Robust Control Design and Analysis

NASA Workshop: Winter 2025

Lecture 3: Multi-loop Disk Margins

Key Takeaways

This lecture begins with a discussion of closed-loop transfer functions for MIMO feedback systems.

We then present loop-at-a-time margins for MIMO systems:

- Uncertainty is introduced at one channel in the feedback while keep all other channels at their nominal value.
- This is an easy extension of gain/phase/delay/disk margins.

Loop-at-a-time margins can be overly optimistic (as we show by example). This motivates the use of multi-loop disk margins. This assesses robustness to (gain/phase) disk margin uncertainty introduced simultaneously in multiple channels.

Closed-Loop MIMO Transfer Functions

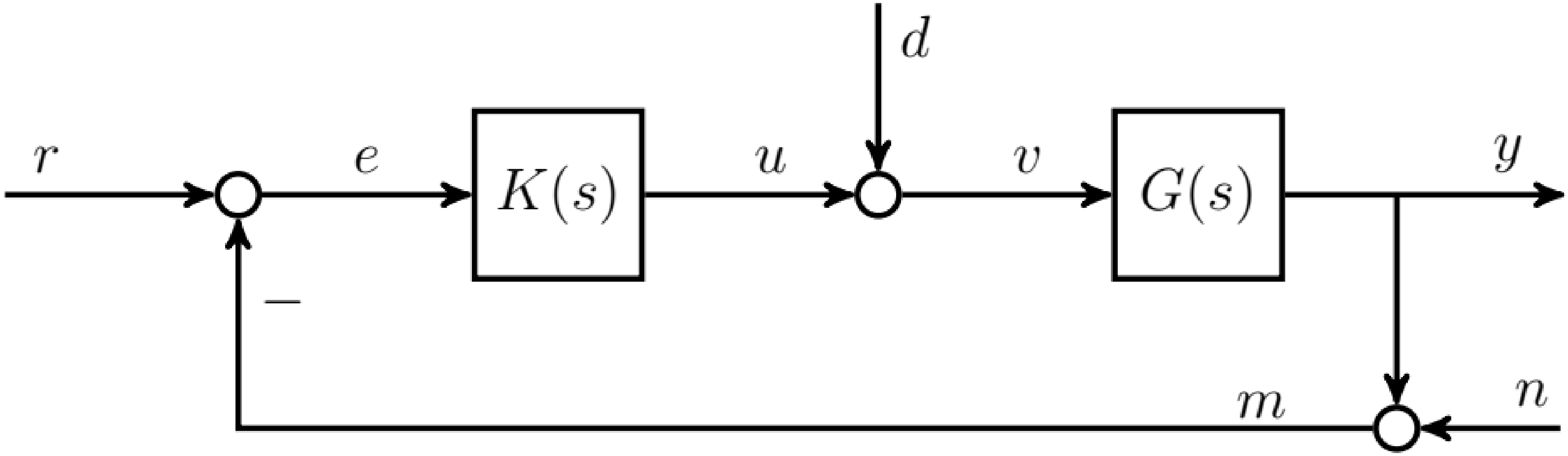
Notes: Section 1 of “Introduction to MIMO Robustness”

Closed-Loop Transfer Functions

Consider the MIMO feedback system where $G(s)$ is $n_y \times n_u$ and $K(s)$ is $n_u \times n_y$.

For the SISO systems ($n_u = n_y = 1$) we have:

- (Open) Loop: $L(s) = G(s)K(s)$
- Sensitivity: $S(s) = \frac{1}{1+L(s)}$
- Complementary Sensitivity: $T(s) = \frac{L(s)}{1+L(s)}$
- Gang of Four: $S(s), T(s), K(s)S(s), S(s)G(s)$.



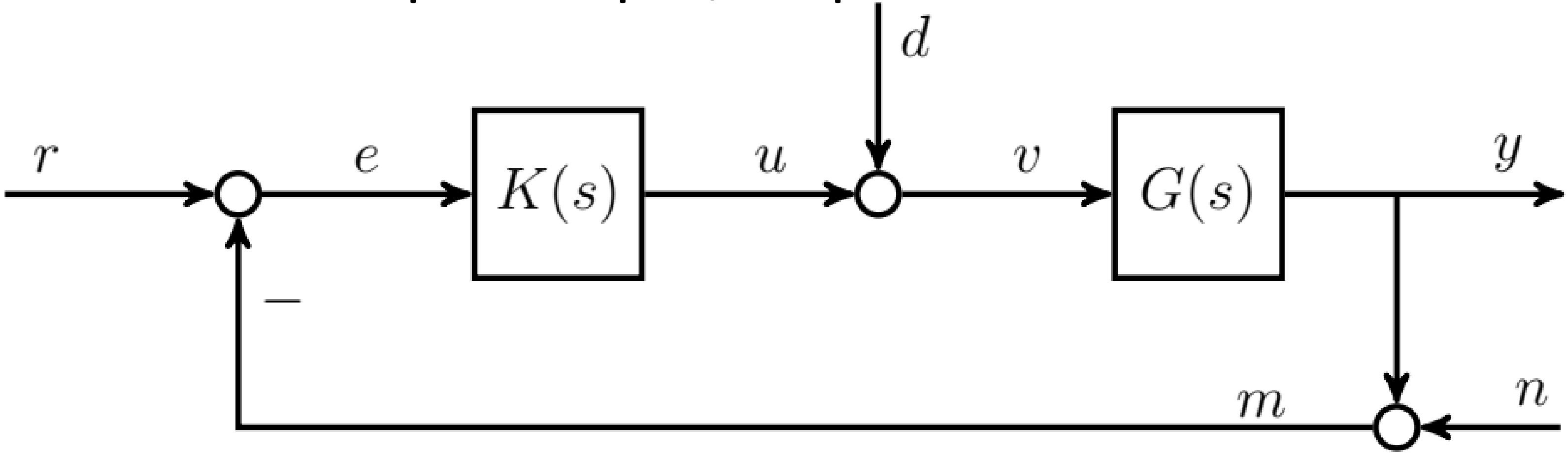
Closed-Loop Transfer Functions

Consider the MIMO feedback system where $G(s)$ is $n_y \times n_u$ and $K(s)$ is $n_u \times n_y$.

In general $G(s)K(s) \neq K(s)G(s)$ for MIMO systems ($n_u > 1$ and/or $n_y > 1$). We need to distinguish between:

- Loop at Plant Output: $L_O(s) = G(s)K(s)$ is $n_y \times n_y$
- Loop at Plant Input: $L_I(s) = K(s)G(s)$ is $n_u \times n_u$

We also need to distinguish between closed-loop transfer functions at the plant input/output.



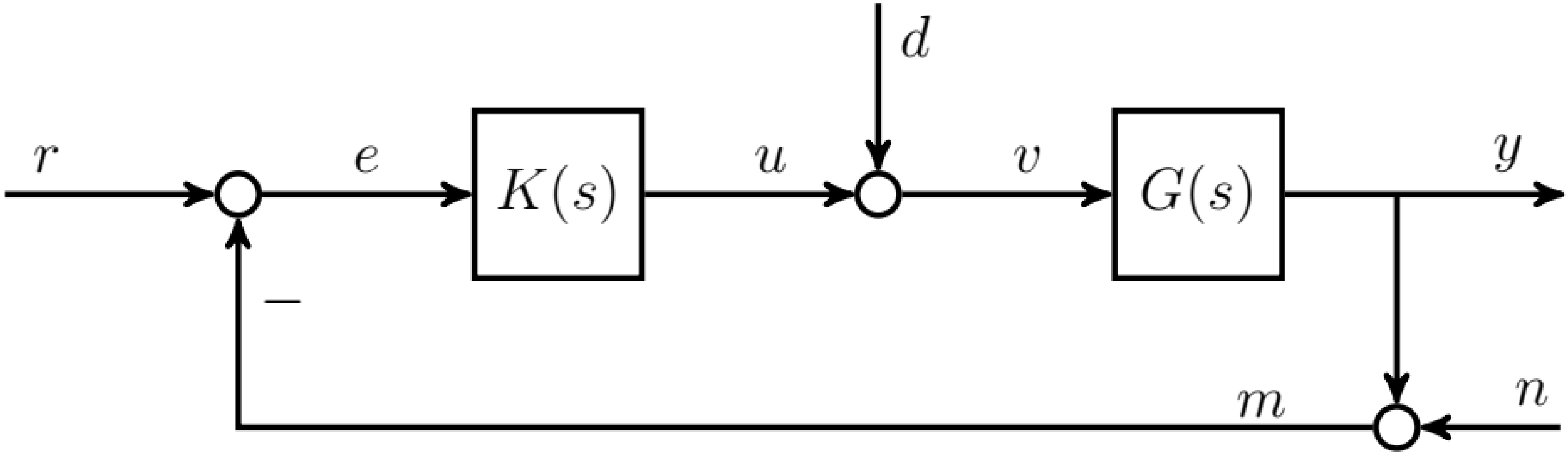
Closed-Loop Transfer Functions

Example ($r \rightarrow e$ assuming $d = 0$ and $n = 0$):

$$E(s) = R(s) - G(s)K(s)E(s) \Rightarrow E(s) = S_O(s)R(s)$$

where $S_O(s) = \left(I + L_O(s)\right)^{-1}$ is the output sensitivity.

