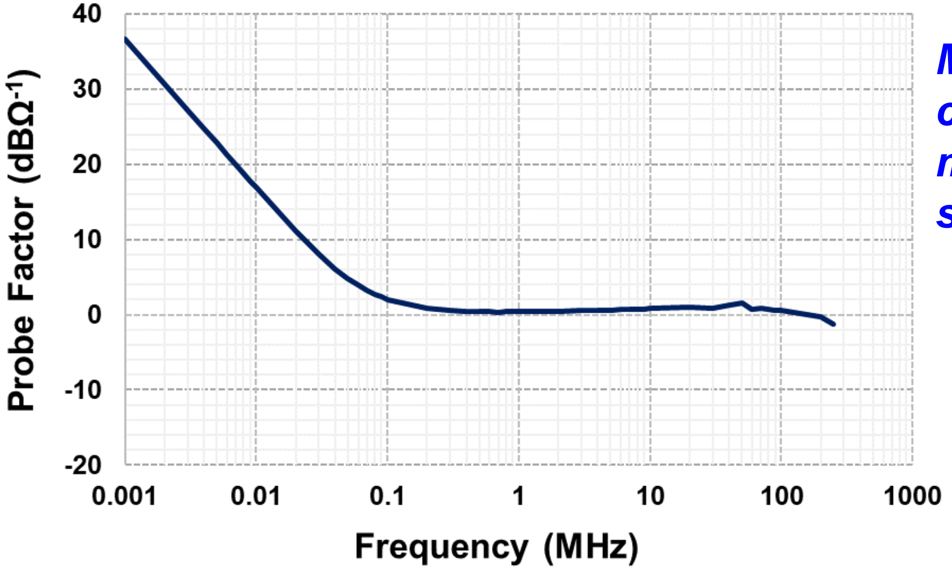
$$Z_t(dB\Omega) = S21(dB) + 34\ \mathrm{dB\Omega}$$



Adding transfer impedance instead of probe factor to measured potential can give significant errors

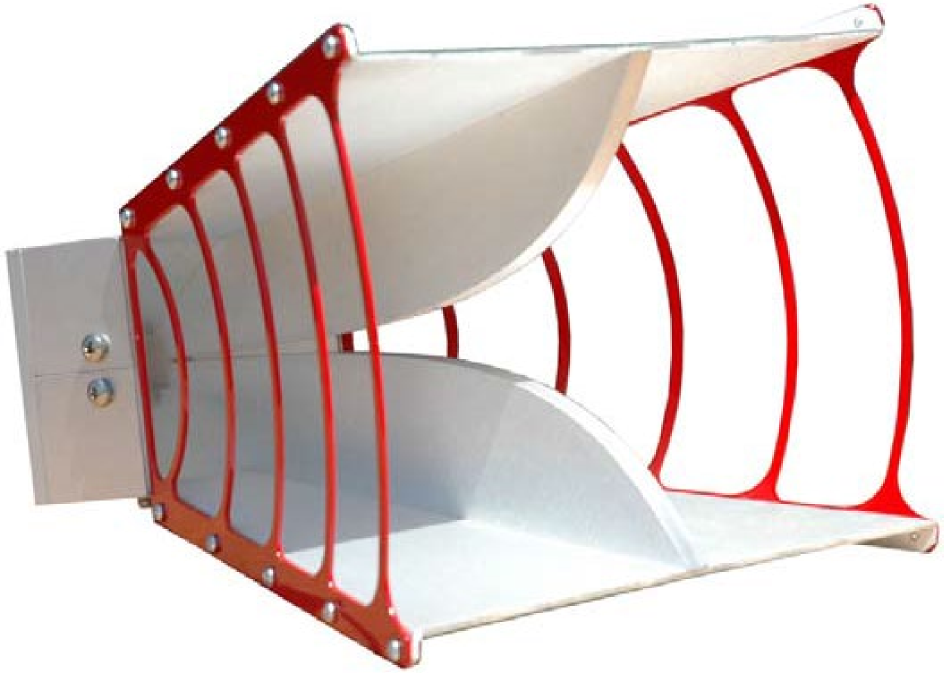
Probe Factor:

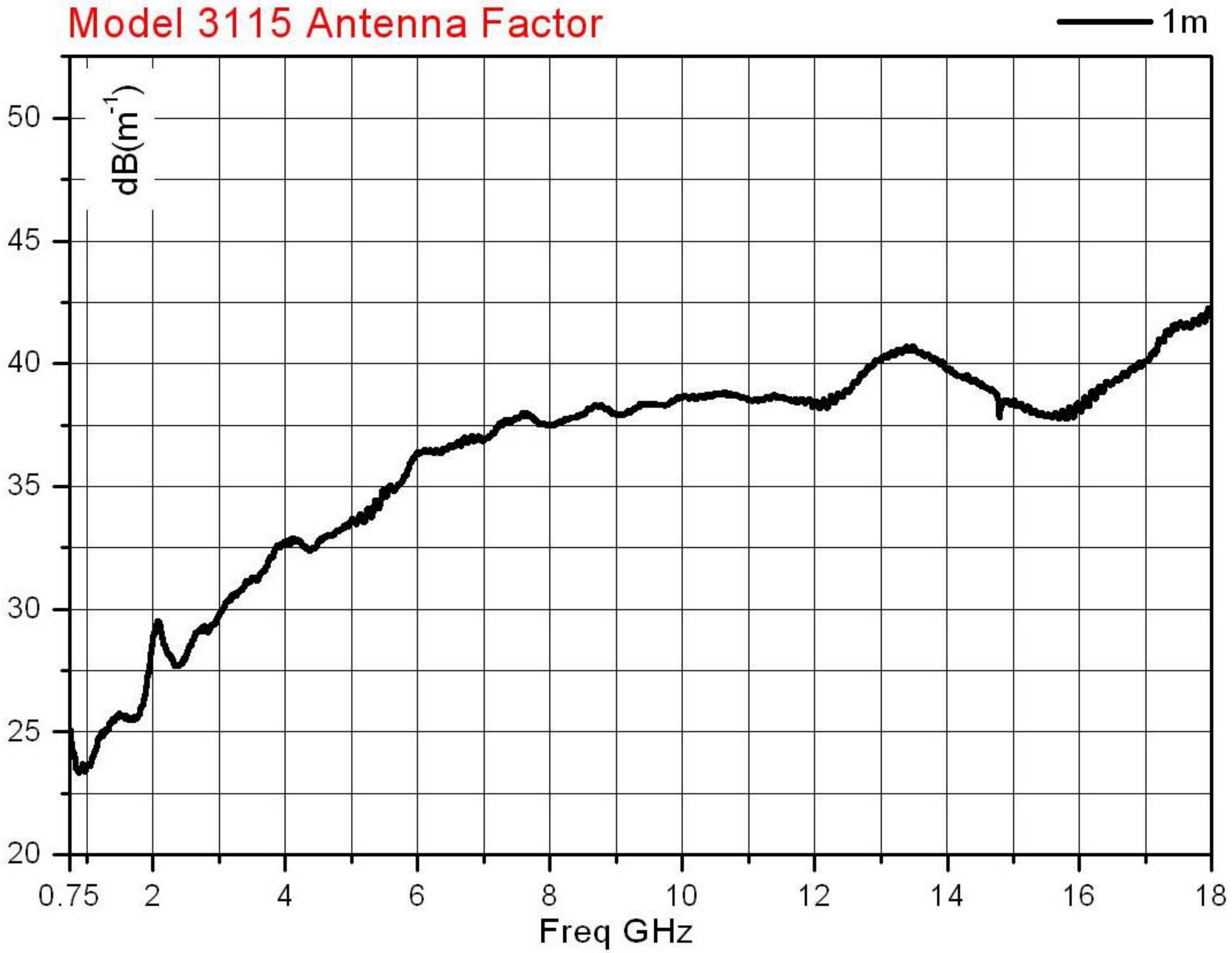
$$PF(dB\Omega^{-1}) = -Z_t(dB\Omega)$$



Make sure you are adding correct value according to your measurement device's or software's needs

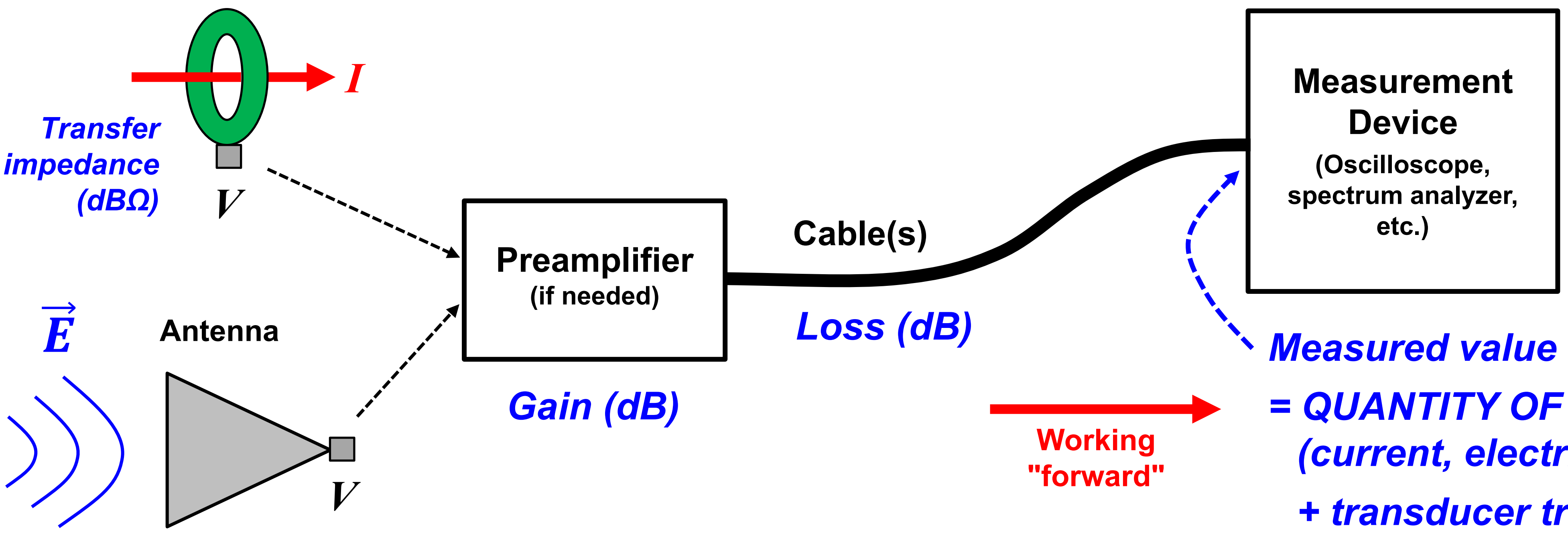
ETS Lindgren 3115 Double Ridged Guide Antenna





Basic Measurement Setup

Transducer



Current probe

Transfer impedance (dBΩ)

I

V

$\vec{E}$

Antenna

V

Effective height (dBmeter)

Preamplifier
(if needed)

Gain (dB)

Cable(s)

Loss (dB)

Measurement Device
(Oscilloscope, spectrum analyzer, etc.)

Working "forward"

Measured value (dBµV, dBm)

= QUANTITY OF INTEREST (current, electric field, etc.)

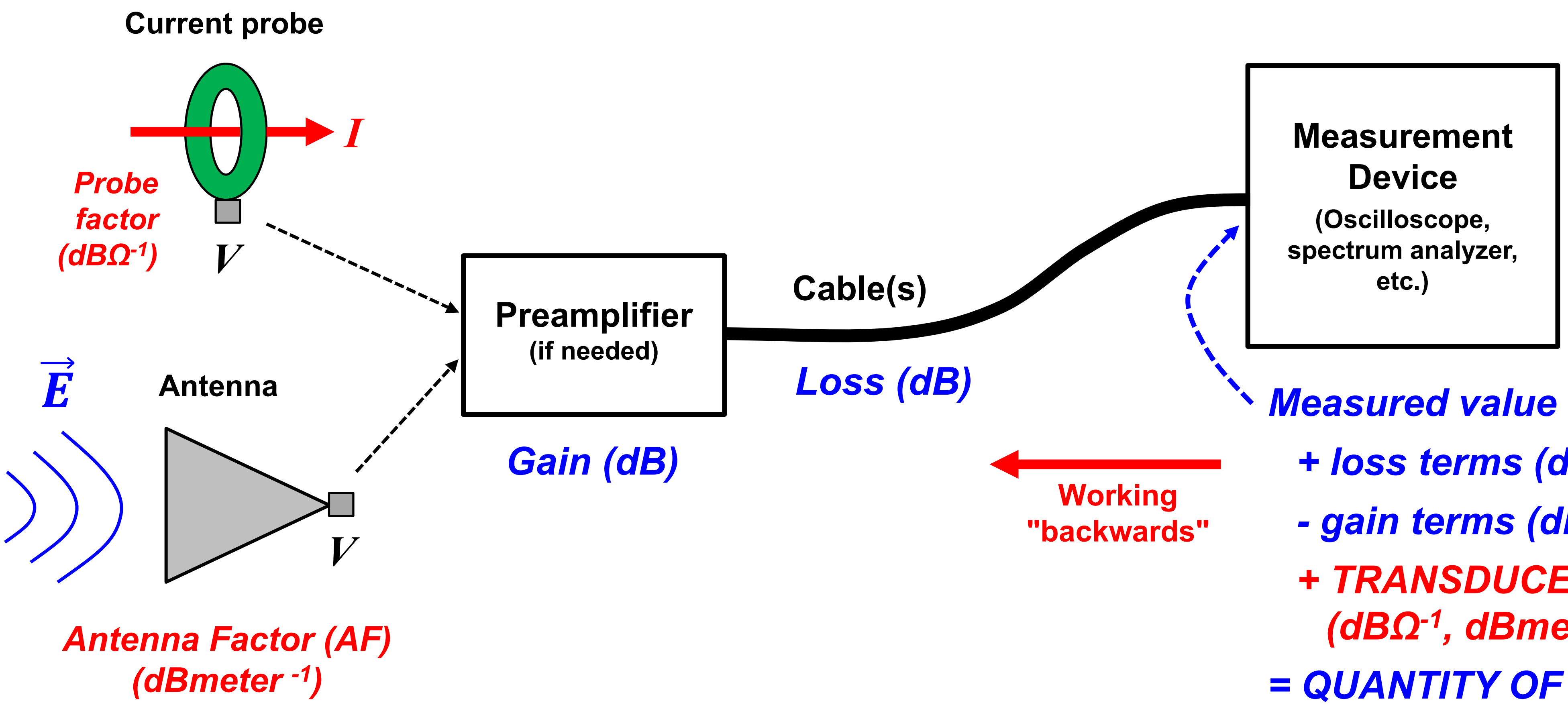
+ transducer transfer function (dBΩ, dBmeter, etc.)

+ gain terms (dB)

- loss terms (dB)

Basic Measurement Setup (cont.)

Transducer



Measured value (dBμV, dBm)

+ loss terms (dB)

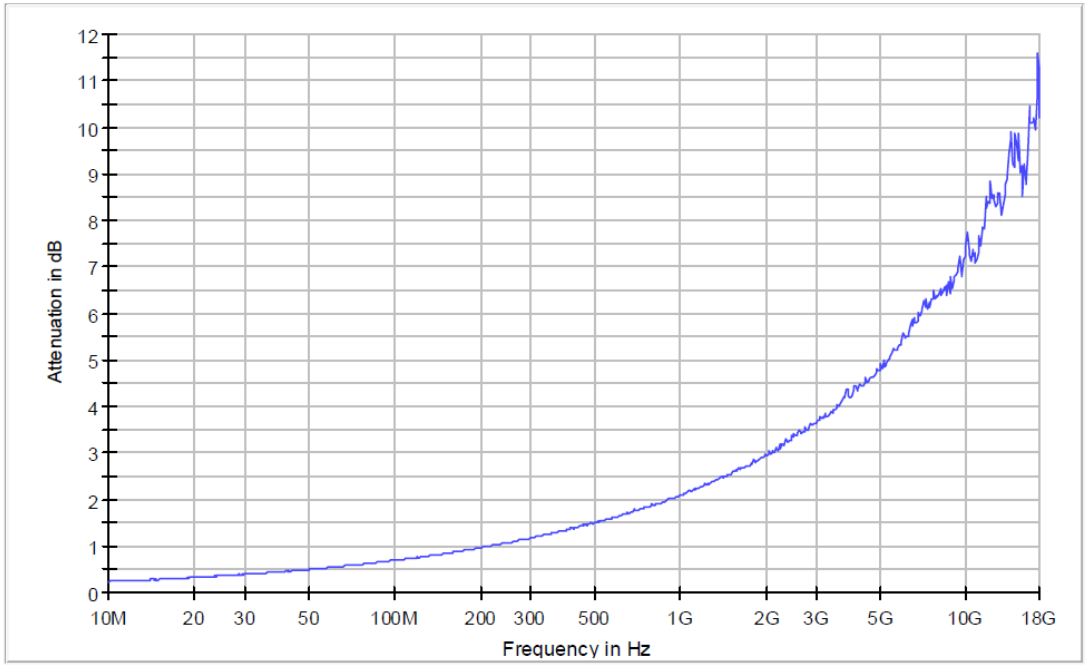
- gain terms (dB)

+ TRANSDUCER FACTOR ($dB\Omega^{-1}$, $dBmeter^{-1}$, etc.)

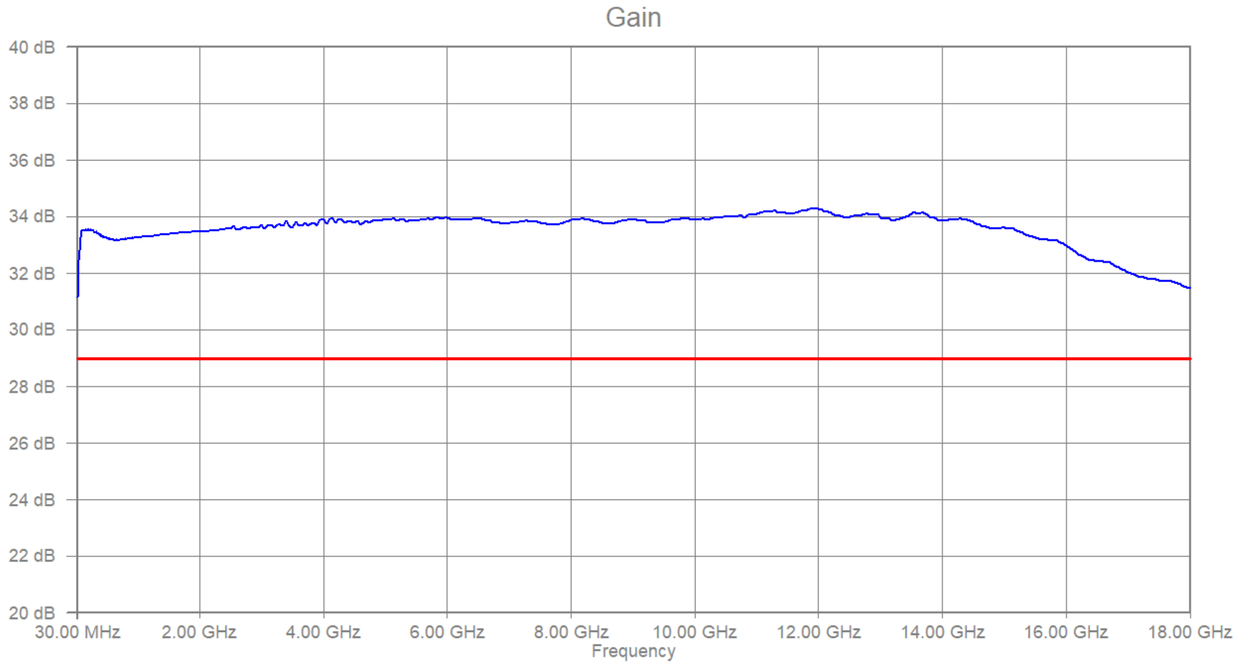
= QUANTITY OF INTEREST (current, electric field, etc.)