Note: The order in the product $L_O(s) = G(s)K(s)$ is important.



Closed-Loop Transfer Functions

Example ($r \to e$ assuming $d = 0$ and $n = 0$):

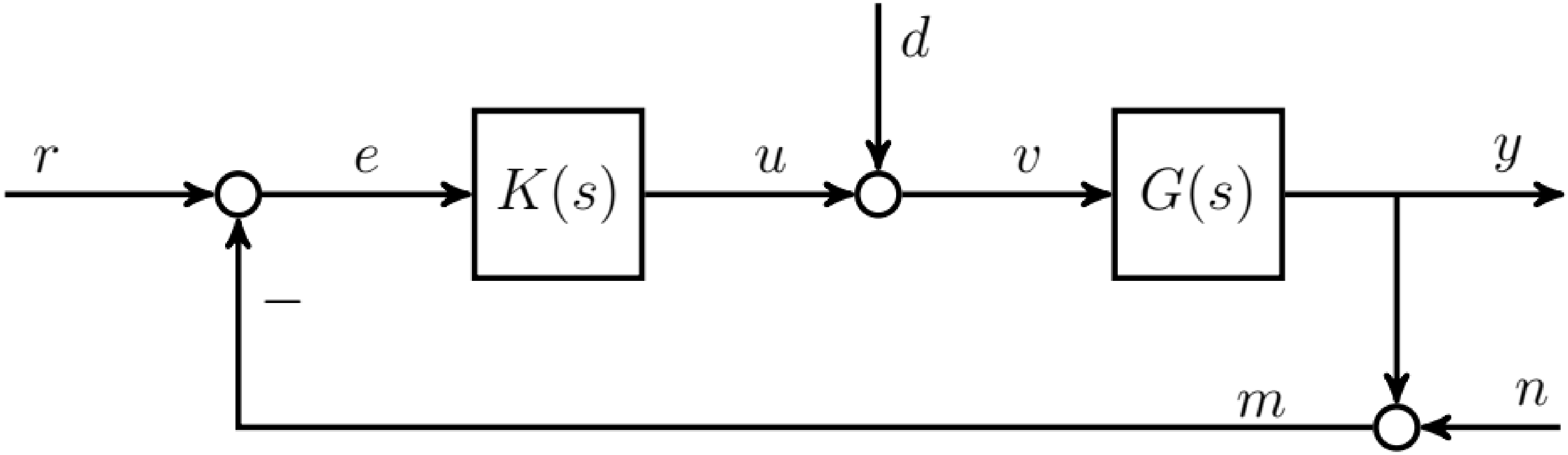
$$E(s) = R(s) - G(s)K(s)E(s) \Rightarrow E(s) = S_O(s)R(s)$$

where $S_O(s) = \big(I + L_O(s)\big)^{-1}$ is the output sensitivity.

Example ($d \to v$ assuming $r = 0$ and $n = 0$):

$$V(s) = D(s) - K(s)G(s)V(s) \Rightarrow V(s) = S_I(s)D(s)$$

where $S_I(s) = \big(I + L_I(s)\big)^{-1}$ is the input sensitivity.



Closed-Loop Transfer Functions

Similarly, we have an input/output complementary sensitivity:

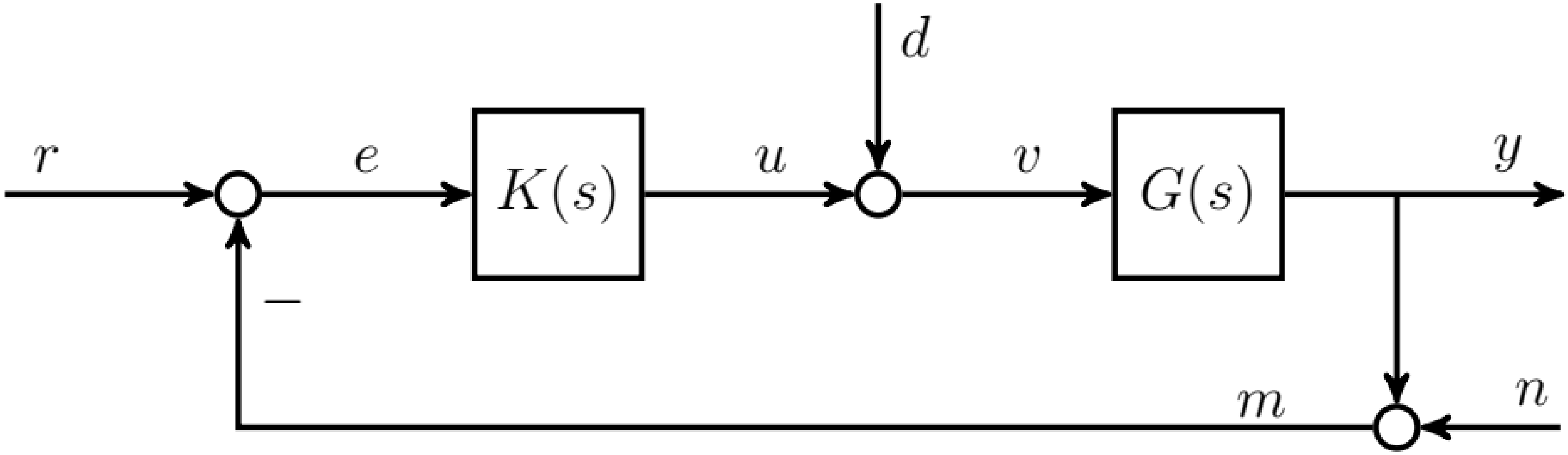
$Y(s) = T_O(s)R(s)$ where $T_O(s) = \big(I + L_O(s)\big)^{-1} L_O(s) = L_O(s)\big(I + L_O(s)\big)^{-1}$

$U(s) = T_I(s)D(s)$ where $T_I(s) = \big(I + L_I(s)\big)^{-1} L_I(s) = L_I(s)\big(I + L_I(s)\big)^{-1}$

Any input/output closed-loop TF is one of the "gang of six":

$$S_I(s), S_O(s), T_I(s), T_O(s), K(s)S_O(s), G(s)S_I(s)$$

In the SISO case, we have $L_I(s) = L_O(s)$, $S_I(s) = S_O(s)$, and $T_O(s) = T_O(s)$ so the closed-loop TFs are a "gang of four".



Closed-Loop Transfer Functions

Finally, we have similar fundamental constraints:

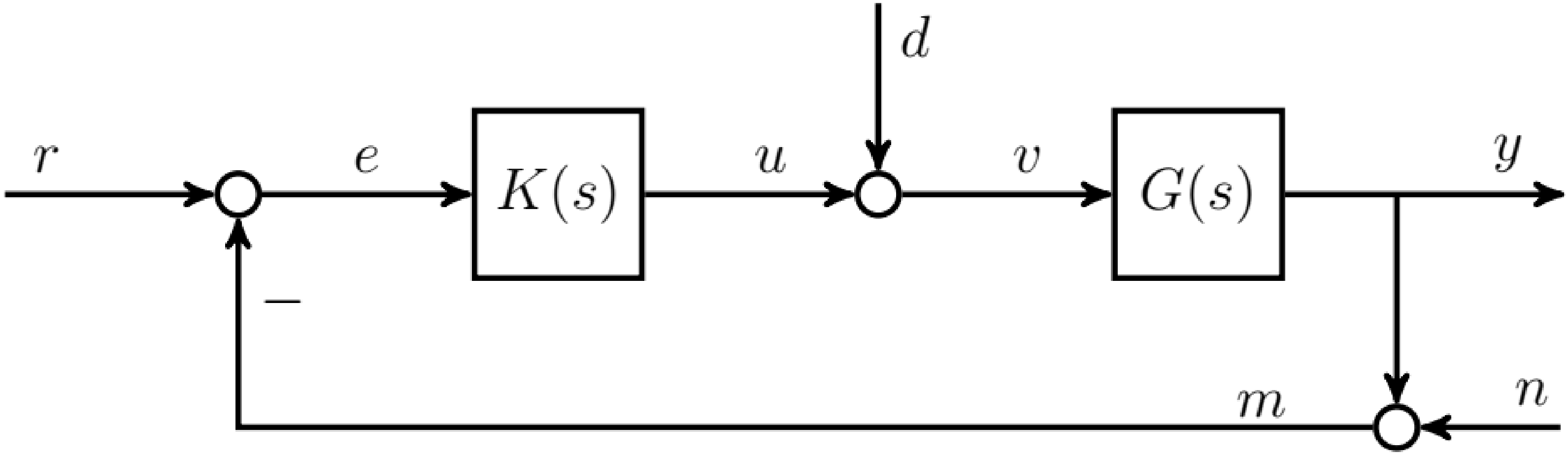
$$T_O(s) = \left(I + L_O(s)\right)^{-1} L_O(s) \text{ and } S_O(s) = \left(I + L_O(s)\right)^{-1}$$

$$\Rightarrow T_O(s) + S_O(s) = I \quad \forall s$$

We cannot simultaneously have good reference tracking and noise rejection at the same frequency.

We have a similar constraint at the plant input:

$$T_I(s) + S_I(s) = I \quad \forall s$$

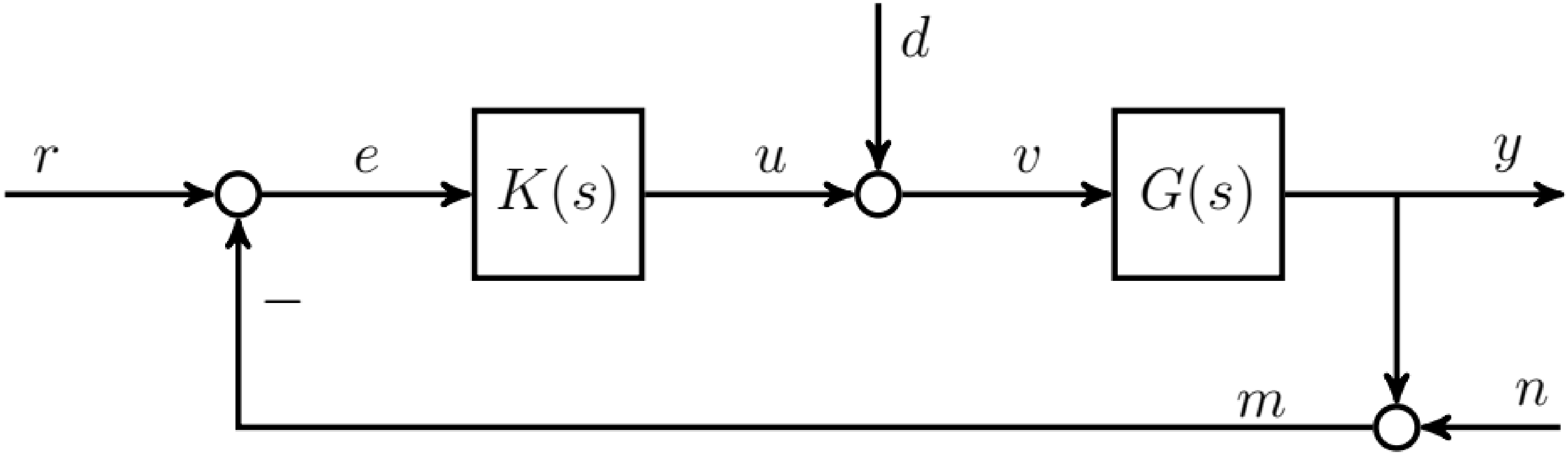


Problem 1

Consider the MIMO feedback system below.

A) Derive the transfer function from r to u.

B) Derive the transfer function from d to y.



Solution 1

Consider the MIMO feedback system below.

A) Derive the transfer function from r to u.

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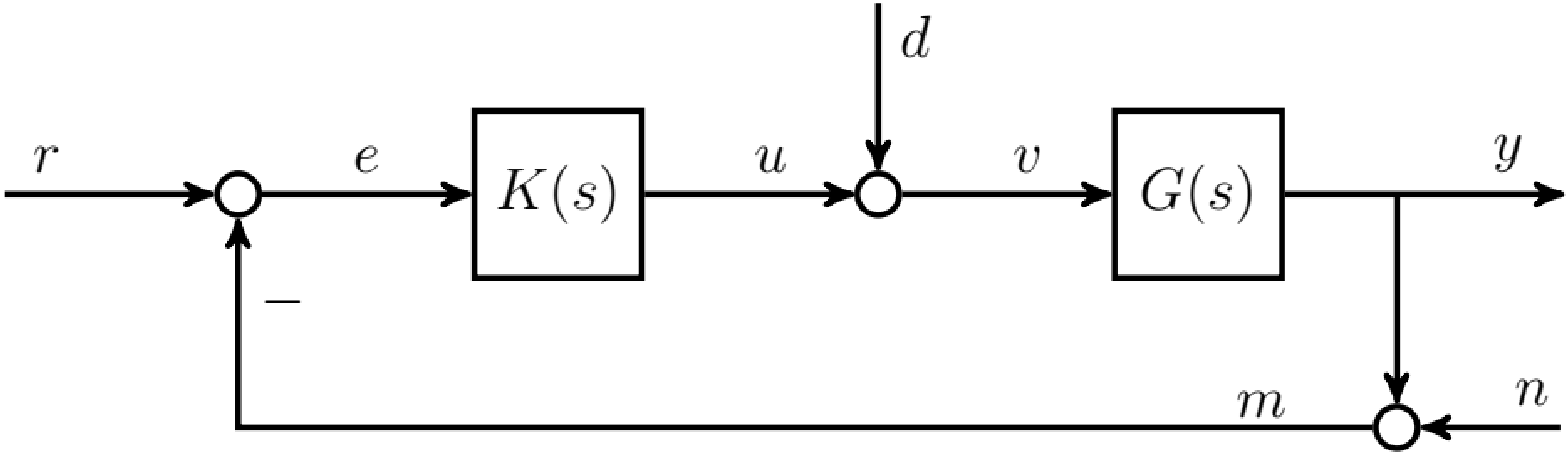


Solution 1-Extra Space

Loop-at-a-Time Margins

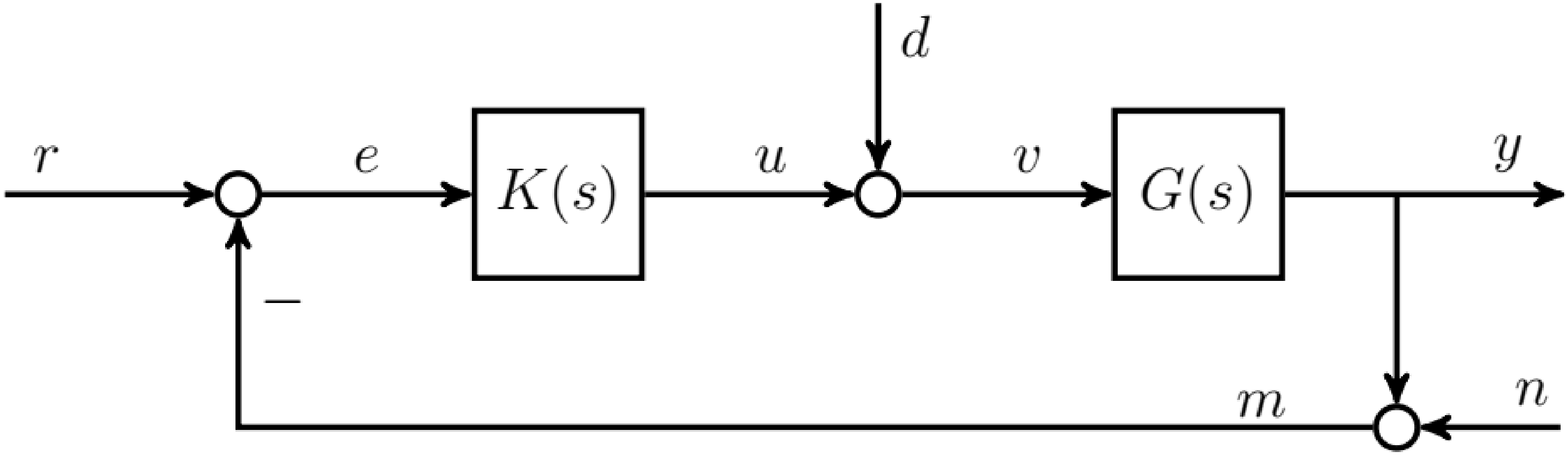
Notes: Section 2 of "Introduction to MIMO Robustness"

MIMO Stability Margins

We reviewed a variety of stability margins for SISO systems in a previous lecture, e.g.