Example Loss and Gain Curves

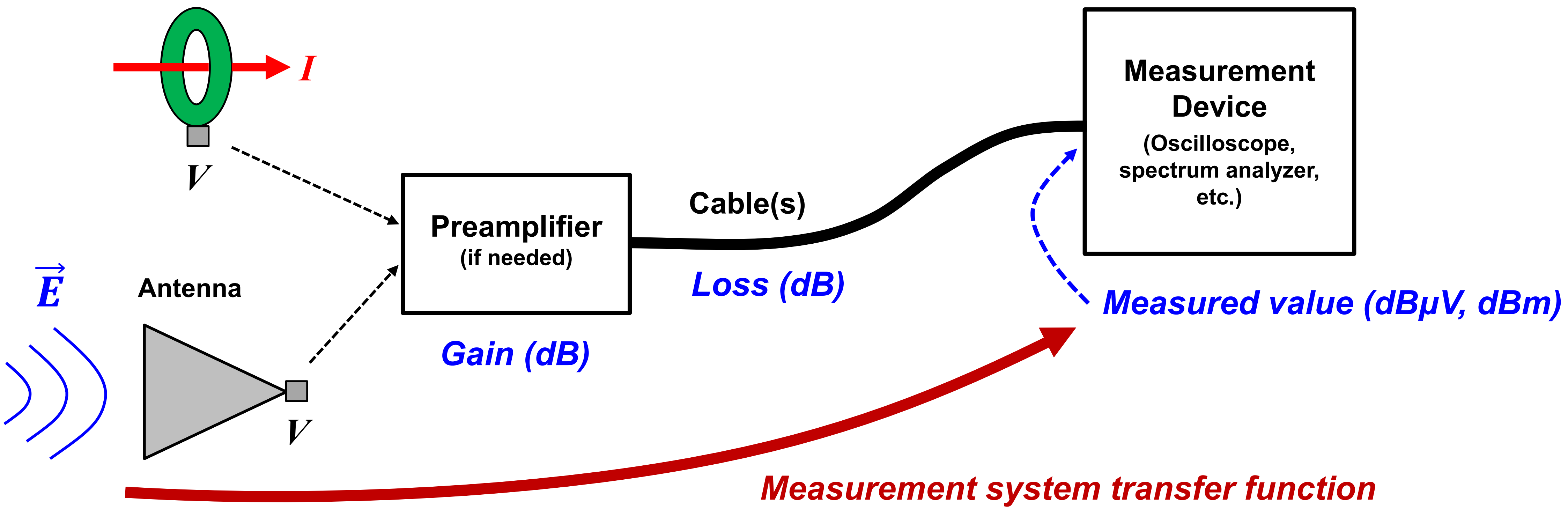
GORE 42' Cable Loss, SN 504-811



RE102 Pre-Amplifier Gain, Model TS-PR18



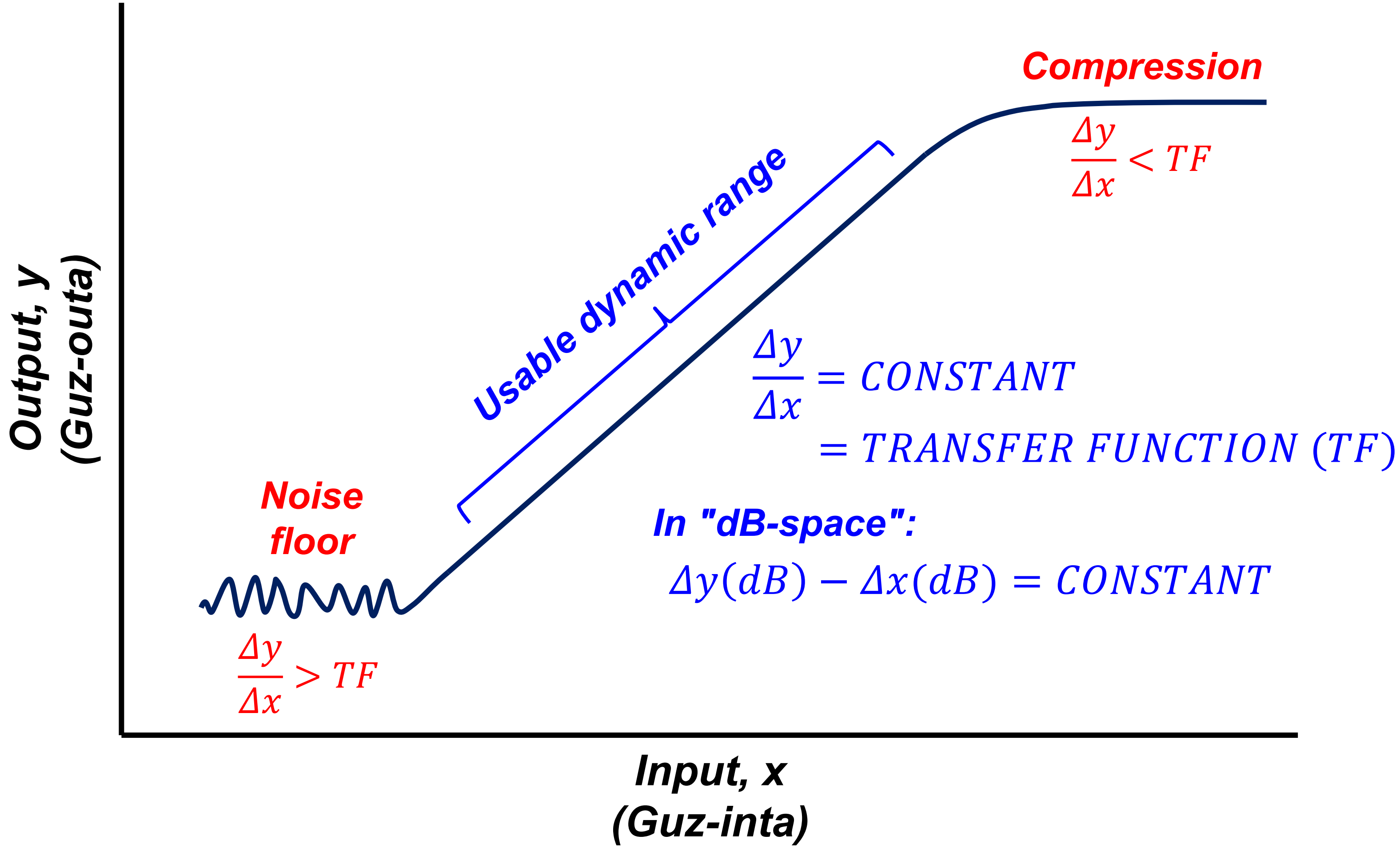
Measurement System Transfer Function



Transducer

Measurement system transfer function
Function of frequency
Linearity desired

Linearity and Dynamic Range



Compression

$\frac{\Delta y}{\Delta x} < TF$

Usable dynamic range

$\frac{\Delta y}{\Delta x} = CONSTANT$

$= TRANSFER\ FUNCTION\ (TF)$

In "dB-space":

$\Delta y(dB) - \Delta x(dB) = CONSTANT$

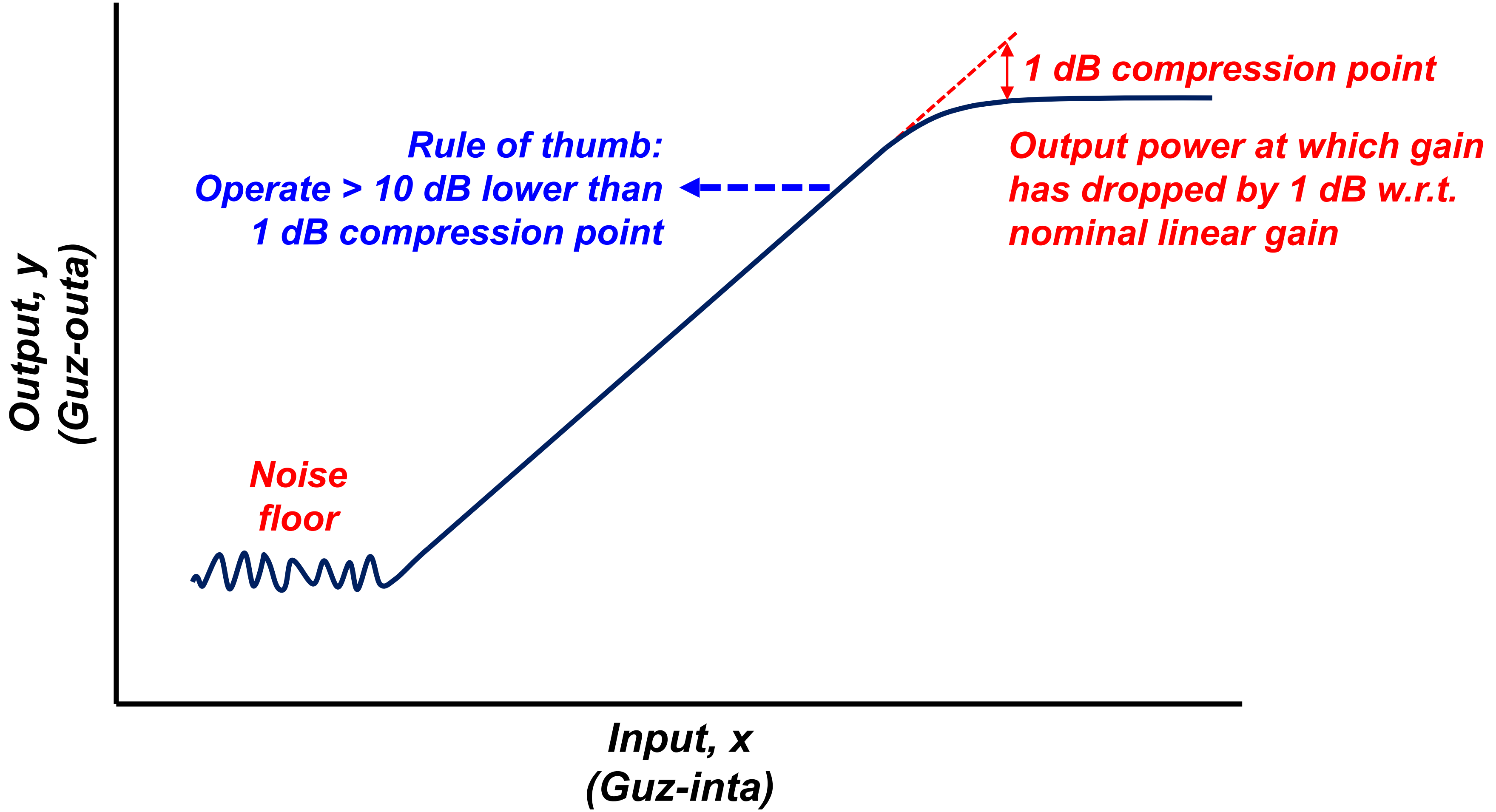
Noise floor

$\frac{\Delta y}{\Delta x} > TF$

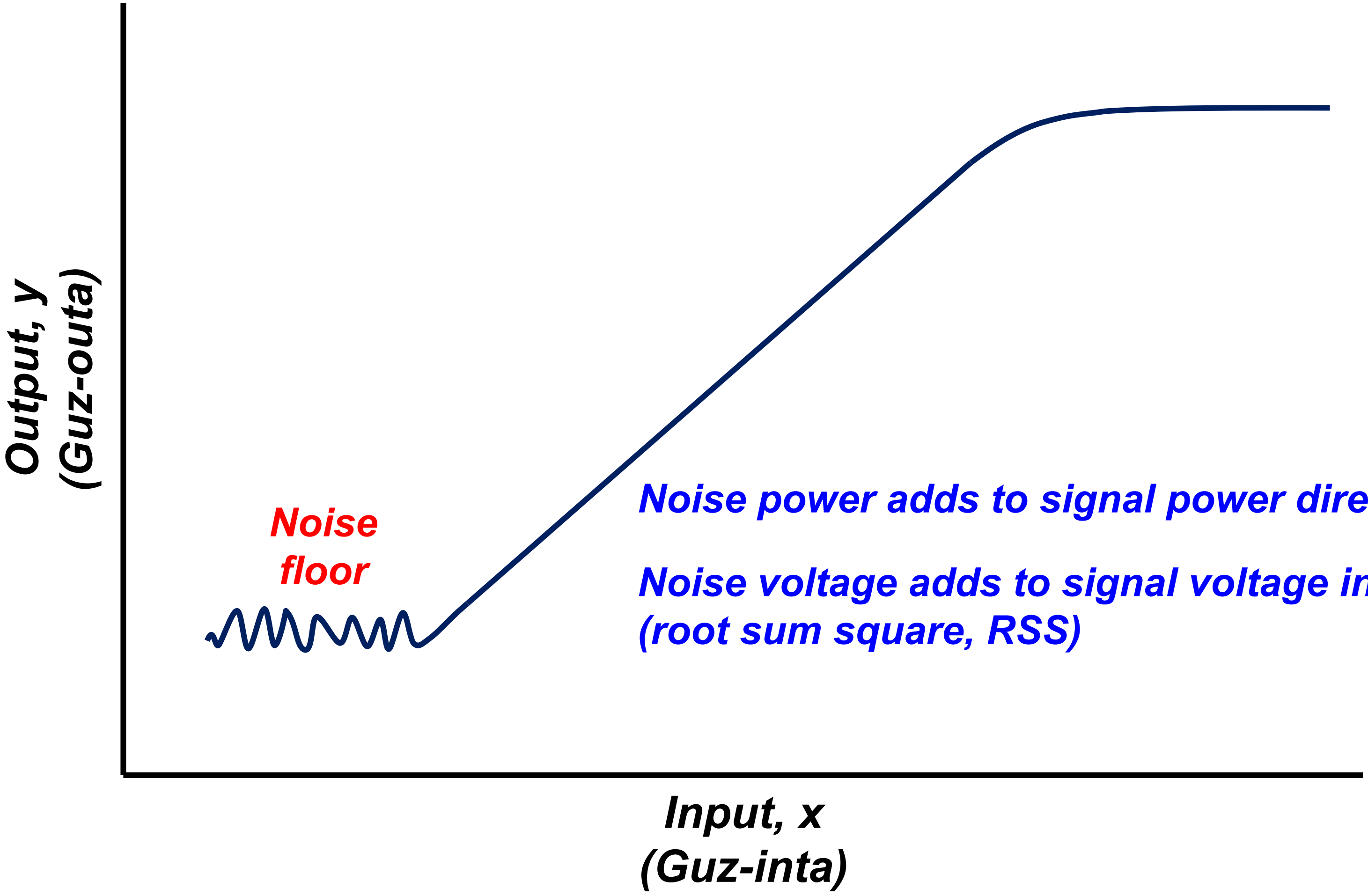
Output, y (Guz-outa)

Input, x (Guz-inta)

Linearity and Dynamic Range (cont.)



Linearity and Dynamic Range (cont.)



Noise power adds to signal power directly

Noise voltage adds to signal voltage in quadrature (root sum square, RSS)

Linearity and Dynamic Range (cont.)

For signal 6 dB (4x power, 2x voltage/current) above noise floor:

- *Noise has 0.25x power level of signal*
- *Noise will increase total level by factor of 1.25*

$$10\log_{10}1.25 \approx 1\,dB$$

- *Noise has 0.5x voltage/current level of signal*
- *Noise will increase total level by:*

$$\sqrt{1^2+0.5^2}=1.12$$

$$20\log_{10}1.12 \approx 1\,dB$$

For signal 10 dB (10x power, 3.16x voltage/current) above noise floor:

- *Noise has 0.1x power level of signal*
- *Noise will increase total level by factor of 1.1*

$$10\log_{10}1.1 \approx 0.4\,dB$$

- *Noise has 0.316x voltage/current level of signal*
- *Noise will increase total level by:*

$$\sqrt{1^2+0.316^2}=1.05$$

$$20\log_{10}1.05 \approx 0.4\,dB$$

For signal 20 dB (100x power, 10x voltage/current) above noise floor:

- *Noise has 0.01x power level of signal*
- *Noise will increase total level by factor of 1.01*

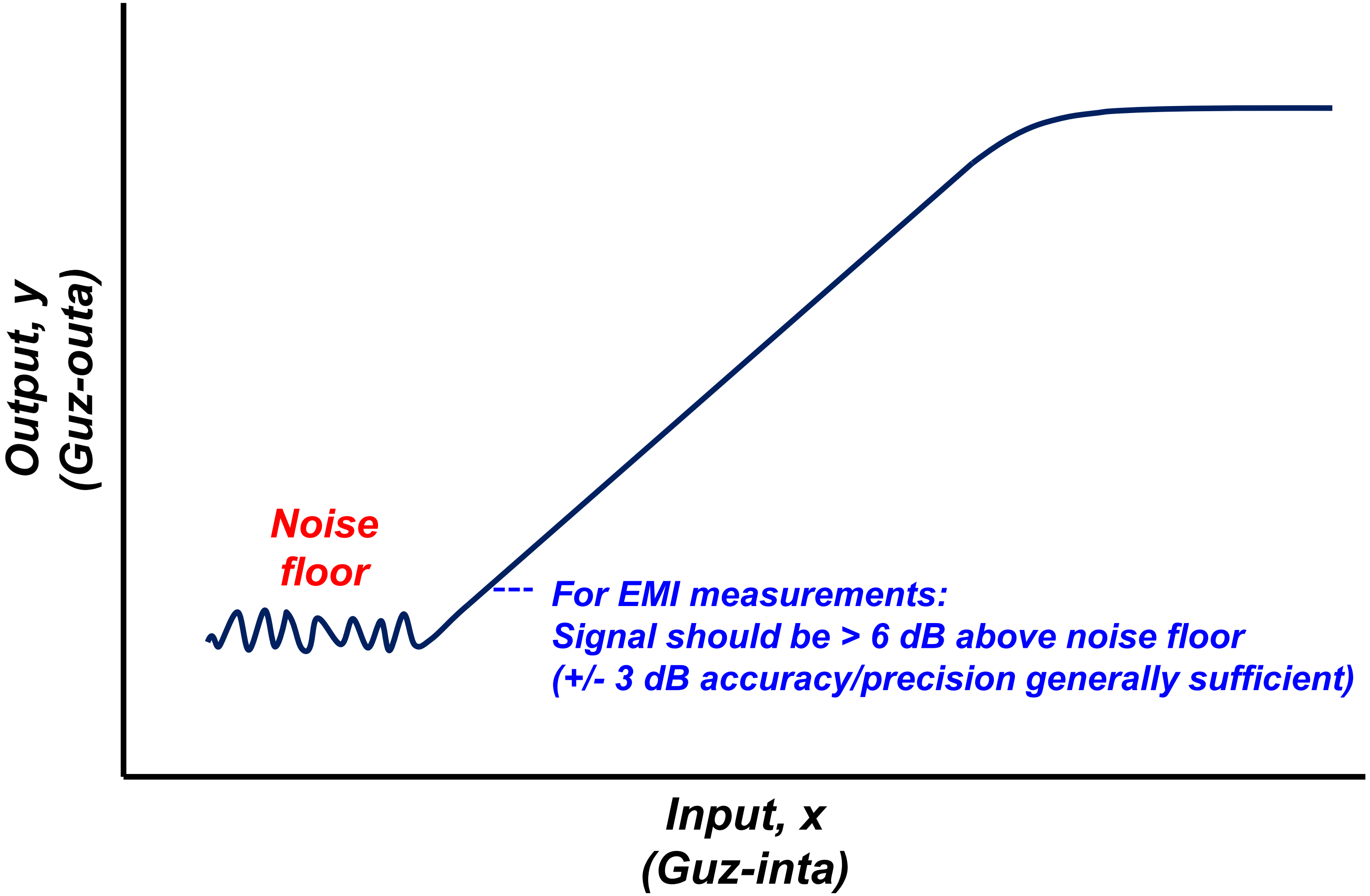
$$10\log_{10}1.01 \approx 0.04\,dB$$

- *Noise has 0.1x voltage/current level of signal*
- *Noise will increase total level by:*

$$\sqrt{1^2+10^2}=1.005$$

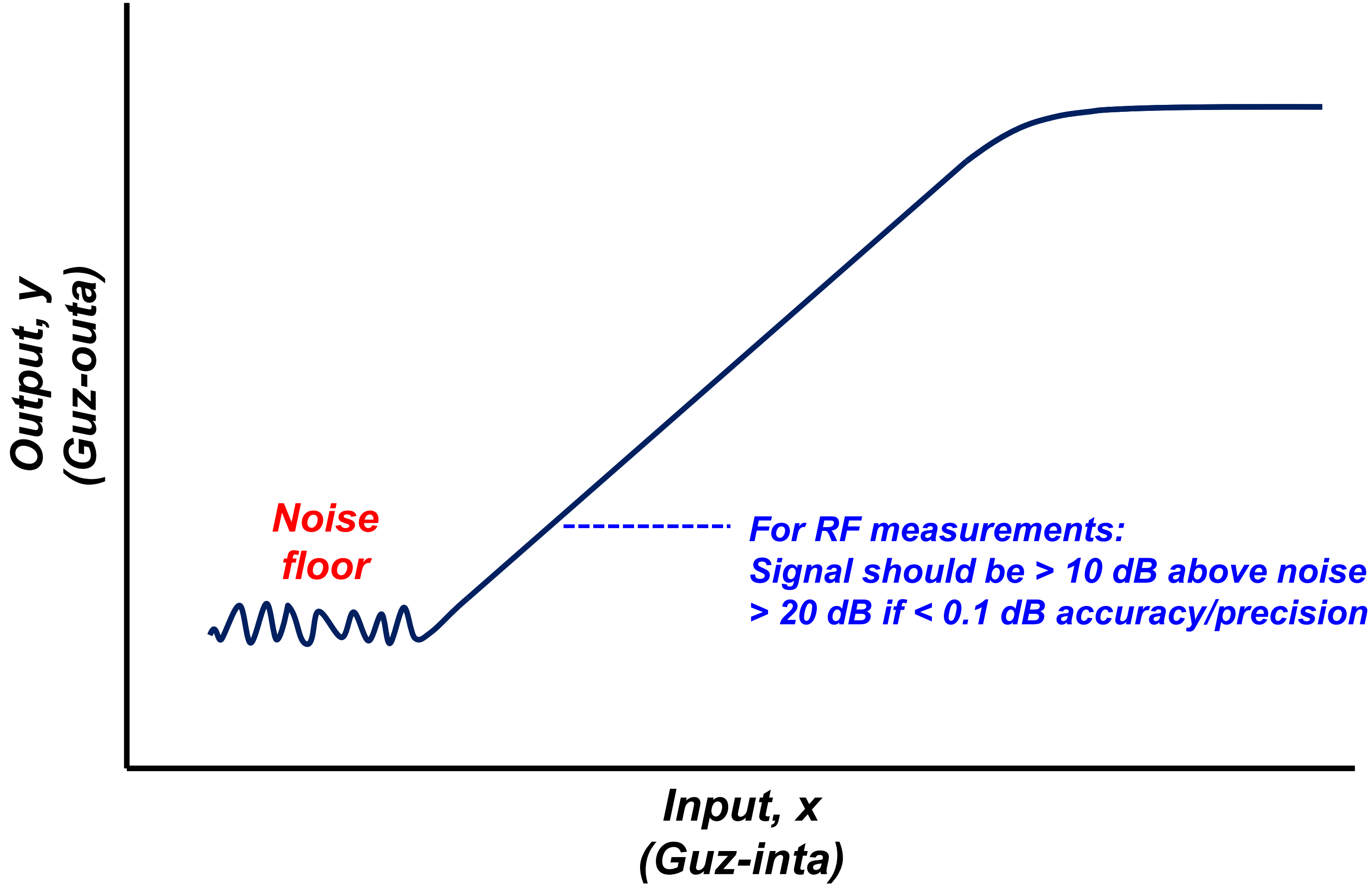
$$20\log_{10}1.005 \approx 0.04\,dB$$

Linearity and Dynamic Range (cont.)



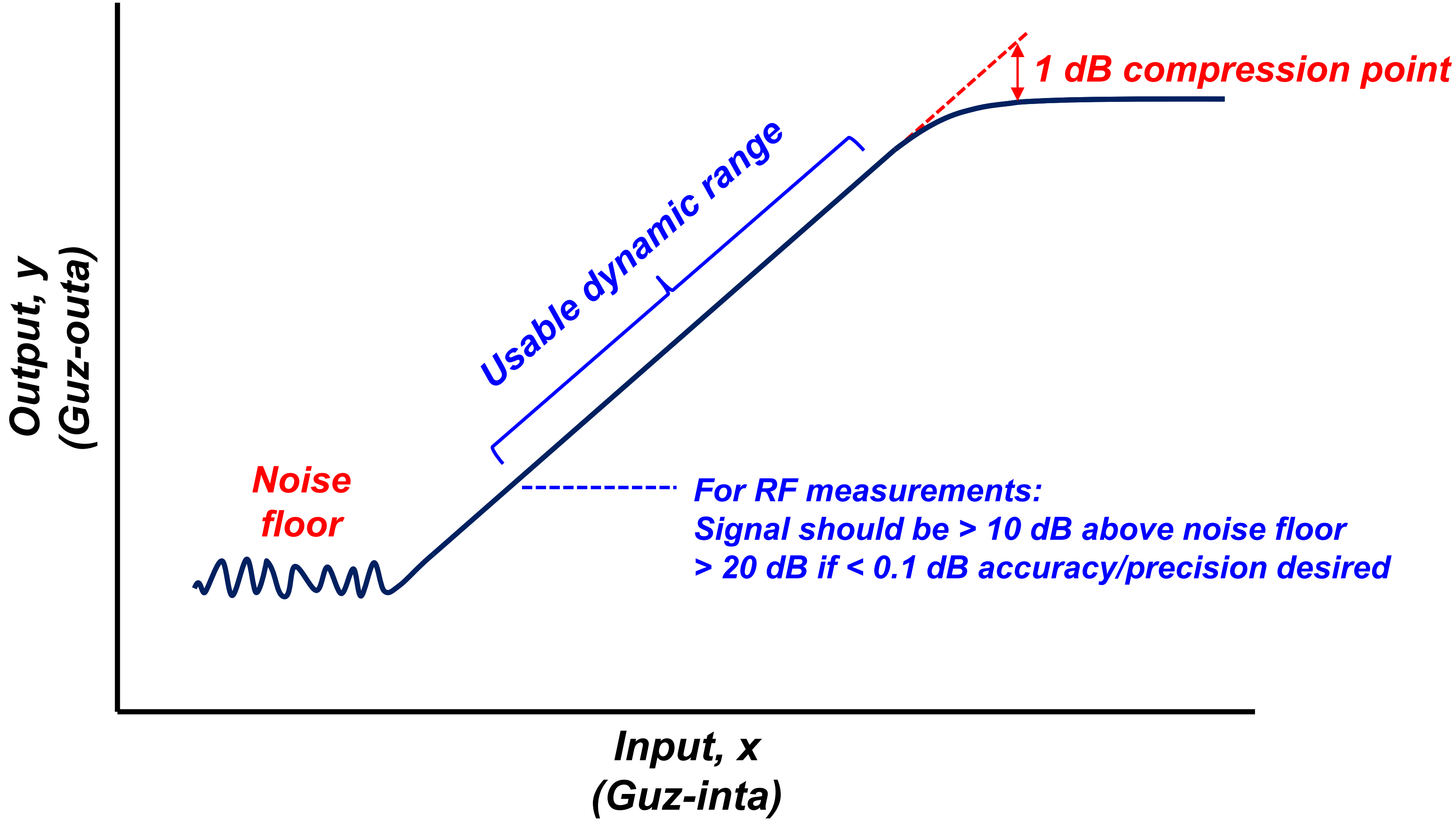
For EMI measurements:
Signal should be > 6 dB above noise floor
(+/- 3 dB accuracy/precision generally sufficient)

Linearity and Dynamic Range (cont.)



For RF measurements:
Signal should be > 10 dB above noise floor
> 20 dB if < 0.1 dB accuracy/precision desired

Linearity and Dynamic Range (cont.)



For RF measurements:
Signal should be > 10 dB above noise floor
> 20 dB if < 0.1 dB accuracy/precision desired

Thermal Noise (Power)

Theoretical noise power in Watts delivered by a thermal source into an impedance matched load:

$$P_N = k_B T B$$

k_B = Boltzmann's constant = 1.38×10^{-23} J/K

T = temperature, K

B = bandwidth, Hz

$$W = \frac{J}{sec} \qquad J = \frac{W}{Hz}$$

For T = 290 K:

$$P_N = \underbrace{\left(4 \times 10^{-21} \; \frac{W}{Hz}\right)}_{\textit{Power spectral density}} B$$

$$\begin{aligned} 10^{-21}\,W &= -210\;dBW \\ &= -180\;dB\text{m} \\ 4\;numeric &= \quad +6\;dB\;(power) \end{aligned}$$

Converting to dBm: $(P_N)_{dB} = -174\;dBm/Hz + 10\log_{10} B$

Thermal Noise (Voltage)

Theoretical noise voltage delivered by a thermal source into an impedance matched load:

$$P_N = \frac{(V_N)^2}{R} \qquad V_N = \sqrt{k_B T B R}$$

For T = 290 K: $V_N = \left(6.33 \times 10^{-11} \sqrt{\frac{W}{Hz}}\right)\sqrt{BR}$

R = 50 Ω (50 Ω system - typical for RF receiver inputs): $V_N = \underbrace{\left(4.48 \times 10^{-10} \frac{V}{\sqrt{Hz}}\right)}_{\text{Noise spectral density}}\sqrt{B}$

Converting to dBμV: $(V_N)_{dB} = -67\ dB\mu V/\sqrt{Hz} + 10\log_{10} B$

↓ *- 107 for 50 Ω system*

$$(P_N)_{dB} = -174\ dBm/Hz + 10\log_{10} B$$

Remember these

Thermal Noise into Open Circuit

You will sometimes see...

Theoretical noise delivered by a thermal source into an open circuit:

$$(P_N)_{OC} = 4k_B TB \qquad (V_N)_{OC} = \sqrt{4k_B TBR}$$

$$= 2(V_N)_{matched}$$

We will mostly be dealing with the previous expressions for an impedance matched source in a 50 Ω system, but you'll need to recognize these expressions when you see them

Noise Factor and Noise Figure

Noise factor (F)

Numeric (non-dB) ratio of device's actual output noise floor to theoretical thermal noise floor limit, with both quantities expressed in terms of power:

$$F = \frac{Noise\ floor\ power}{k_B T B}$$

Noise Figure (NF)

Noise factor expressed in dB:

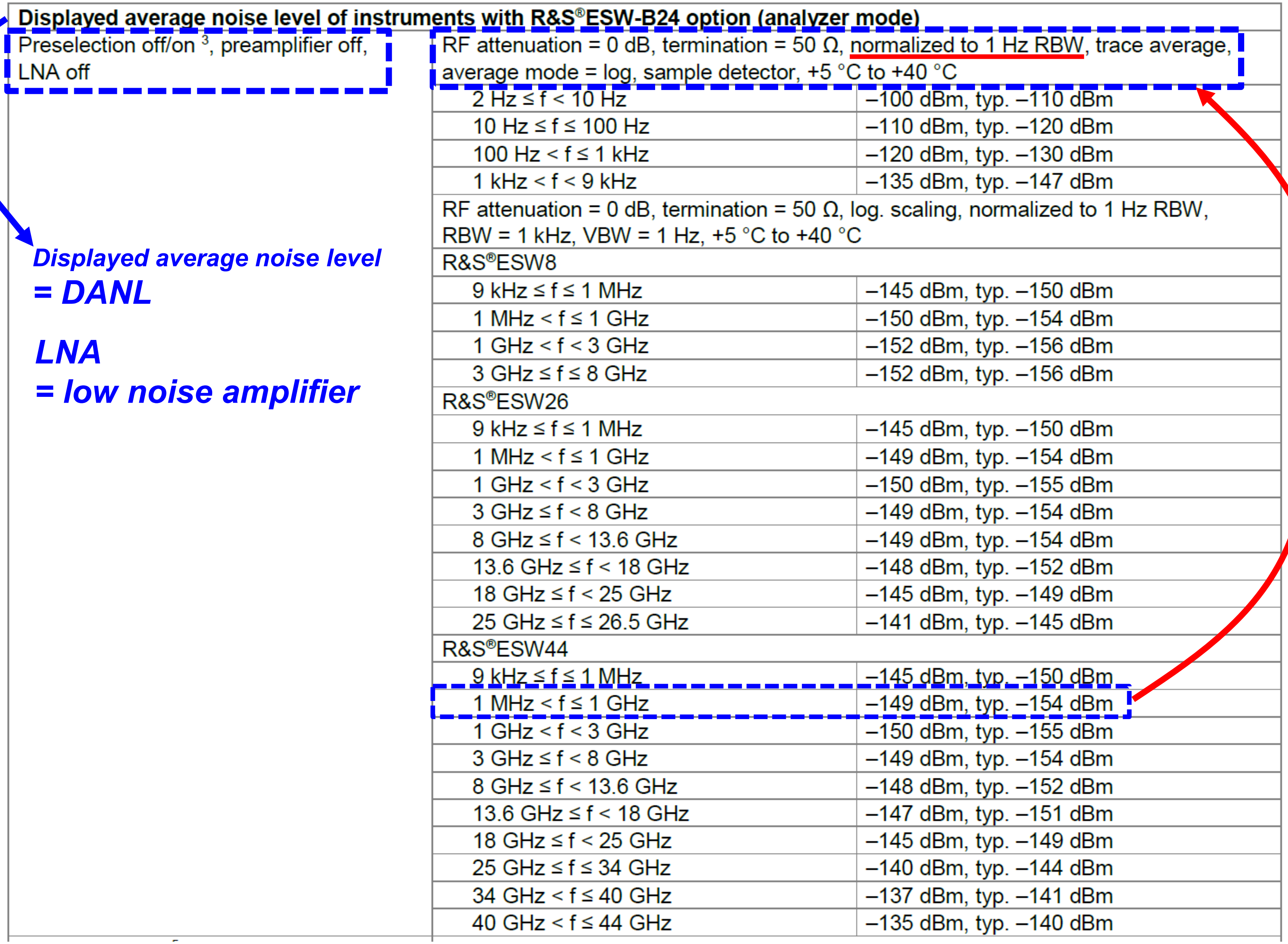
$$NF = 10 \log_{10} F$$

Noise floor (power): $P_{NF} = -174\ dB\,m/H\,z + 10log_{10}B + NF$

Noise floor (voltage): $V_{NF} = -67\ dB\mu\,V/\sqrt{Hz} + 10log_{10}B + NF$

For good low noise amplifier: $NF \leq 2\ dB$

Excerpt from R&S ESW Datasheet



Displayed average noise level of instruments with R&S®ESW-B24 option (analyzer mode)		
Preselection off/on[3], preamplifier off, LNA off	RF attenuation = 0 dB, termination = 50 Ω, normalized to 1 Hz RBW, trace average, average mode = log, sample detector, +5 °C to +40 °C	
	2 Hz ≤ f < 10 Hz	–100 dBm, typ. –110 dBm
	10 Hz ≤ f ≤ 100 Hz	–110 dBm, typ. –120 dBm
	100 Hz < f ≤ 1 kHz	–120 dBm, typ. –130 dBm
	1 kHz < f < 9 kHz	–135 dBm, typ. –147 dBm
	RF attenuation = 0 dB, termination = 50 Ω, log. scaling, normalized to 1 Hz RBW, RBW = 1 kHz, VBW = 1 Hz, +5 °C to +40 °C	
	R&S®ESW8	
	9 kHz ≤ f ≤ 1 MHz	–145 dBm, typ. –150 dBm
	1 MHz < f ≤ 1 GHz	–150 dBm, typ. –154 dBm
	1 GHz < f < 3 GHz	–152 dBm, typ. –156 dBm
	3 GHz ≤ f ≤ 8 GHz	–152 dBm, typ. –156 dBm
	R&S®ESW26	
	9 kHz ≤ f ≤ 1 MHz	–145 dBm, typ. –150 dBm
	1 MHz < f ≤ 1 GHz	–149 dBm, typ. –154 dBm
	1 GHz < f < 3 GHz	–150 dBm, typ. –155 dBm
	3 GHz ≤ f < 8 GHz	–149 dBm, typ. –154 dBm
	8 GHz ≤ f < 13.6 GHz	–149 dBm, typ. –154 dBm
	13.6 GHz ≤ f < 18 GHz	–148 dBm, typ. –152 dBm
	18 GHz ≤ f < 25 GHz	–145 dBm, typ. –149 dBm
	25 GHz ≤ f ≤ 26.5 GHz	–141 dBm, typ. –145 dBm
	R&S®ESW44	
	9 kHz ≤ f ≤ 1 MHz	–145 dBm, typ. –150 dBm
	1 MHz < f ≤ 1 GHz	–149 dBm, typ. –154 dBm
	1 GHz < f < 3 GHz	–150 dBm, typ. –155 dBm
	3 GHz ≤ f < 8 GHz	–149 dBm, typ. –154 dBm
	8 GHz ≤ f < 13.6 GHz	–148 dBm, typ. –152 dBm
	13.6 GHz ≤ f < 18 GHz	–147 dBm, typ. –151 dBm
	18 GHz ≤ f < 25 GHz	–145 dBm, typ. –149 dBm
	25 GHz ≤ f ≤ 34 GHz	–140 dBm, typ. –144 dBm
	34 GHz < f ≤ 40 GHz	–137 dBm, typ. –141 dBm
	40 GHz < f ≤ 44 GHz	–135 dBm, typ. –140 dBm

Displayed average noise level = DANL

LNA = low noise amplifier

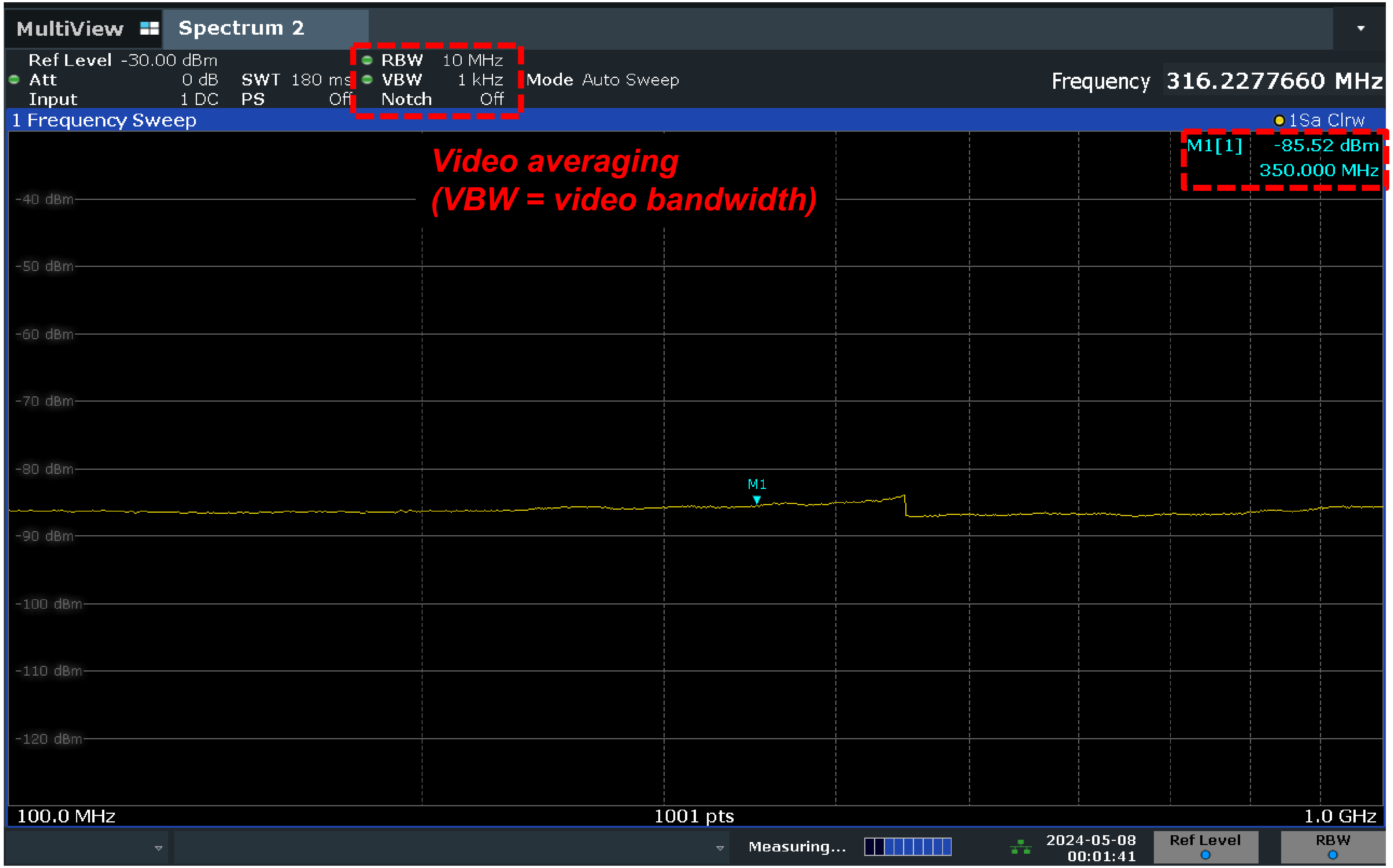
RBW = resolution bandwidth

per Hz

$+10\log_{10} B$

NF ≈ 20-25 dB (not great...)