- Classical gain, phase, and delay margins.
- *S*-based, *T*-based, and symmetric disk margins.
- Robustness to general multiplicative uncertainty.

Next, we study stability margins for MIMO systems.



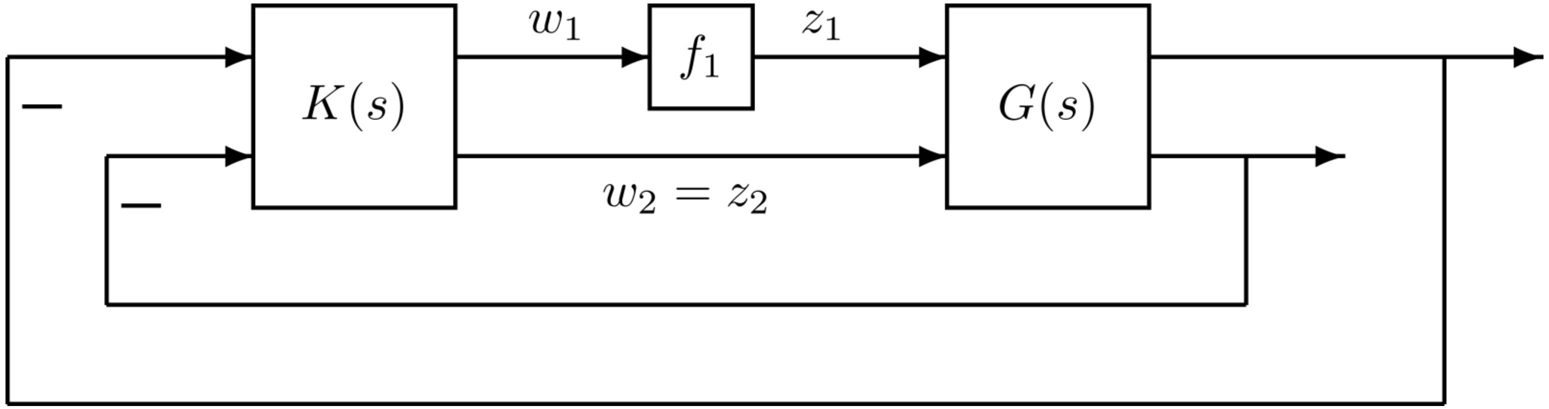
Loop-at-a-Time Margins

Loop-at-a-time analysis is a simple extension of classical margins to assess the robustness of a MIMO feedback system.

We illustrate the procedure for the 2-by-2 system below.

- A scalar (gain, phase, or disk) perturbation f_1 is introduced at the first input of the plant $G(s)$.
- The other loop remains at its nominal (unperturbed) value.

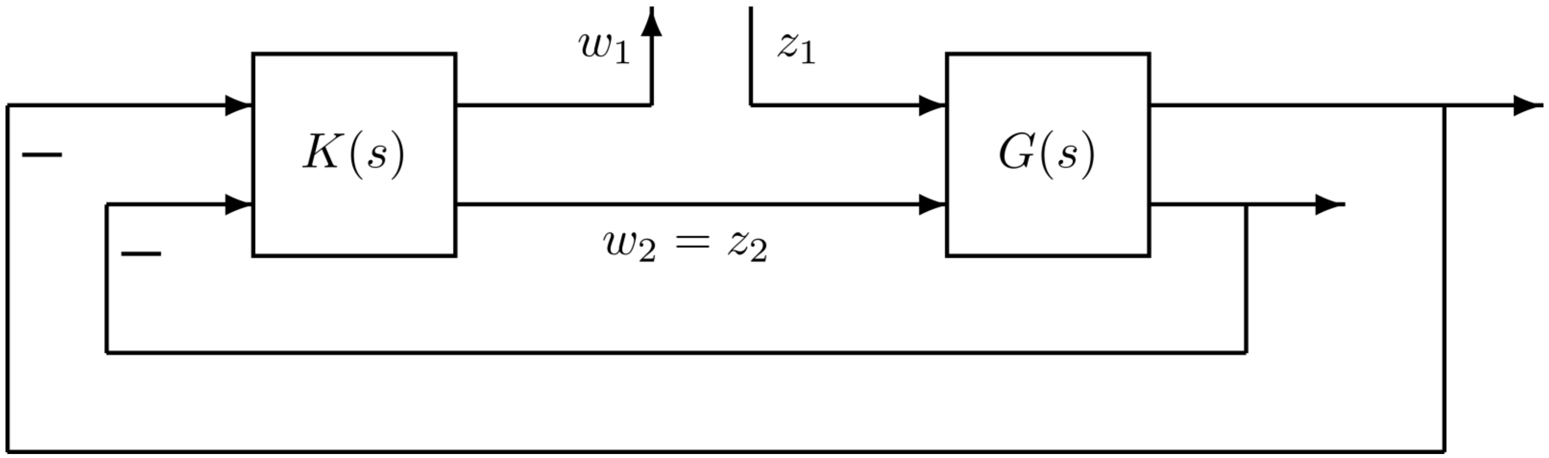
Question: How much (gain, phase, or disk) variation f_1 can be tolerated while remaining stable?



Loop-at-a-Time Margins

We can convert this to an equivalent SISO (gain, phase, or disk) margin problem.

- Break the loop at the location of the perturbation.

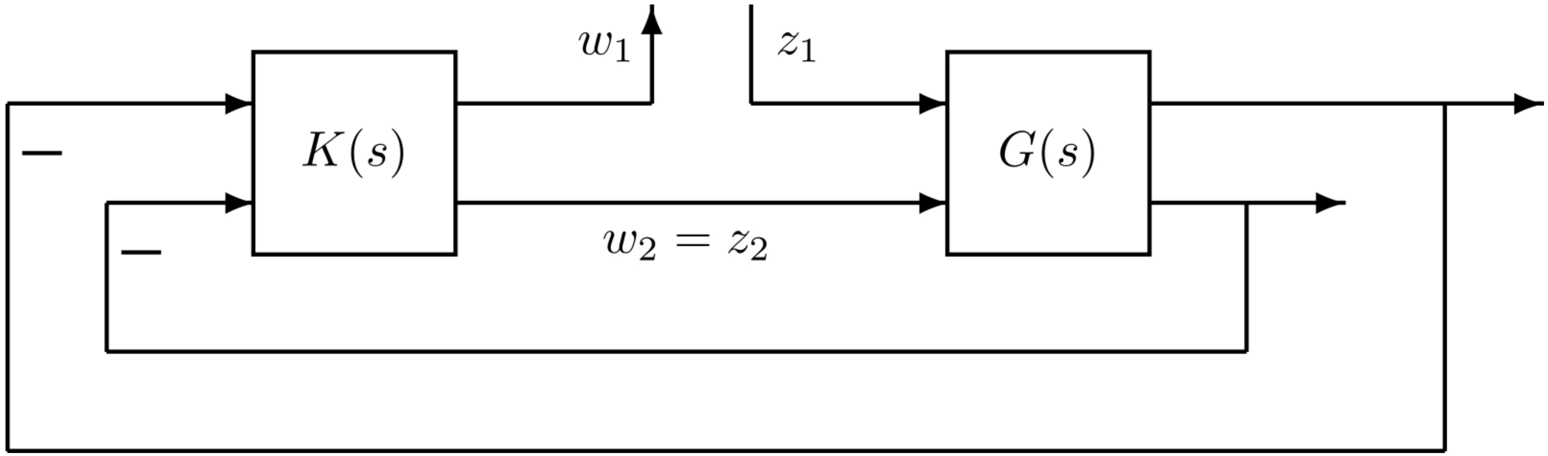


Loop-at-a-Time Margins

We can convert this to an equivalent SISO (gain, phase, or disk) margin problem.