Noise Floor, Video Averaging, 10 MHz RBW



MultiView Spectrum 2

Ref Level -30.00 dBm
Att 0 dB SWT 180 ms
Input 1 DC PS Off

RBW 10 MHz
VBW 1 kHz
Notch Off

Mode Auto Sweep

Frequency 316.2277660 MHz

1 Frequency Sweep

1Sa Clrw

M1[1] -85.52 dBm
350.000 MHz

Video averaging
(VBW = video bandwidth)

Measuring...

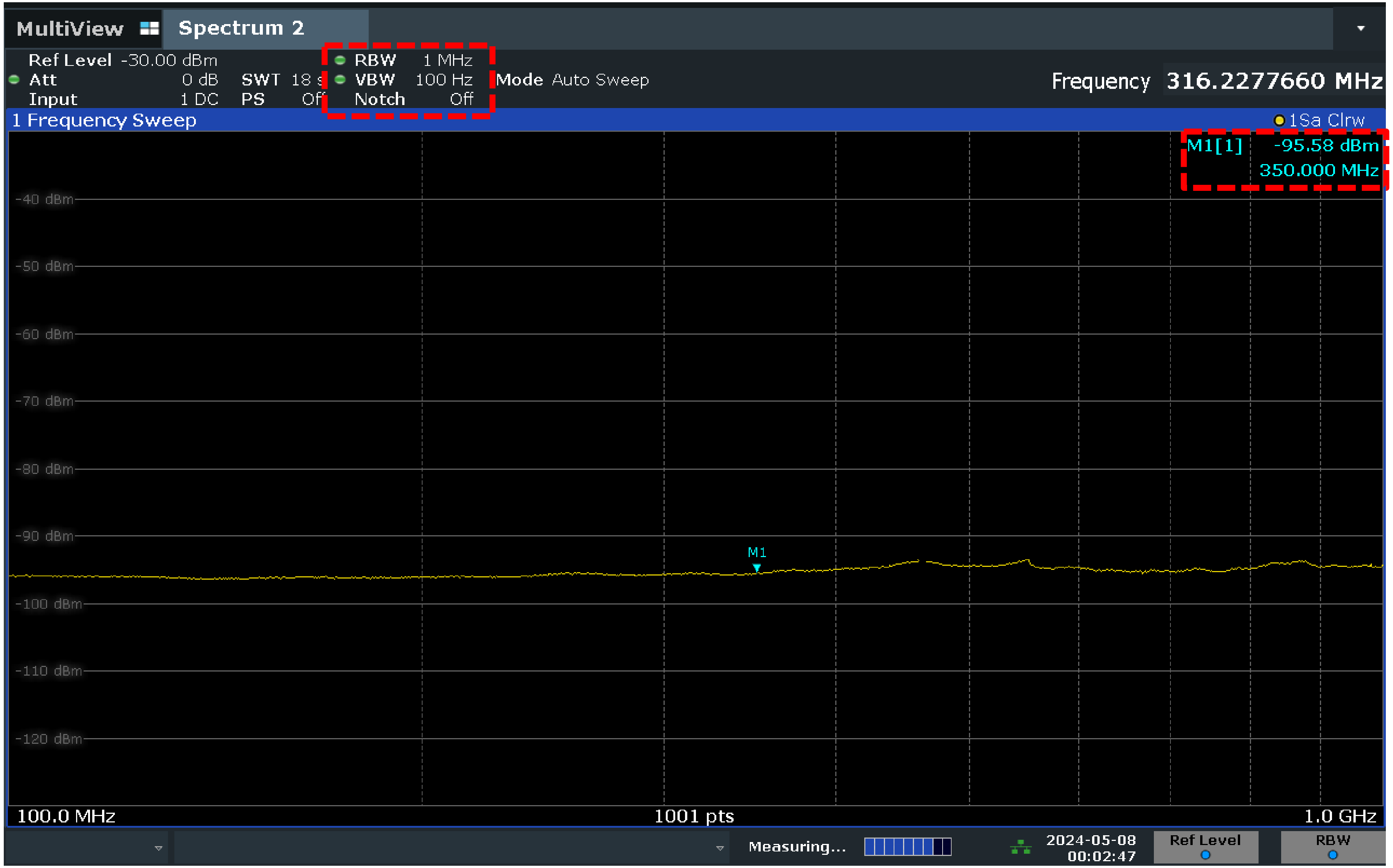
2024-05-08
00:01:41

$10\ MHz = 70\ dBHz$

$-154\ dBm/Hz$
$+70\ dBHz$
$= -84\ dBm$

12:01:41 AM 05/08/2024

Noise Floor, Video Averaging, 1 MHz RBW



Ref Level -30.00 dBm
Att 0 dB SWT 18 s
Input 1 DC PS Off
RBW 1 MHz
VBW 100 Hz
Notch Off
Mode Auto Sweep
Frequency 316.2277660 MHz

1 Frequency Sweep
1Sa Clrw

M1[1] -95.58 dBm
350.000 MHz

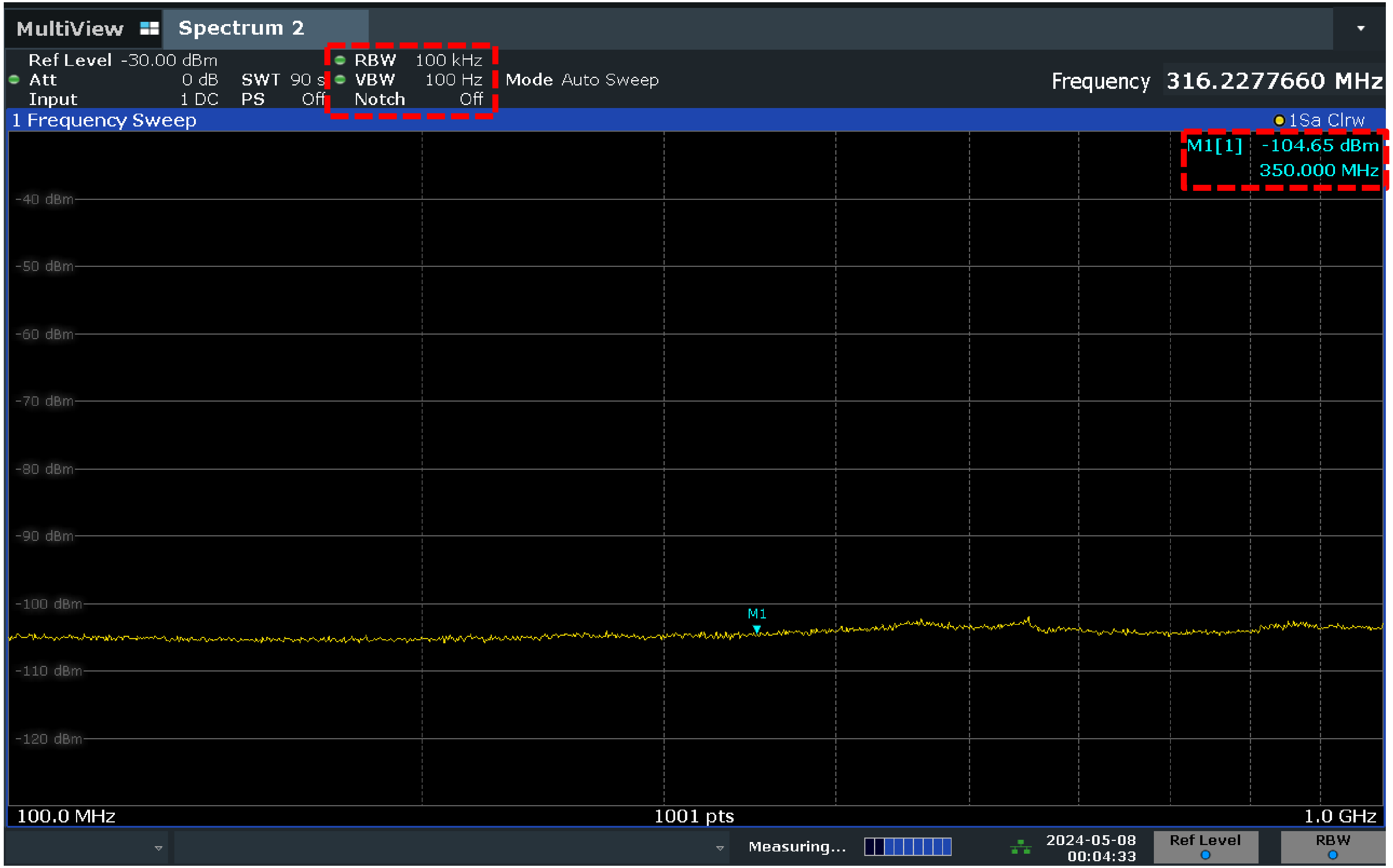
2024-05-08
00:02:47

12:02:48 AM 05/08/2024

$$1\ MHz = 60\ dBHz$$

$$-154\ dBm/Hz$$
$$+60\ dBHz$$
$$= -94\ dBm$$

Noise Floor, Video Averaging, 100 kHz RBW



Ref Level -30.00 dBm

Att 0 dB SWT 90 s

Input 1 DC PS Off

RBW 100 kHz

VBW 100 Hz

Notch Off

Mode Auto Sweep

Frequency 316.2277660 MHz

1 Frequency Sweep

1Sa Clrw

M1[1] -104.65 dBm 350.000 MHz

Measuring...

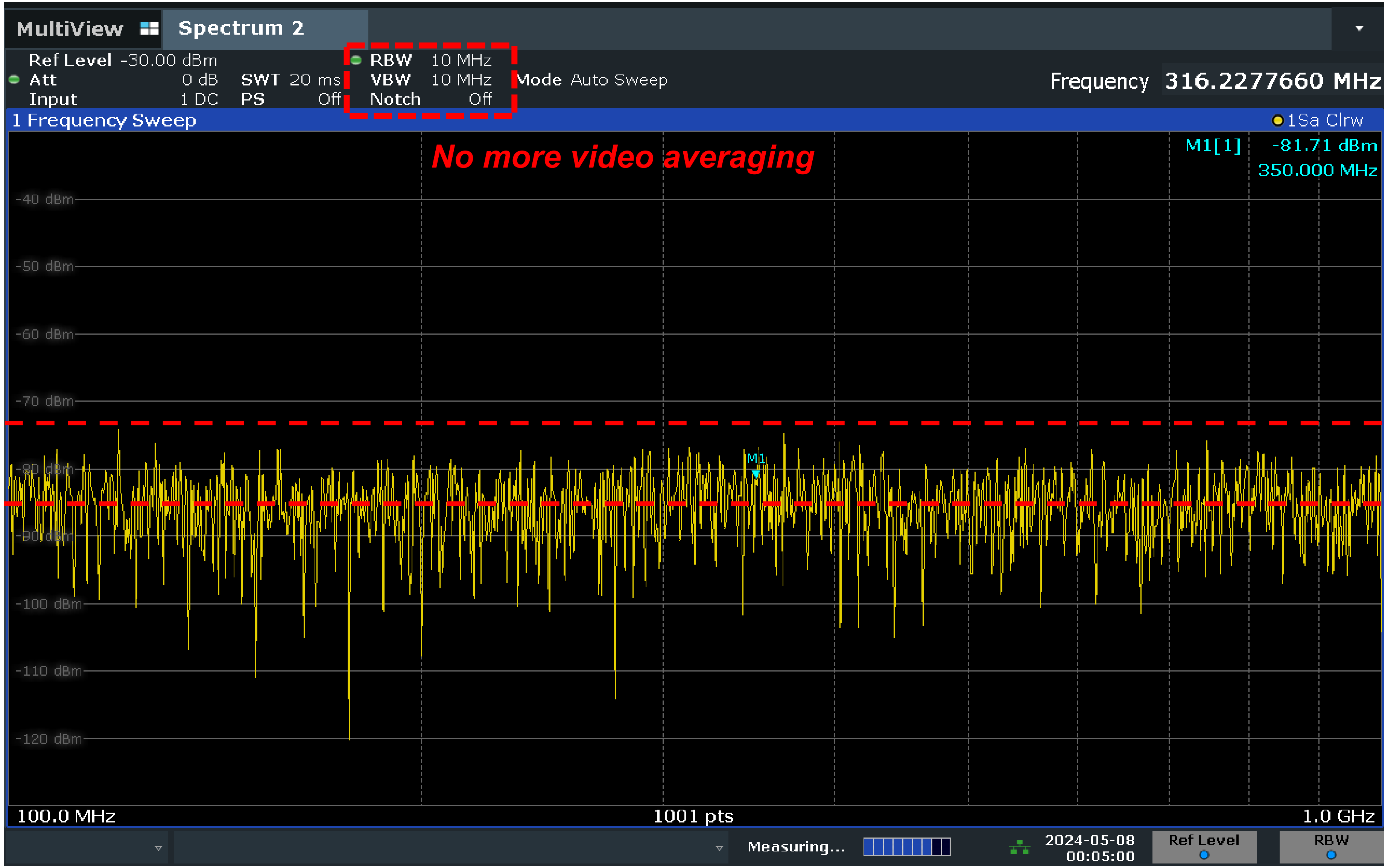
2024-05-08 00:04:33

$100\ kHz =$
$50\ dBHz$

$-154\ dBm/Hz$
$+50\ dBHz$
$= -104\ dBm$

12:04:34 AM 05/08/2024

But noise doesn't look like that in real life...



MultiView Spectrum 2

Ref Level -30.00 dBm
Att 0 dB SWT 20 ms
Input 1 DC PS Off

RBW 10 MHz
VBW 10 MHz
Notch Off

Mode Auto Sweep

Frequency 316.2277660 MHz

1 Frequency Sweep 1Sa Clrw

M1[1] -81.71 dBm
350.000 MHz

No more video averaging

Peak

"Average"

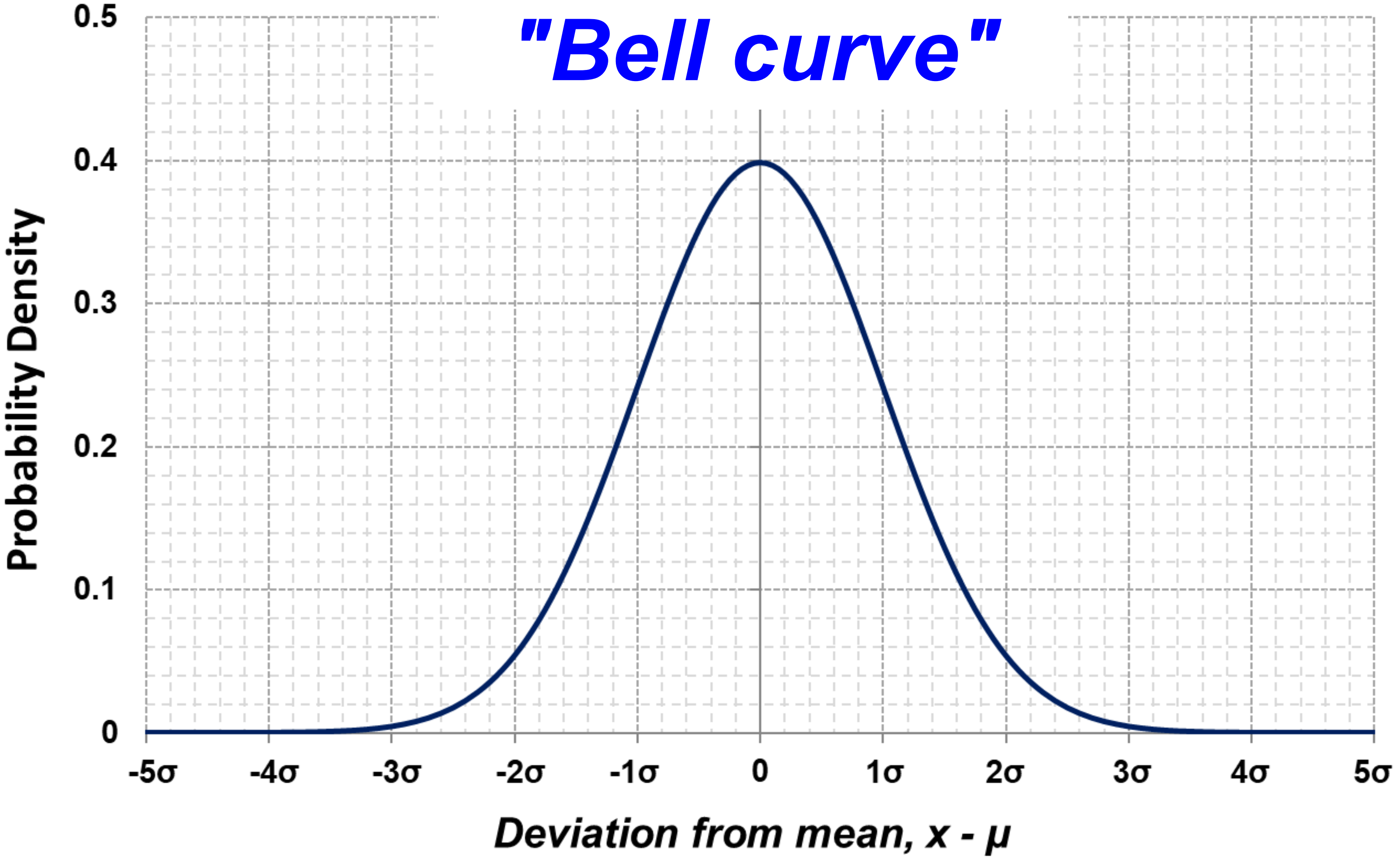
12:05:01 AM 05/08/2024

Gaussian (Normal) Distribution

Thermal noise follows Gaussian (normal) probability distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ = mean
σ = standard deviation



Mean and Standard Deviation

Sample set (N samples) $x_1, x_2, x_3, \ldots x_N$

Mean (arithmetic) $$\mu = \frac{1}{N}\sum_{n=1}^{N} x_n$$

Sample deviation from mean $x_n - \mu$

(Sample deviation)² $(x_n - \mu)^2$

Variance (Mean of deviation²) $$\sigma^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)^2$$

Standard deviation $$\sigma = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)^2}$$

Sounds a bit like root mean square (rms), n'est-ce pas?