- Break the loop at the location of the perturbation.
- Compute the SISO transfer function, denoted $-B_{1,1}(s)$, from z_1 to w_1 (with the other loop closed).

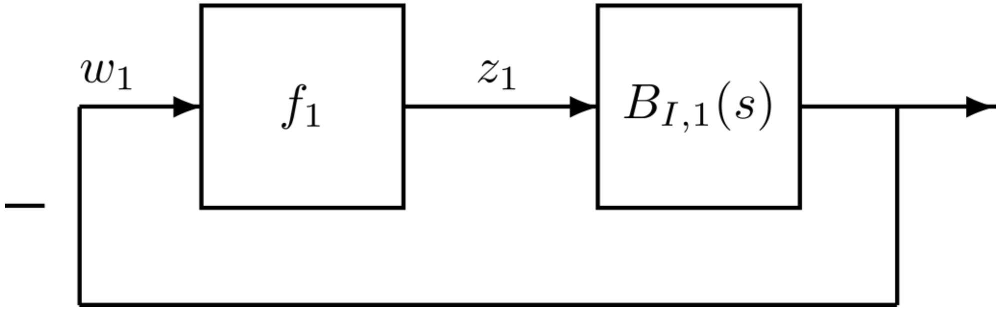
Note: The minus sign in $-B_{1,1}(s)$ is introduced to match the standard negative feedback loop convention in the next step.



Loop-at-a-Time Margins

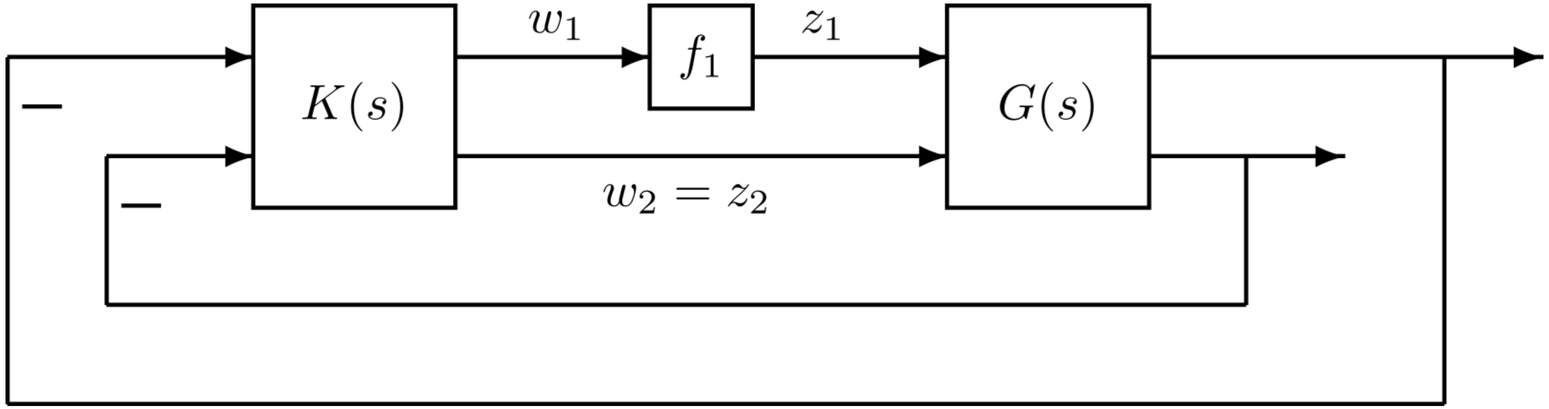
We can convert this to an equivalent SISO (gain, phase, or disk) margin problem.

- Break the loop at the location of the perturbation.
- Compute the SISO transfer function, denoted $-B_{1,1}(s)$, from z_1 to w_1 (with the other loop closed).
- The perturbation f_1 closes the loop from w_1 to z_1.
- The (gain, phase, or disk) margin associated with this loop can be computed using SISO methods.



Loop-at-a-Time Margins

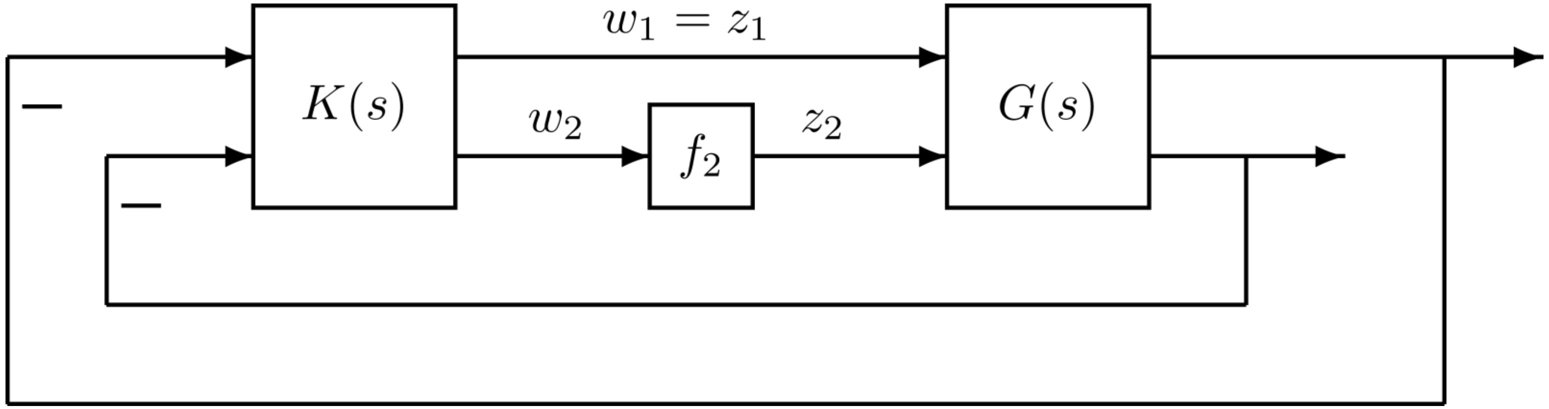
This gives the margin associated with the first input of *G(s)*.



Loop-at-a-Time Margins

This gives the margin associated with the first input of *G(s)*.

This can be repeated to compute the margin at the second input of *G(s).* We can also compute margins at both outputs of *G(s)*.



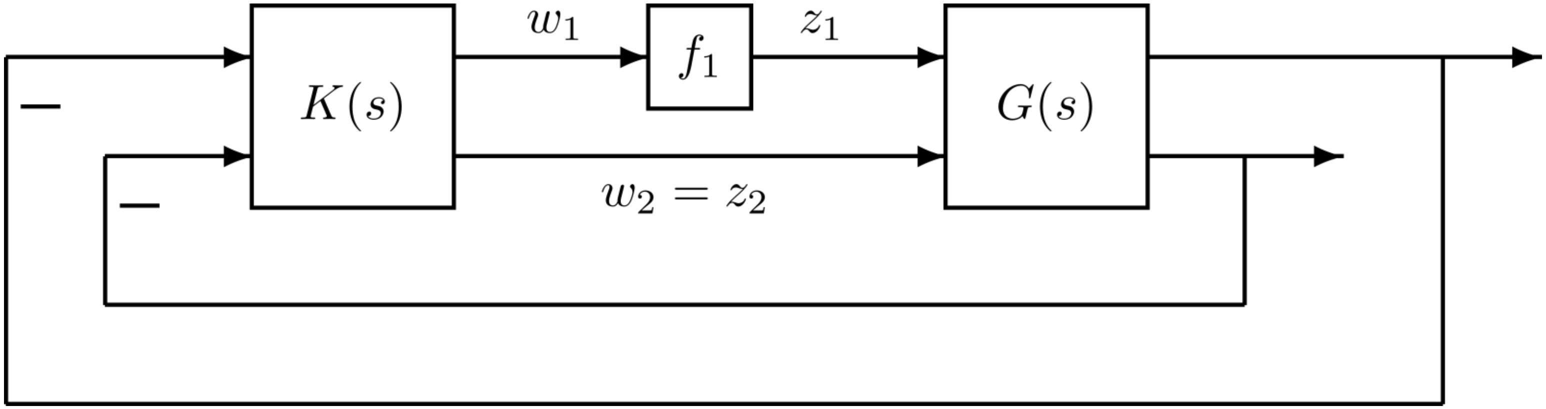
Loop-at-a-Time Margins

In general, if the plant is $n_y \times n_u$ then this gives:

- n_u margins at the inputs of *G(s)*, and
- n_y margins at the outputs of *G(s)*.

Gain, phase, delay, and/or disk margins can be computed at each location.

Unfortunately, loop-at-a-time margins can be overly optimistic as we show next.



Spinning Satellite Example

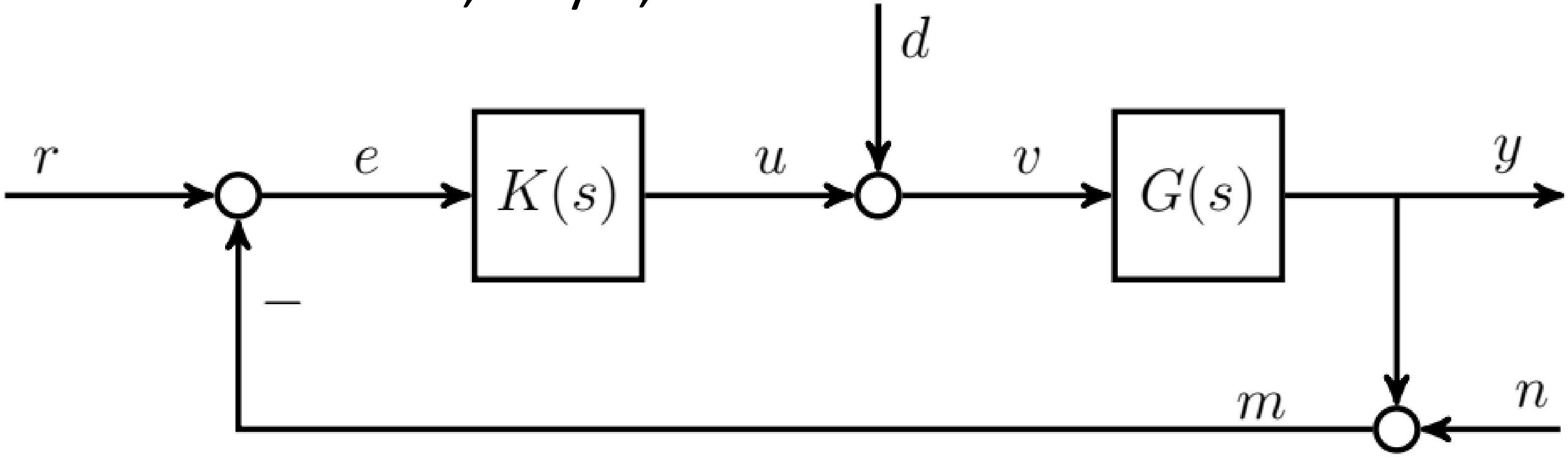
Plant and Controller:

$$G(s) := \frac{1}{s^2+a^2}\begin{bmatrix} s-a^2 & a(s+1) \\ -a(s+1) & s-a^2 \end{bmatrix} \text{ and } K(s) := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

with *a = 10.* The dynamics represent a simplified model for a spinning satellite.

References:

- Doyle, Robustness of Multiloop Linear Feedback Systems, ‘78 CDC.
- Additional details in Section 3.7 of Skogestad and Postlethwaite or Section 9.6 of Zhou, Doyle, Glover.



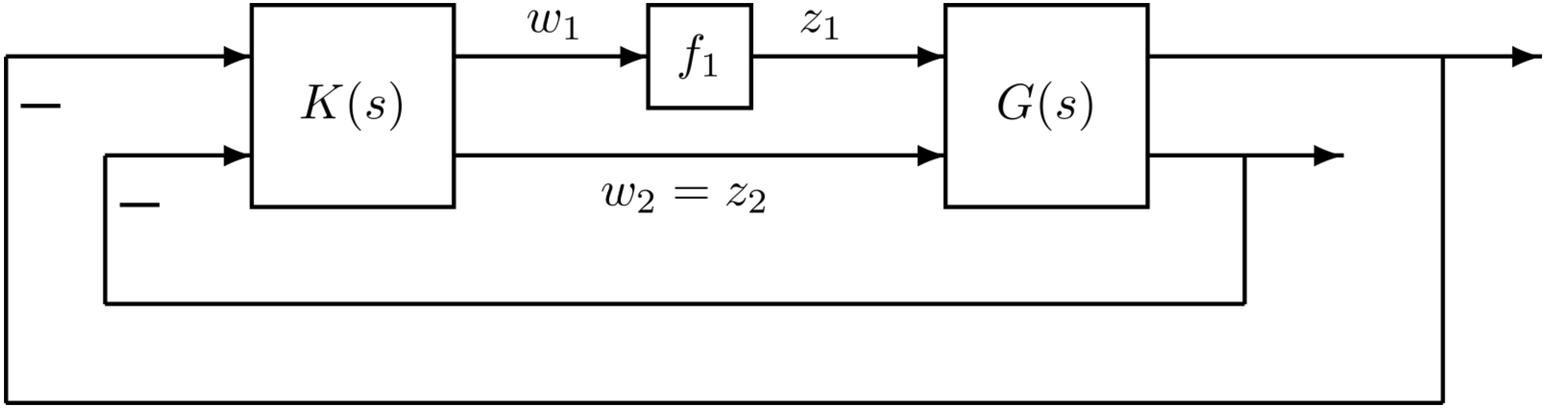
Spinning Satellite Example

Breaking loop at first plant input gives $B_{1,1}(s) = \frac{1}{s}$.

- Gain and phase margin of $[0, \infty)$ and $\pm 90^o$.
- Symmetric disk margin is = 2 implying stability for $Re(f_1) > 0$.

System is very robust to perturbations at the first input of *G(s)* with all other inputs/outputs at their nominal value.

Loop-at-a-time margins at every other input or output give the same (very robust) results.



Spinning Satellite Example

```
% Plant
a=10; A=[0 a; -a 0]; B=eye(2); C=[1 a; -a 1]; D=0;
G = ss(A,B,C,D);
% Controller
K = eye(2);
```

```
% Classical loop-at-a-time
% margins at input
AMI = allmargin(K*G);
AMI(1)      % Same for AMI(2)
    GainMargin: [1×0 double]
   GMFrequency: [1×0 double]
   PhaseMargin: 90
   PMFrequency: 1
   DelayMargin: 1.5708
   DMFrequency: 1
        Stable: 1
```

```
% Disk margins at input
DMI = diskmargin(K*G);
DMI(1)      % Same for DMI(2)
   GainMargin: [0 Inf]
   PhaseMargin: [-90 90]
   DiskMargin: 2
   LowerBound: 2
   UpperBound: 2
   Frequency: 0
   WorstPerturbation: [2×2 ss]
```

Margins at output: `AMO=allmargin(G*K);` `DMO=diskmargin(G*K);`

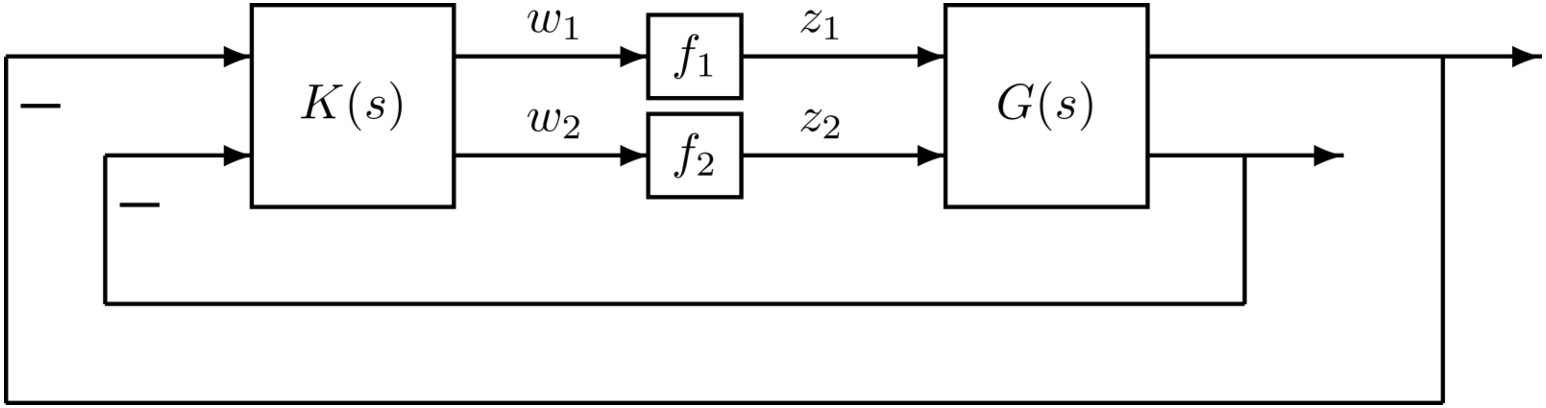
Spinning Satellite Example

However, the system becomes unstable if there are small simultaneous perturbations to both input channels.