For thermal noise: $$\sigma = (V_N)_{rms}$$
$$= \sqrt{k_B TBR}$$ *(minimum)*

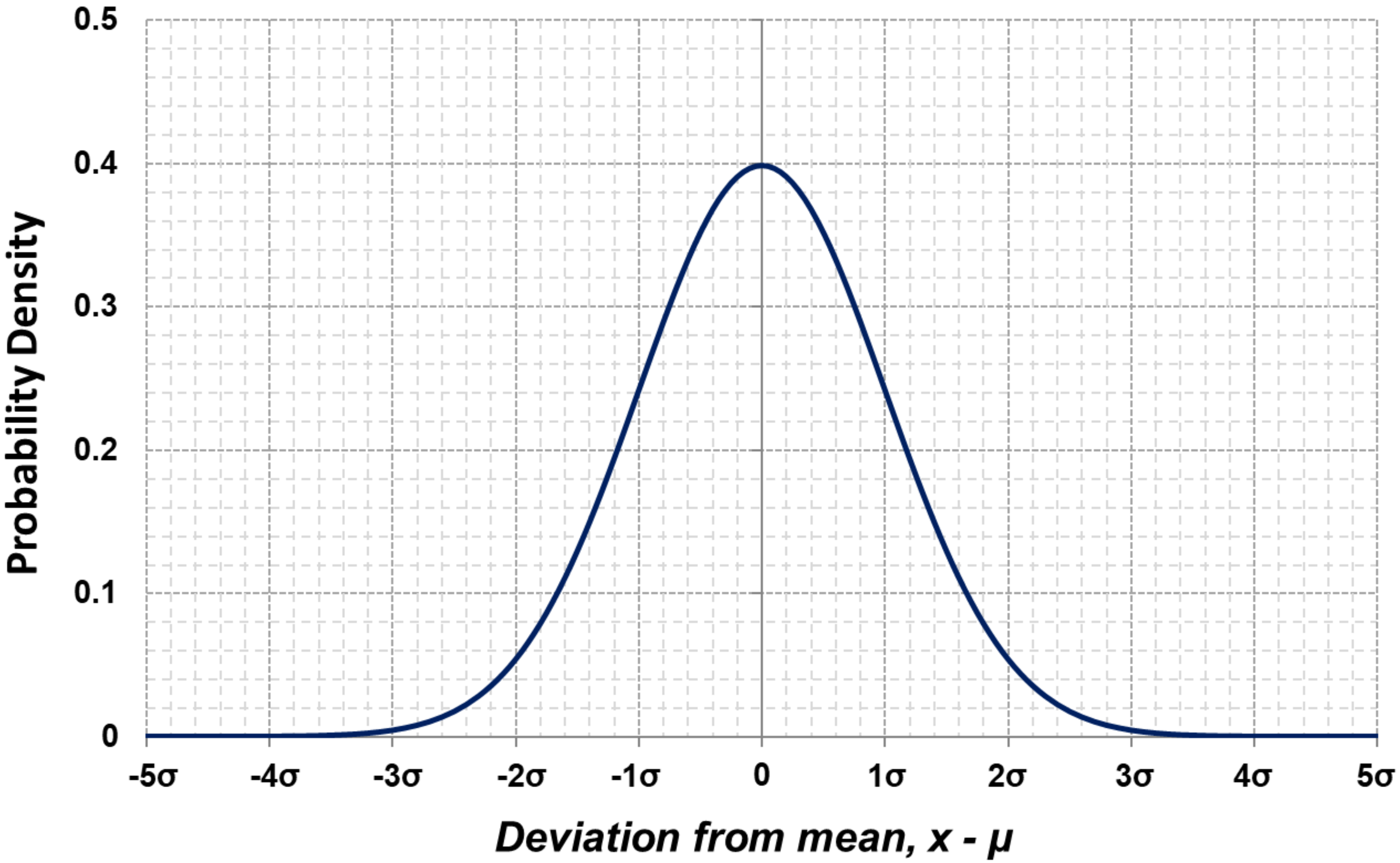
Probability of Occurrence

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Area under curve gives probability of occurrence in given range

Integral not possible in closed form

Must be taken numerically

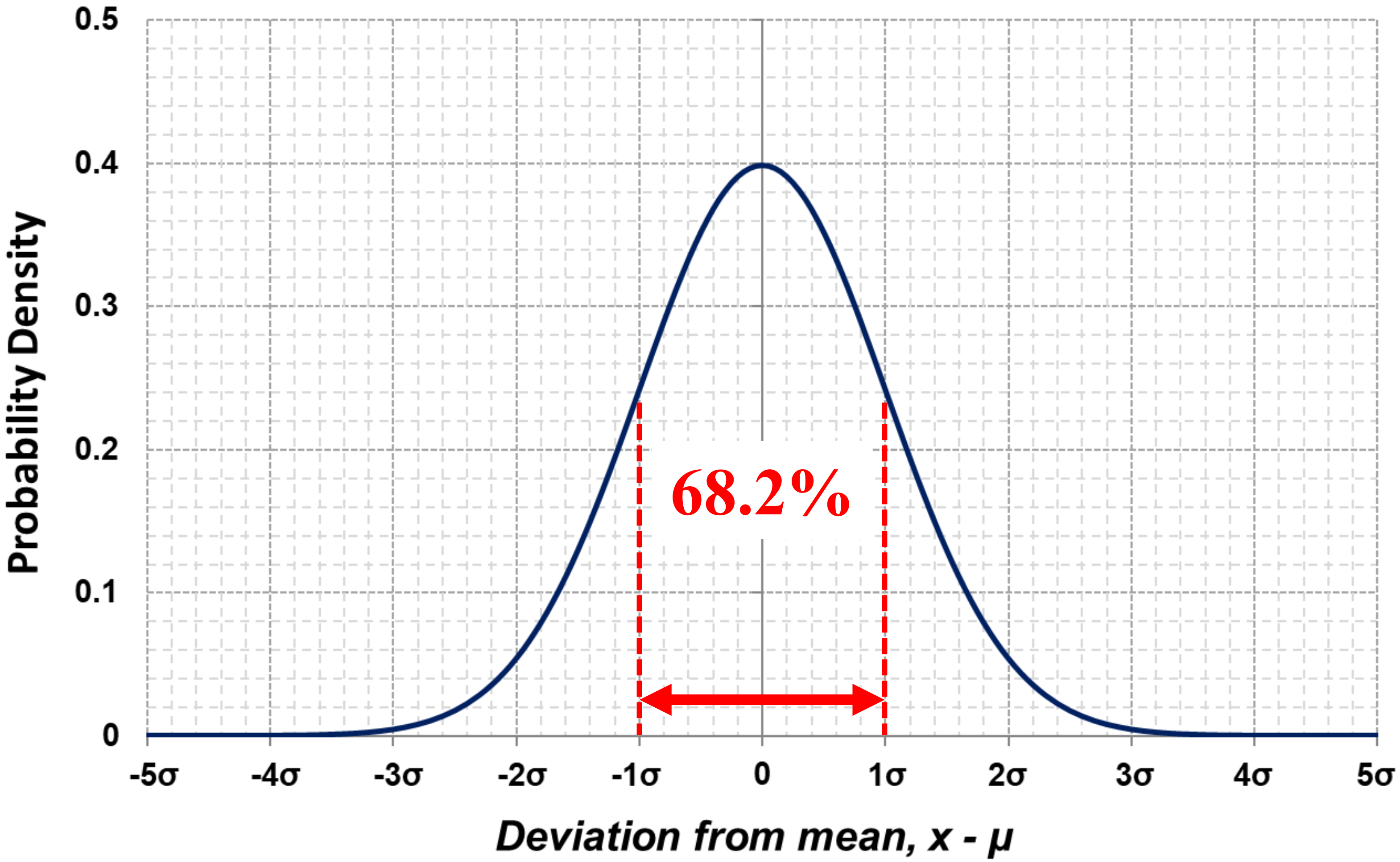


Probability of Occurrence

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Area under curve gives probability of occurrence in given range

$$|x-\mu| \leq 1\sigma: \ 68.2\%$$



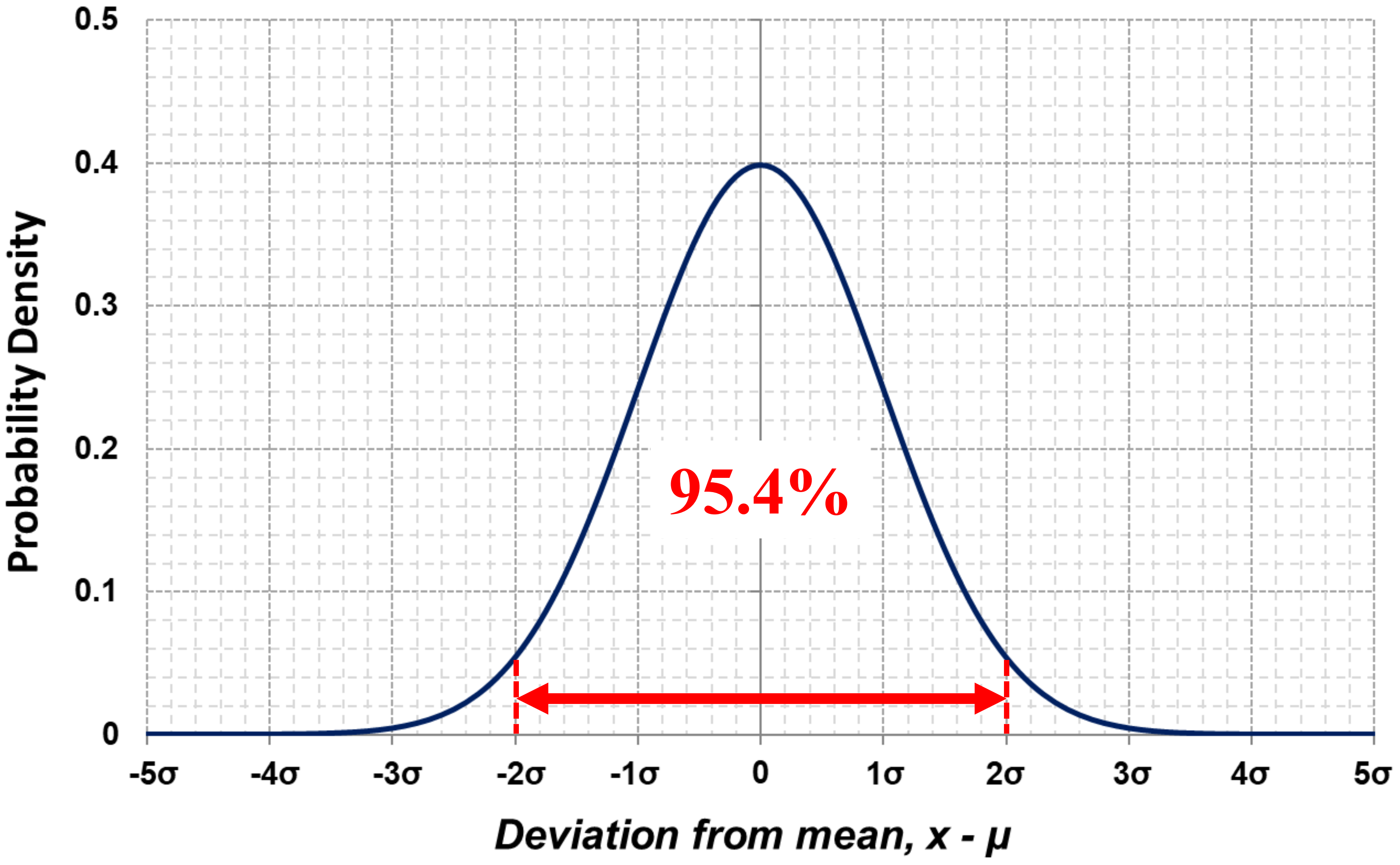
Probability of Occurrence (cont.)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Area under curve gives probability of occurrence in given range

$|x-\mu| \leq 1\sigma$: 68.2%

$|x-\mu| \leq 2\sigma$: 95.4%



Probability of Occurrence (cont.)

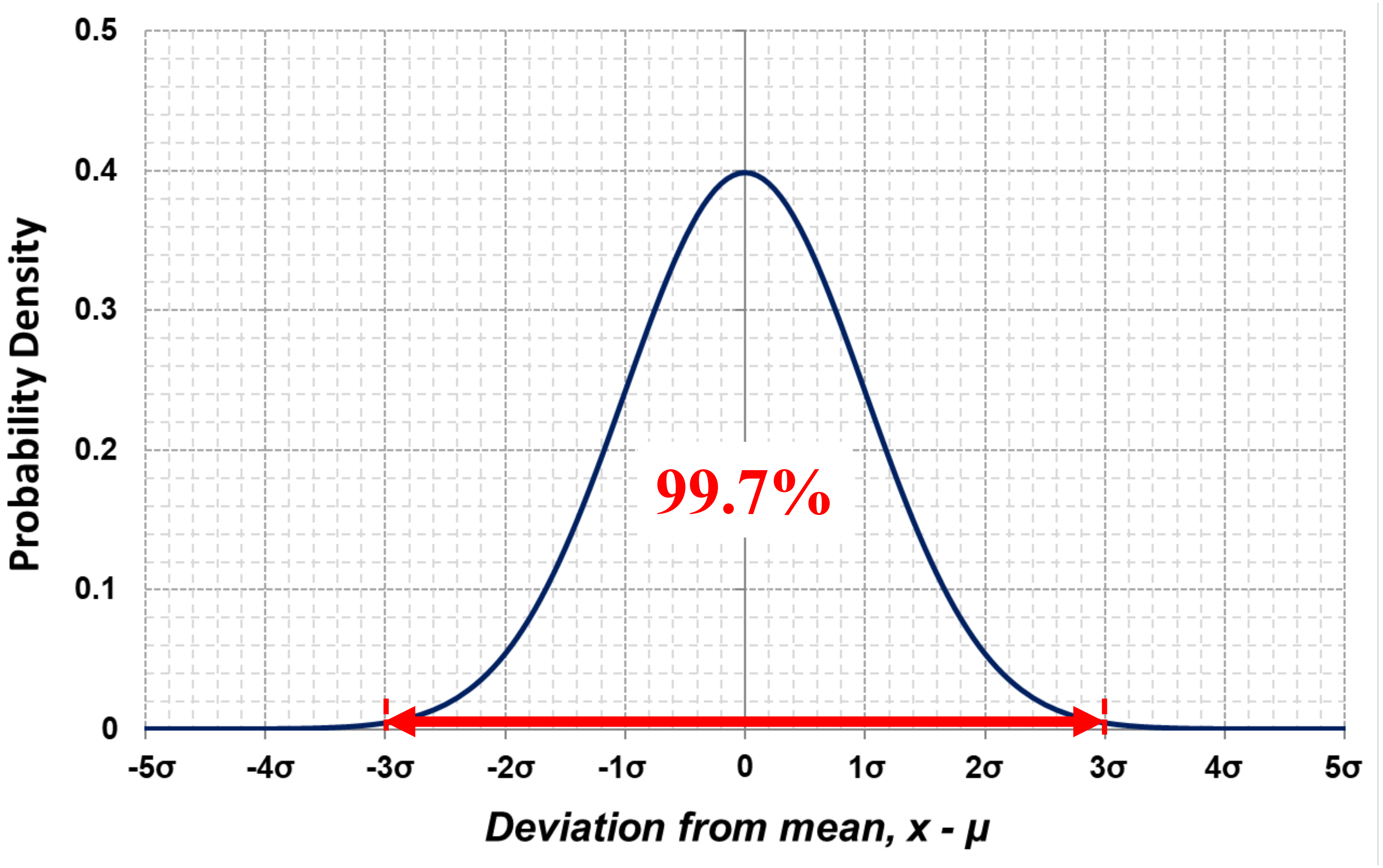
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Area under curve gives probability of occurrence in given range

$|x-\mu| \leq 1\sigma$: 68.2%

$|x-\mu| \leq 2\sigma$: 95.4%

$|x-\mu| \leq 3\sigma$: 99.7%



Probability of Occurrence (cont.)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Area under curve gives probability of occurrence in given range

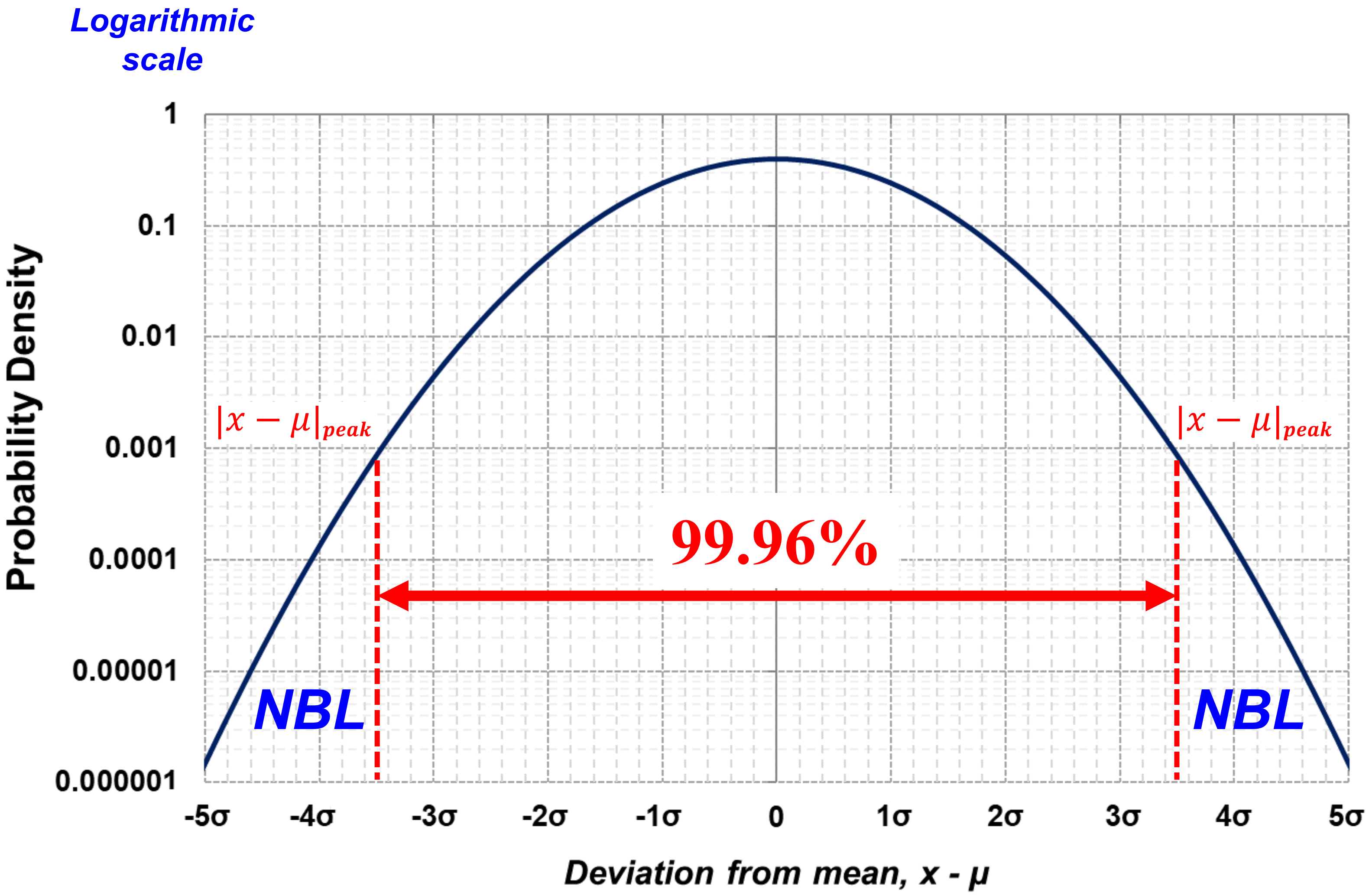
$|x-\mu| \leq 3.5\sigma$: 99.96%

$|x-\mu| \geq 3.5\sigma$:

...NOT BLOODY LIKELY (NBL)

For all practical purposes:

$$|x-\mu|_{peak} \approx 3.5\sigma$$



Thermal Noise (cont.)