For example, the feedback system is unstable if:

$$f_1 = 0.9 \text{ and } f_2 = 1.1$$

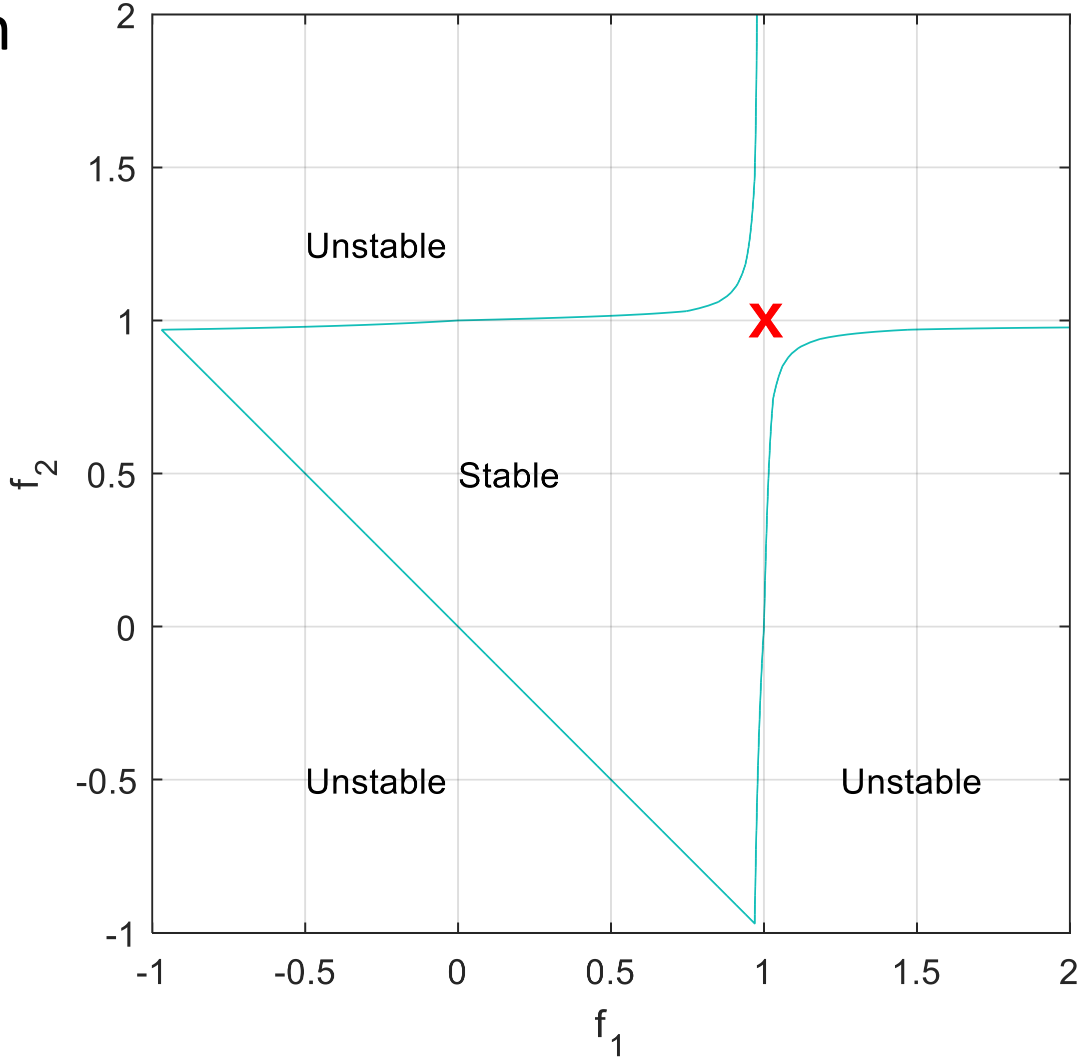
Loop-at-a-time margins can be optimistic. They do not capture the effect of simultaneous variations in multiple channels.



Spinning Satellite Example

This result is understood by plotting the stability domain as a function of (f_1, f_2).

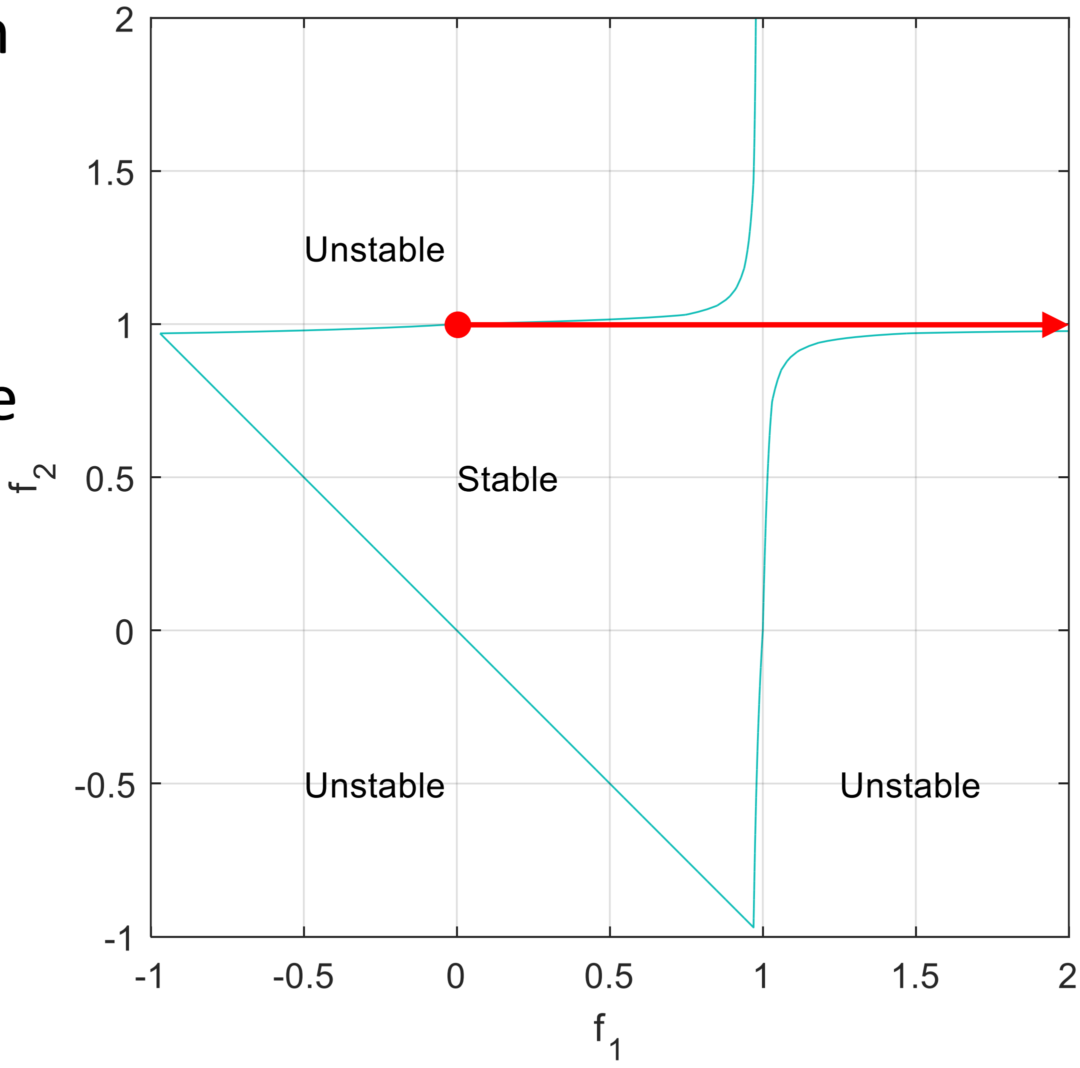
Closed-loop is stable for the nominal: $(f_1, f_2) = (1,1)$.