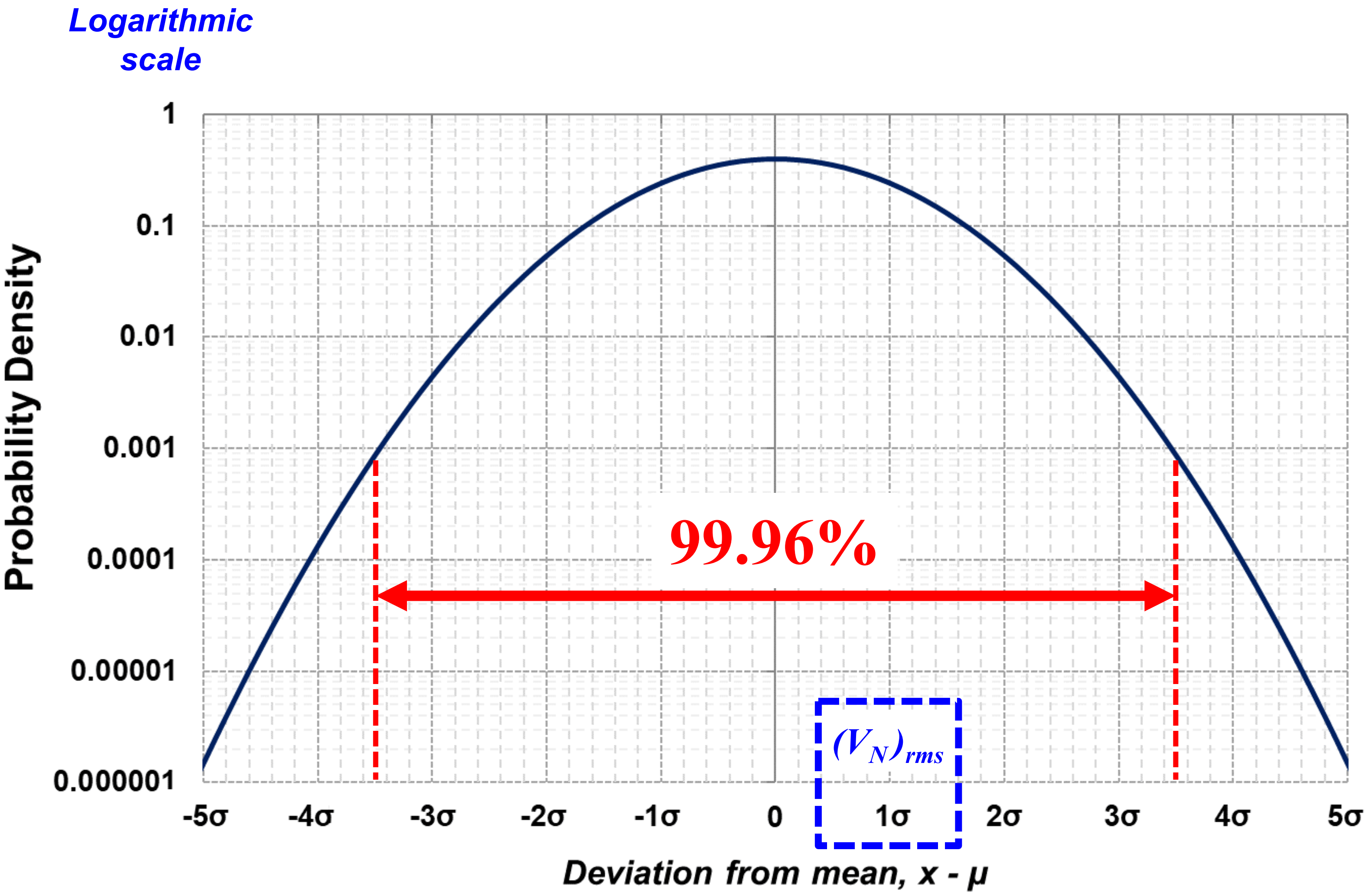
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Recall that σ is defined as the root mean square (rms) of the distribution:

$$\sigma = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)^2}$$

For thermal noise:

$$\sigma = (V_N)_{rms}$$



Back to Thermal Noise...

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

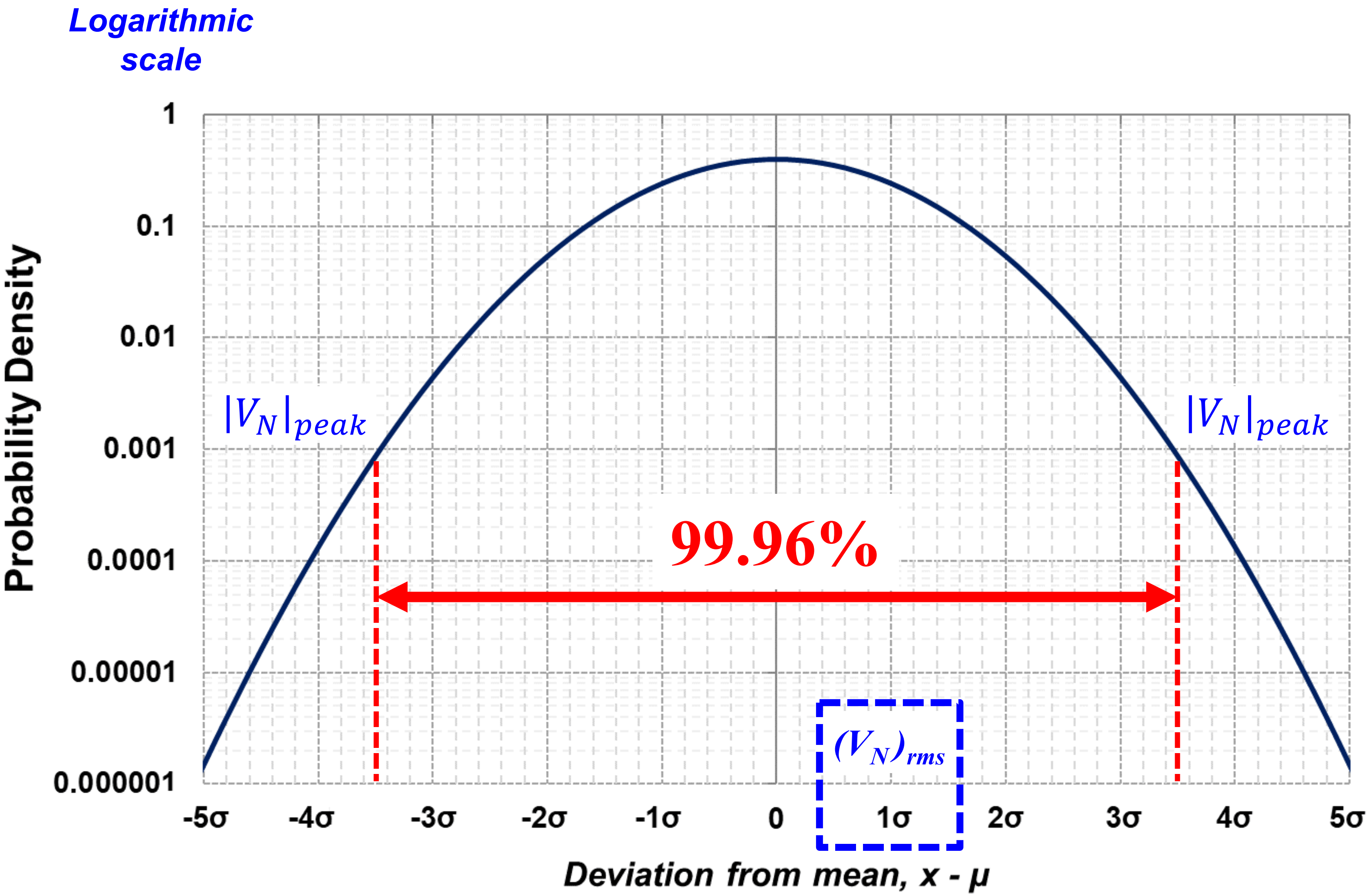
$$\frac{|V_N|_{peak}}{(V_N)_{rms}} \approx 3.5$$

$$20\log_{10}(3.5) \approx \underline{11\ dB}$$

Crest Factor for Thermal Noise

Noise floor peak will be ~11 dB higher than RMS value

(MIL-STD-461G requires peak detector)



More about standard deviation and rms...

- **"How Standard Deviation Relates to Root-Mean-Square Values"**
  - All About Circuits, July 28, 2020
  - *https://www.allaboutcircuits.com/technical-articles/how-standard-deviations-relates-rms-values/*

Averaging and Root Mean Square

Linear average (arithmetic mean)

$$\mu = \frac{1}{N}\sum_{n=1}^{N} x_n$$

$$\frac{1}{N}\sum_{n=1}^{N} \log A_n = \log(A_1 A_2 \ldots A_N)^{\frac{1}{N}} = \log\Big(\underbrace{\sqrt[N]{A_1 A_2 \ldots A_N}}_{\text{Geometric mean of numeric values}}\Big)$$

Logarithmic average (arithmetic mean of logarithmic values)

Root mean square

$$rms = \sqrt{\frac{1}{N}\sum_{n=1}^{N} (x_n)^2}$$

Averaging and Root Mean Square (cont.)

For close distribution of numbers, arithmetic mean, geometric mean, and rms are in close agreement

Example 1: 4, 5, 6

Arithmetic mean = 5

Geometric mean = 4.93

rms = 5.07

$$\frac{5.07}{4.93} = 1.028 = 0.24\ dB$$

Values agree to within 3%

For a wider distribution, linear and logarithmic/geometric means are further apart

Example 2: 1, 2, 3 ... 10

Arithmetic mean = 5.5

Geometric mean = 4.52

rms = 6.2

$$\frac{5.5}{4.52} = 1.22 = 1.7\ dB$$

Arithmetic mean is 1.7 dB higher than geometric mean (this example)

$$\frac{6.2}{5.5} = 1.13 = 1.05\ dB$$

rms is 1.05 higher than arithmetic mean (this example)

Averaging and Root Mean Square (cont.)

From Rohde & Schwarz educational note:
"Measuring with Modern Spectrum Analyzers," Feb. 2013, page 30:
https://scdn.rohde-schwarz.com/ur/pws/dl_downloads/dl_application/application_notes/1ma201_1/1MA201_9e_spectrum_analyzers_meas.pdf

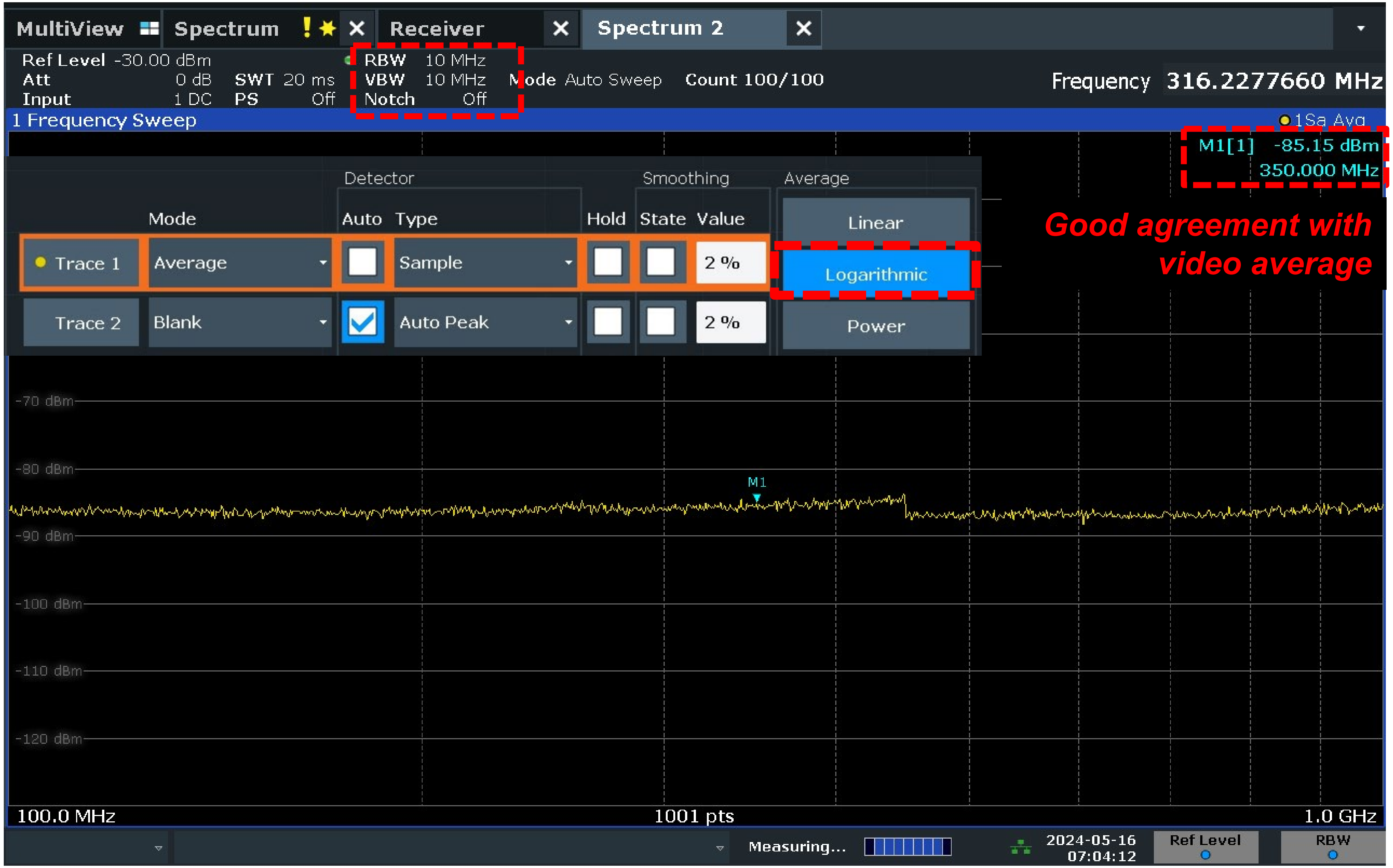
Sample detector:

Since it is always the case that only one sample is used at a defined time, the displayed trace varies due to the distribution of the instantaneous value around the average value for the envelope of the IF signal that results from the noise.

In the case of Gaussian noise, this average value is 1.05 dB below the RMS value (also: using a narrow video bandwidth in the logarithmic scale results in display values that are lower by an additional 1.45 dB).

Thus, the displayed noise is a total of 2.5 dB below the RMS value.

Noise Floor, Sample Average - Logarithmic, 10 MHz RBW



Ref Level -30.00 dBm
Att 0 dB SWT 20 ms
Input 1 DC PS Off
RBW 10 MHz
VBW 10 MHz
Notch Off
Mode Auto Sweep
Count 100/100
Frequency 316.2277660 MHz

M1[1] -85.15 dBm
350.000 MHz

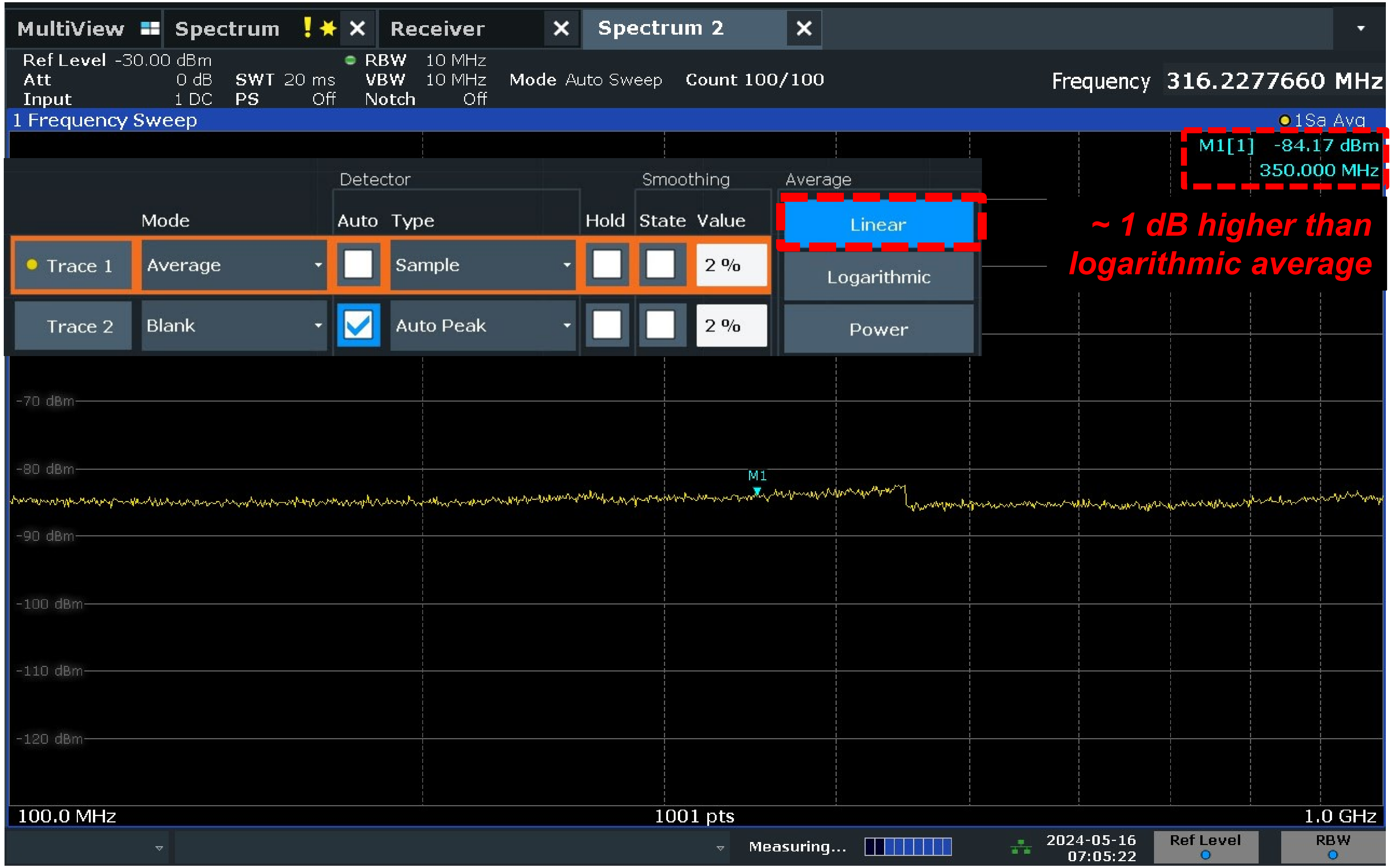
Mode		Auto	Type	Hold	State	Value
● Trace 1	Average	☐	Sample	☐	☐	2 %
Trace 2	Blank	☑	Auto Peak	☐	☐	2 %

2024-05-16
07:04:12

Good agreement with video average

07:04:13 AM 05/16/2024

Noise Floor, Sample Average - Linear, 10 MHz RBW



Ref Level -30.00 dBm
Att 0 dB
Input 1 DC
SWT 20 ms
PS Off
RBW 10 MHz
VBW 10 MHz
Notch Off
Mode Auto Sweep
Count 100/100
Frequency 316.2277660 MHz

M1[1] -84.17 dBm
350.000 MHz

		Detector			Smoothing	
	Mode	Auto	Type	Hold	State	Value
● Trace 1	Average	☐	Sample	☐	☐	2 %
Trace 2	Blank	☑	Auto Peak	☐	☐	2 %

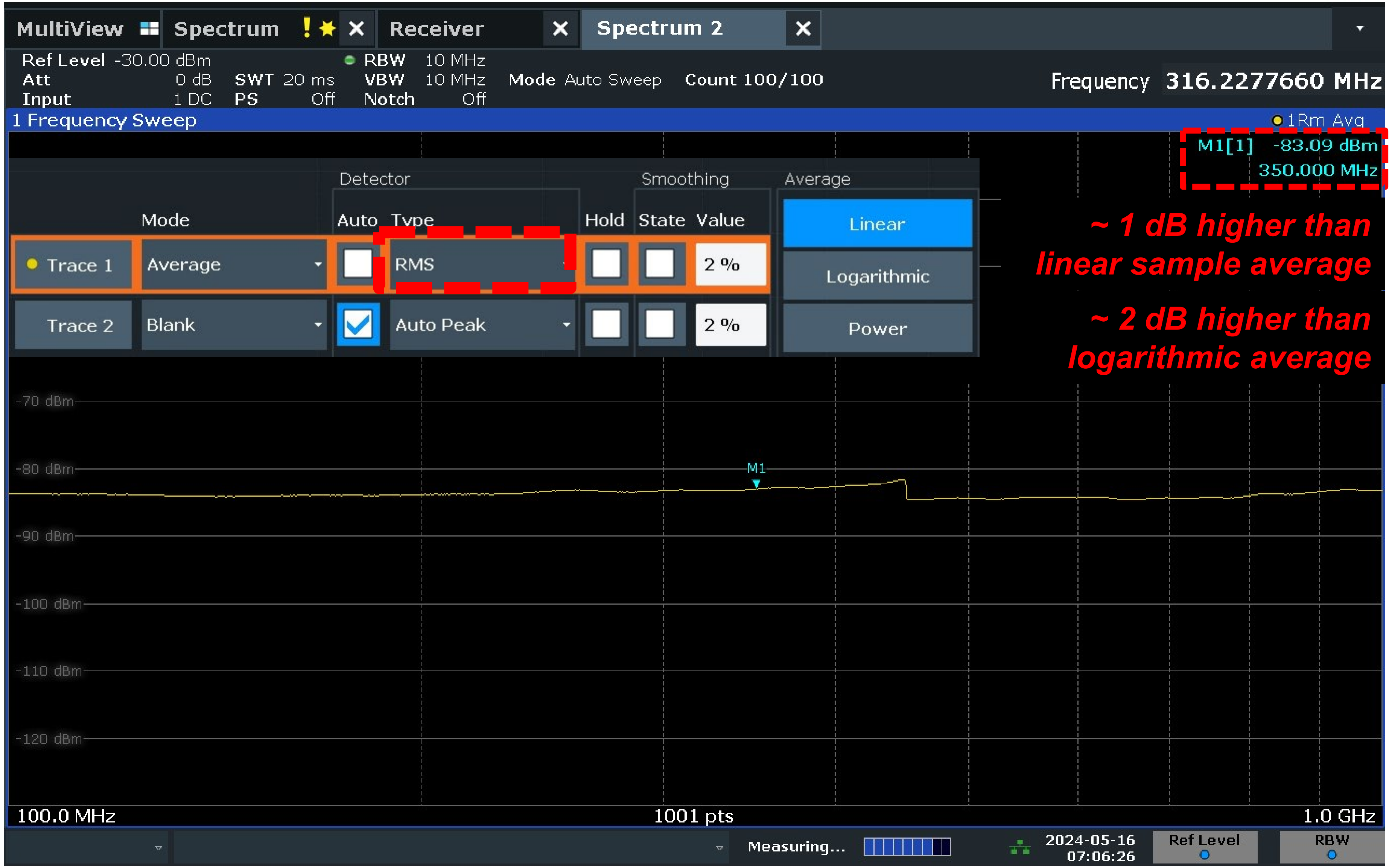
Average: Linear

~ 1 dB higher than logarithmic average

2024-05-16
07:05:22

07:05:23 AM 05/16/2024

Noise Floor, RMS Detect, 10 MHz RBW



Ref Level -30.00 dBm
Att 0 dB
Input 1 DC
SWT 20 ms
PS Off
RBW 10 MHz
VBW 10 MHz
Notch Off
Mode Auto Sweep
Count 100/100
Frequency 316.2277660 MHz

1 Frequency Sweep

Mode		Detector Auto	Detector Type	Smoothing Hold	Smoothing State	Smoothing Value
Trace 1	Average	☐	RMS	☐	☐	2 %
Trace 2	Blank	☑	Auto Peak	☐	☐	2 %

Average: Linear / Logarithmic / Power

M1[1] -83.09 dBm
350.000 MHz

100.0 MHz 1001 pts 1.0 GHz

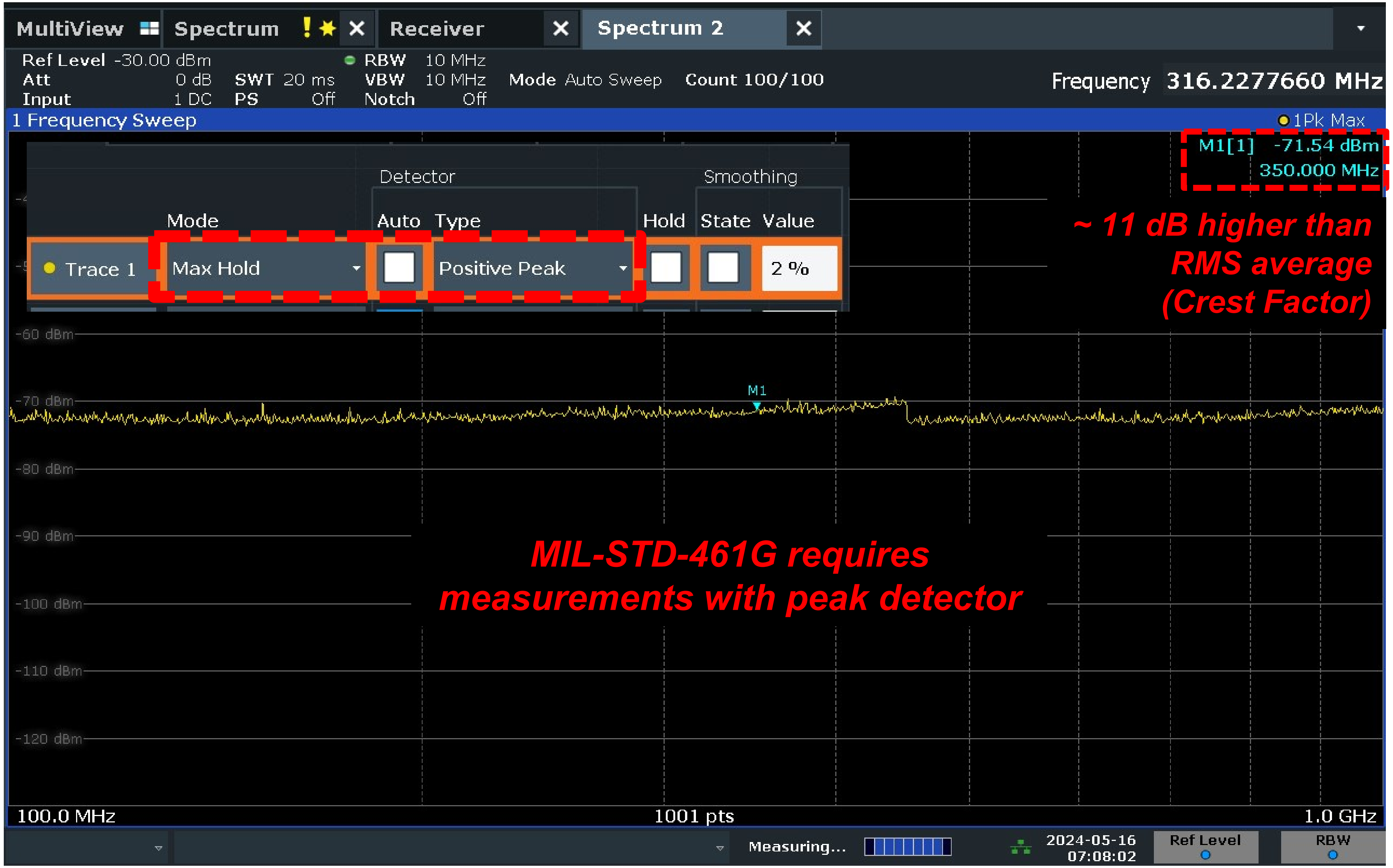
Measuring...
2024-05-16
07:06:26

~ 1 dB higher than linear sample average

~ 2 dB higher than logarithmic average

07:06:26 AM 05/16/2024

Noise Floor, Peak Detect, Max Hold, 10 MHz RBW



Ref Level -30.00 dBm
Att 0 dB
Input 1 DC
SWT 20 ms
PS Off
RBW 10 MHz
VBW 10 MHz
Notch Off
Mode Auto Sweep
Count 100/100
Frequency 316.2277660 MHz

Trace 1: Max Hold; Detector Auto ☐; Type Positive Peak; Hold ☐; Smoothing State ☐; Value 2 %

M1[1] -71.54 dBm
350.000 MHz

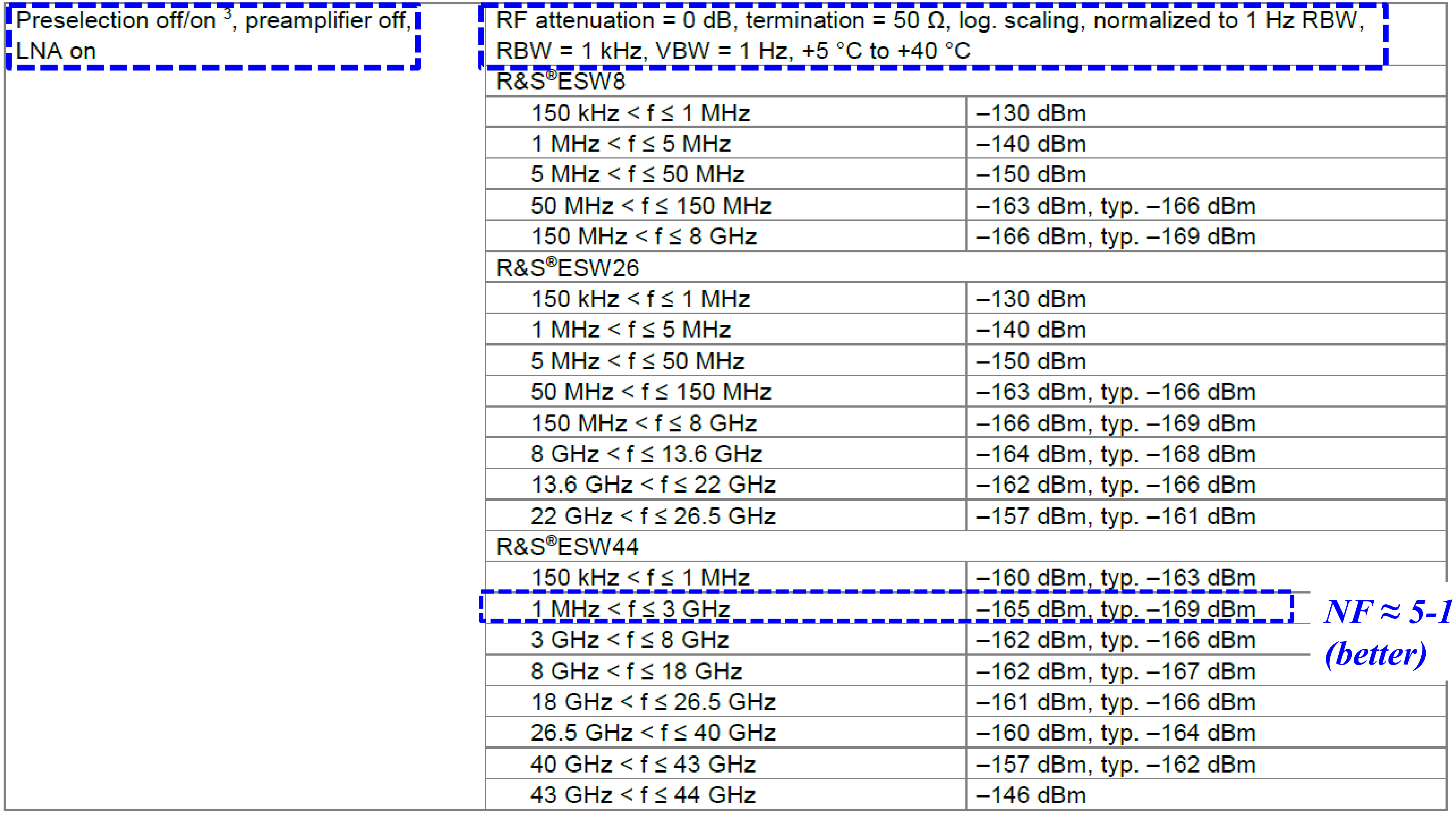
2024-05-16
07:08:02

~ 11 dB higher than RMS average (Crest Factor)

MIL-STD-461G requires measurements with peak detector

07:08:03 AM 05/16/2024

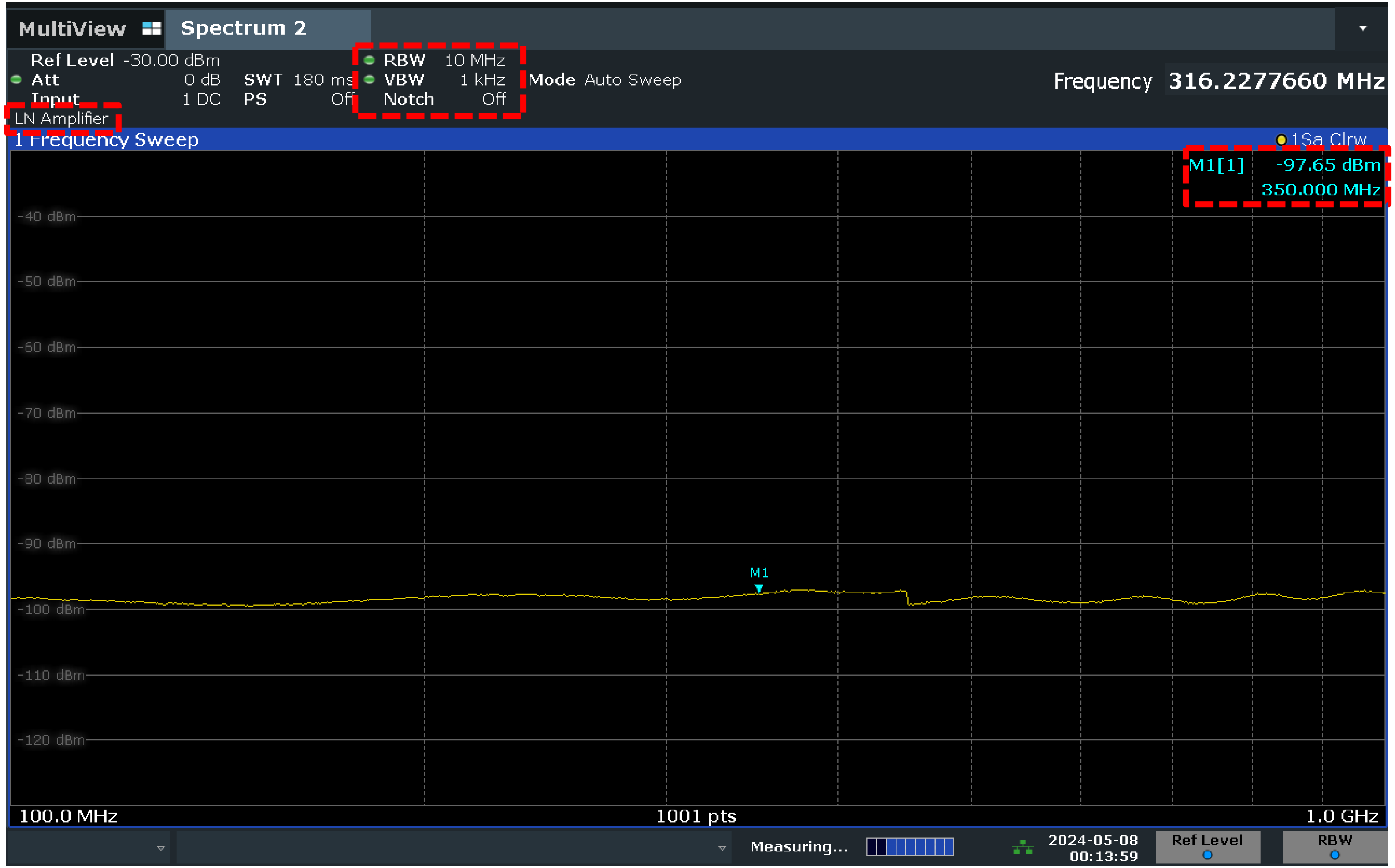
Another Excerpt from R&S ESW Datasheet



Preselection off/on [3], preamplifier off, LNA on	RF attenuation = 0 dB, termination = 50 Ω, log. scaling, normalized to 1 Hz RBW, RBW = 1 kHz, VBW = 1 Hz, +5 °C to +40 °C	
	R&S®ESW8	
	150 kHz < f ≤ 1 MHz	–130 dBm
	1 MHz < f ≤ 5 MHz	–140 dBm
	5 MHz < f ≤ 50 MHz	–150 dBm
	50 MHz < f ≤ 150 MHz	–163 dBm, typ. –166 dBm
	150 MHz < f ≤ 8 GHz	–166 dBm, typ. –169 dBm
	R&S®ESW26	
	150 kHz < f ≤ 1 MHz	–130 dBm
	1 MHz < f ≤ 5 MHz	–140 dBm
	5 MHz < f ≤ 50 MHz	–150 dBm
	50 MHz < f ≤ 150 MHz	–163 dBm, typ. –166 dBm
	150 MHz < f ≤ 8 GHz	–166 dBm, typ. –169 dBm
	8 GHz < f ≤ 13.6 GHz	–164 dBm, typ. –168 dBm
	13.6 GHz < f ≤ 22 GHz	–162 dBm, typ. –166 dBm
	22 GHz < f ≤ 26.5 GHz	–157 dBm, typ. –161 dBm
	R&S®ESW44	
	150 kHz < f ≤ 1 MHz	–160 dBm, typ. –163 dBm
	1 MHz < f ≤ 3 GHz	–165 dBm, typ. –169 dBm
	3 GHz < f ≤ 8 GHz	–162 dBm, typ. –166 dBm
	8 GHz < f ≤ 18 GHz	–162 dBm, typ. –167 dBm
	18 GHz < f ≤ 26.5 GHz	–161 dBm, typ. –166 dBm
	26.5 GHz < f ≤ 40 GHz	–160 dBm, typ. –164 dBm
	40 GHz < f ≤ 43 GHz	–157 dBm, typ. –162 dBm
	43 GHz < f ≤ 44 GHz	–146 dBm

NF ≈ 5-10 dB (better)

Noise Floor, LNA ON, 10 MHz RBW



Ref Level -30.00 dBm
Att 0 dB
SWT 180 ms
Input 1 DC
PS Off
RBW 10 MHz
VBW 1 kHz
Notch Off
Mode Auto Sweep
Frequency 316.2277660 MHz
LN Amplifier

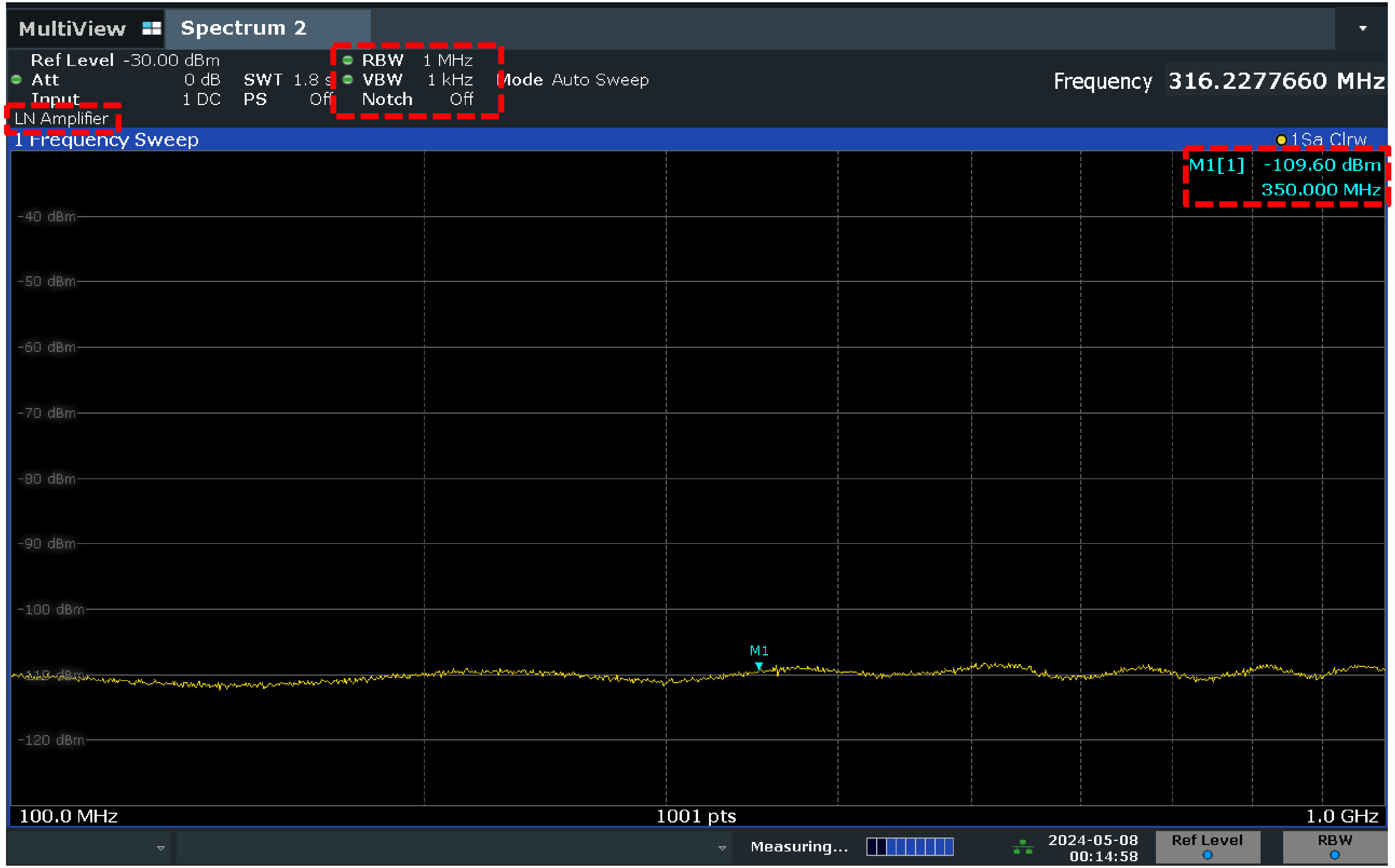
M1[1] -97.65 dBm
350.000 MHz

12:13:59 AM 05/08/2024

$10\ MHz =$
$70\ dBHz$

$-169\ dBm/Hz$
$+70\ dBHz$
$= -99\ dBm$

Noise Floor, LNA ON, 1 MHz RBW



MultiView | Spectrum 2

Ref Level -30.00 dBm
Att 0 dB SWT 1.8 s
Input 1 DC PS Off
LN Amplifier

RBW 1 MHz
VBW 1 kHz Mode Auto Sweep
Notch Off

Frequency 316.2277660 MHz

1 Frequency Sweep

M1[1] -109.60 dBm
350.000 MHz

100.0 MHz 1001 pts 1.0 GHz

Measuring...

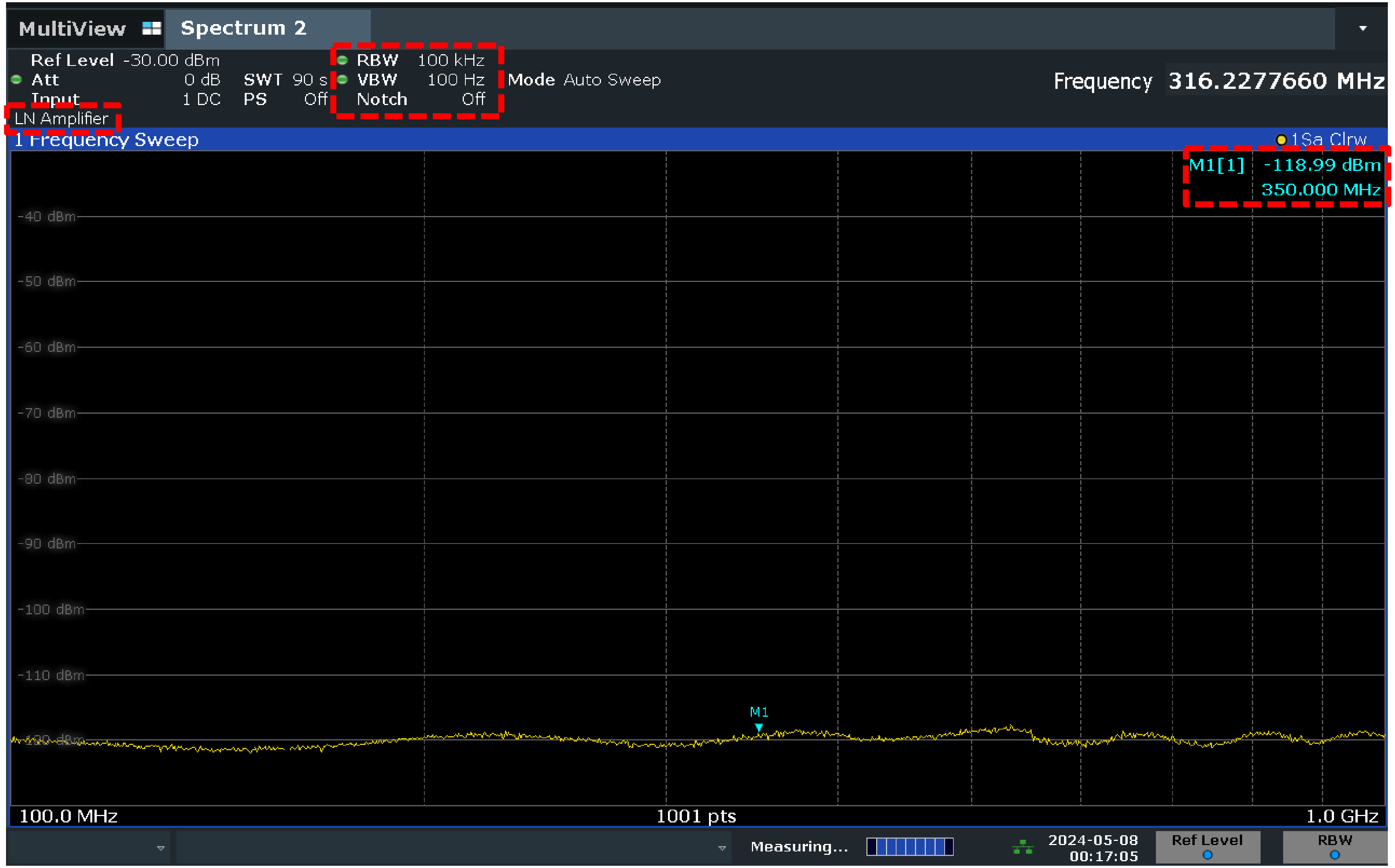
2024-05-08
00:14:58

12:14:58 AM 05/08/2024

$10\ MHz =$
$60\ dBHz$

$-169\ dBm/Hz$
$+60\ dBHz$
$= -109\ dBm$

Noise Floor, LNA ON, 100 kHz RBW



MultiView Spectrum 2

Ref Level -30.00 dBm
Att 0 dB SWT 90 s
Input 1 DC PS Off
LN Amplifier

RBW 100 kHz
VBW 100 Hz Mode Auto Sweep
Notch Off

Frequency 316.2277660 MHz

1 Frequency Sweep

1Sa Clrw

M1[1] -118.99 dBm
350.000 MHz

100.0 MHz 1001 pts 1.0 GHz

Measuring...

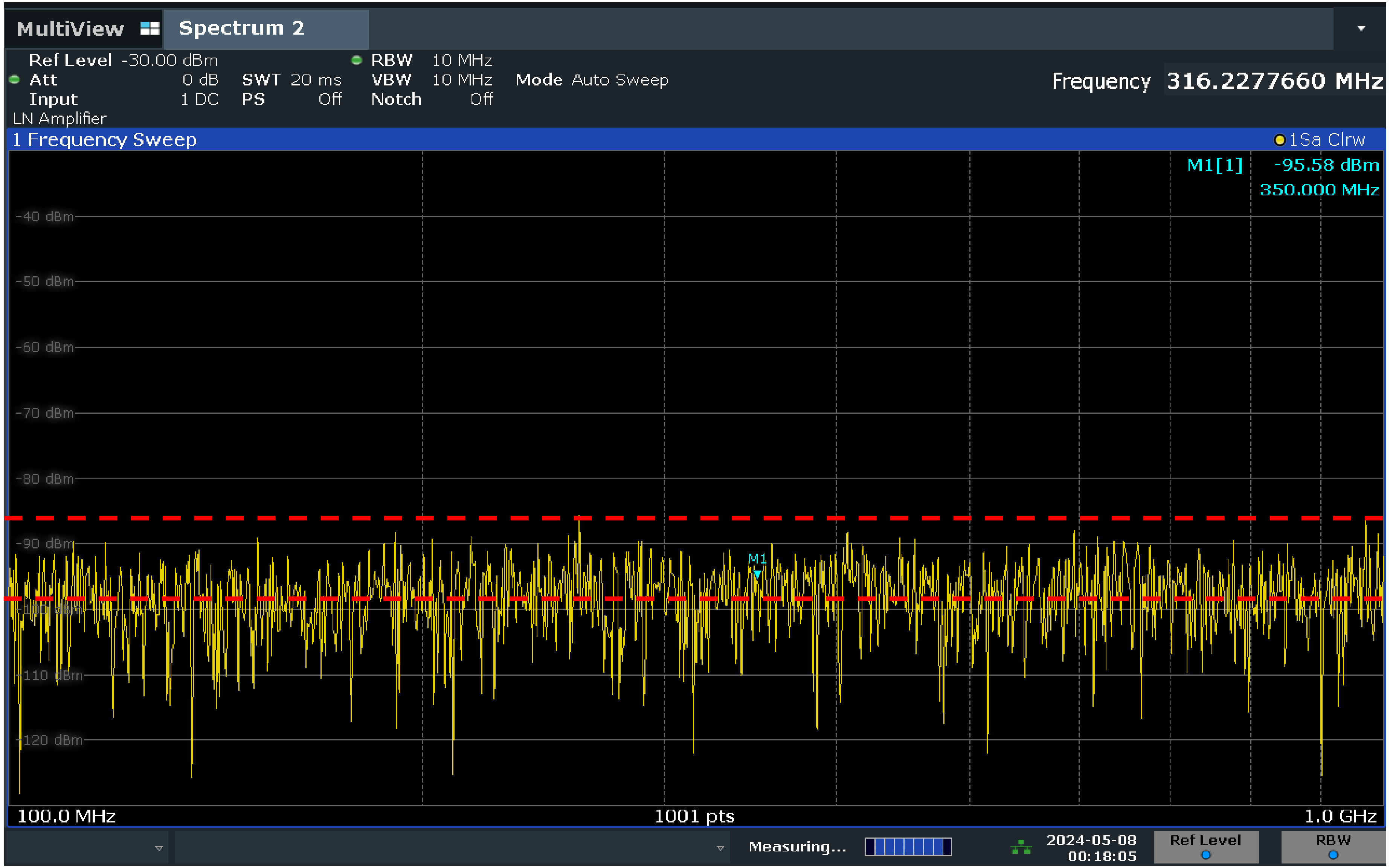
2024-05-08
00:17:05

12:17:06 AM 05/08/2024

$10\ MHz =$
$50\ dBHz$

$-169\ dBm/Hz$
$+50\ dBHz$
$= -119\ dBm$

Again, noise in real life...



MultiView Spectrum 2

Ref Level -30.00 dBm
Att 0 dB SWT 20 ms
Input 1 DC PS Off
LN Amplifier
RBW 10 MHz
VBW 10 MHz Mode Auto Sweep
Notch Off

Frequency 316.2277660 MHz

1 Frequency Sweep

M1[1] -95.58 dBm
350.000 MHz

100.0 MHz 1001 pts 1.0 GHz

Measuring...

2024-05-08
00:18:05

Peak

"Average"

12:18:06 AM 05/08/2024

Noise Floor, Sample Average - Log, LNA ON, 10 MHz RBW (cont.)

Ref Level -30.00 dBm
Att 0 dB
Input 1 DC
LN Amplifier
SWT 20 ms
PS Off
RBW 10 MHz
VBW 10 MHz
Notch Off
Mode Auto Sweep
Count 100/100
Frequency 316.2277660 MHz

1 Frequency Sweep
1Sa Avg
M1[1] -97.57 dBm
350.000 MHz

		Detector		Smoothing	
	Mode	Auto	Type	Hold	State	Value
Trace 1	Average	☐	Sample	☐	☐	2 %
Trace 2	Blank	☑	Auto Peak	☐	☐	2 %

Average: Logarithmic

100.0 MHz
1001 pts
1.0 GHz
Measuring...
2024-05-16
07:13:58

Good agreement with video average

07:13:59 AM 05/16/2024

Noise Floor, Sample Average - Linear, LNA ON, 10 MHz RBW (cont.)

Ref Level -30.00 dBm
Att 0 dB
Input 1 DC
SWT 20 ms
PS Off
RBW 10 MHz
VBW 10 MHz
Notch Off
Mode Auto Sweep
Count 100/100
LN Amplifier
Frequency 316.2277660 MHz

M1[1] -95.91 dBm
350.000 MHz

	Mode	Detector Auto	Detector Type	Hold	Smoothing State	Smoothing Value
Trace 1	Average	☐	Sample	☐	☐	2 %
Trace 2	Blank	☑	Auto Peak	☐	☐	2 %

Average: **Linear** / Logarithmic / Power

2024-05-16
07:14:28

~ 1.6 dB higher than logarithmic average

07:14:28 AM 05/16/2024

Noise Floor, RMS Detect, LNA ON, 10 MHz RBW (cont.)

Ref Level -30.00 dBm
Att 0 dB
Input 1 DC
LN Amplifier
SWT 20 ms
PS Off
RBW 10 MHz
VBW 10 MHz
Notch Off
Mode Auto Sweep
Count 100/100
Frequency 316.2277660 MHz

M1[1] -95.09 dBm
350.000 MHz

Mode		Detector		Smoothing		
		Auto	Type	Hold	State	Value
Trace 1	Average	☐	RMS	☐	☐	2 %

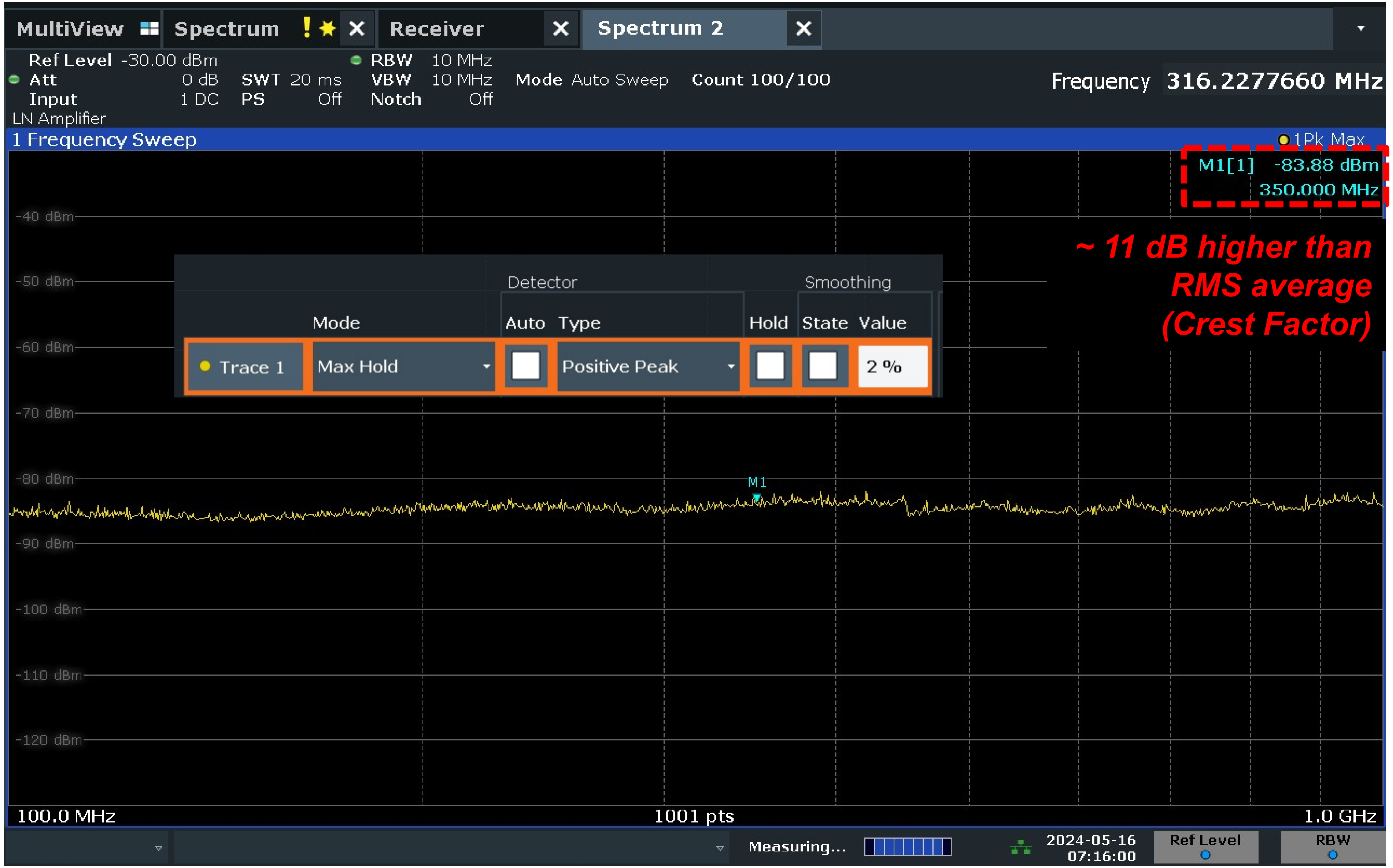
2024-05-16
07:15:07

~ 0.9 dB higher than linear sample average

~ 2.5 dB higher than logarithmic average

07:15:07 AM 05/16/2024

Noise Floor, Peak Detect & Max Hold, LNA ON, 10 MHz RBW (cont.)



Ref Level -30.00 dBm
Att 0 dB
Input 1 DC
LN Amplifier
SWT 20 ms
PS Off
RBW 10 MHz
VBW 10 MHz
Notch Off
Mode Auto Sweep
Count 100/100
Frequency 316.2277660 MHz

M1[1] -83.88 dBm
350.000 MHz

		Detector		Smoothing		
	Mode	Auto	Type	Hold	State	Value
Trace 1	Max Hold	☐	Positive Peak	☐	☐	2 %

~ 11 dB higher than RMS average (Crest Factor)

07:16:01 AM 05/16/2024

Thermal Noise Summary

- **Noise floor determines lowest value that can be measured by a given system**
  - Low end of dynamic range
- **Is thermal noise "random"?**
  - Yes and no...
  - It follows statistical Gaussian distribution about RMS level defined by k_BTB
- **Noise is noise, but measured values depend directly on measurement parameters**
  - Scales with measurement bandwidth, a.k.a. resolution bandwidth (RBW)
  - All "averages" not created equal
    - Linear average ~1.45 dB higher than logarithmic average
    - RMS ~1.04 dB higher than linear average
  - Peak/max hold measurements will be ~11 dB higher than RMS measurements (Crest Factor for Thermal Noise)
  - MIL-STD-461G requires measurements with peak detector; must account for 11 dB Crest Factor to determine noise floor of measurement system