Spinning Satellite Example

This result is understood by plotting the stability domain as a function of (f_1, f_2).

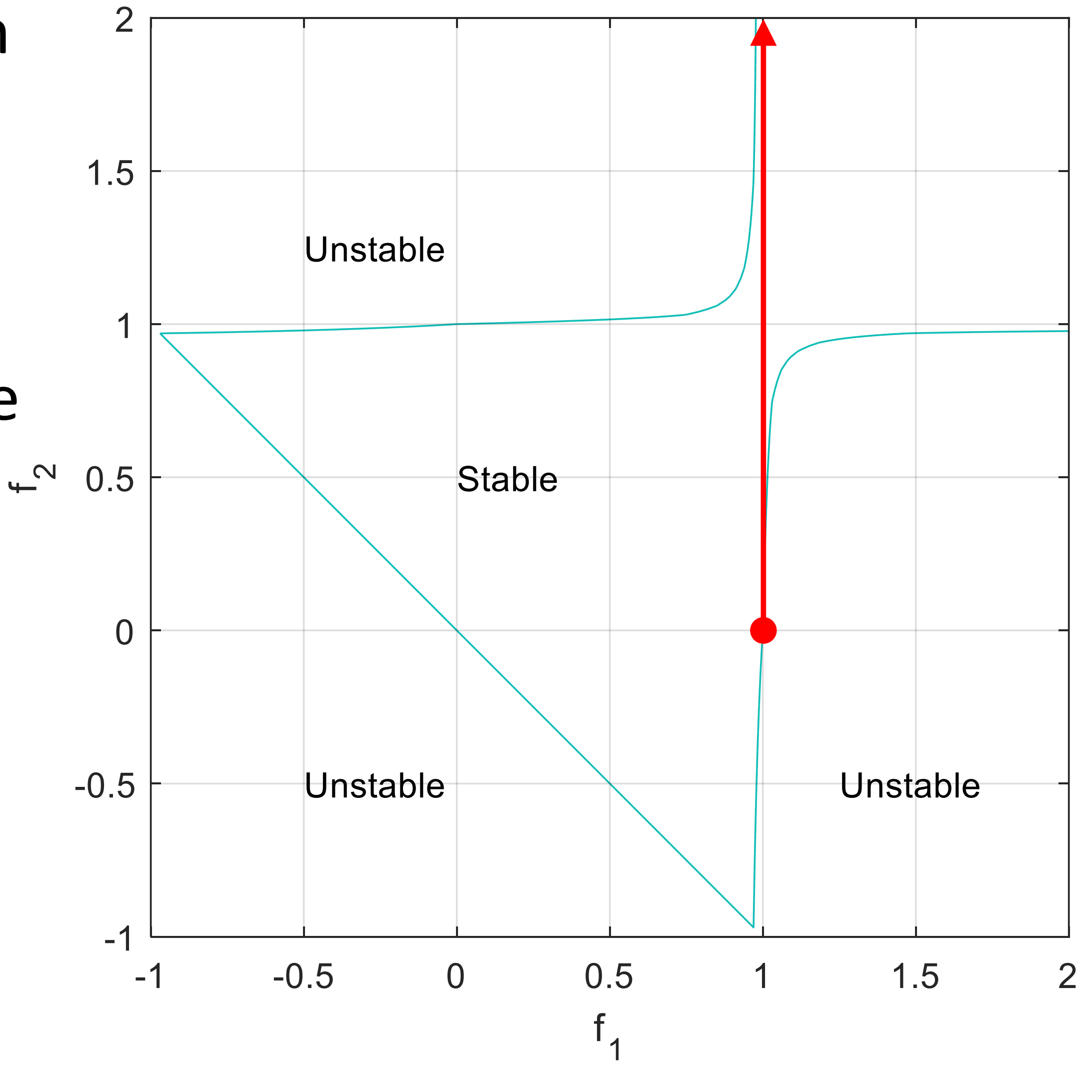
Closed-loop is stable if $f_2 = 1$ and $f_1 \in [0, \infty)$. This is the loop-at-a-time gain margin on input 1.



Spinning Satellite Example

This result is understood by plotting the stability domain as a function of (f_1, f_2).

Closed-loop is stable if $f_1 = 1$ and $f_2 \in [0, \infty)$. This is the loop-at-a-time gain margin on input 2.

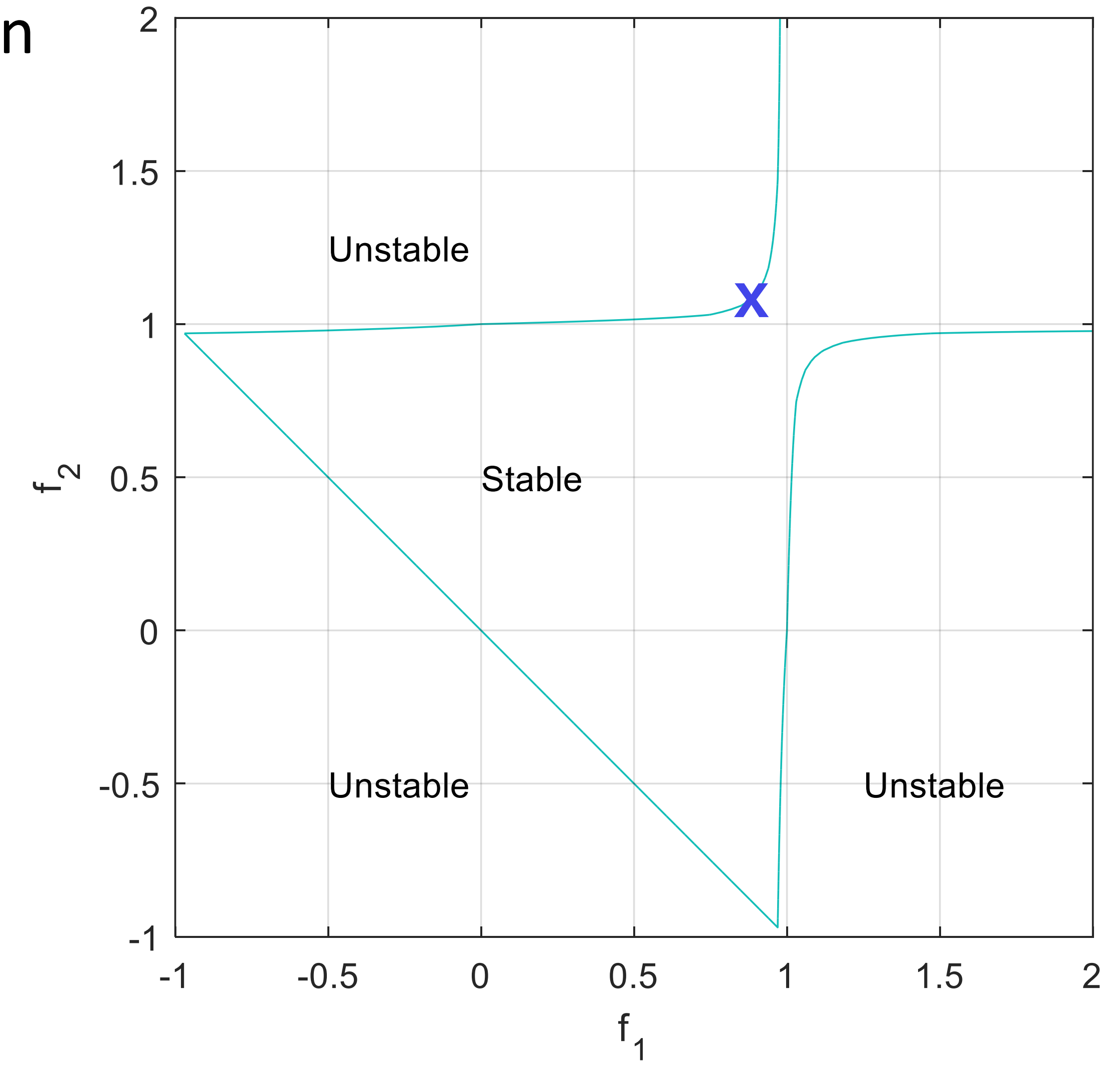


Spinning Satellite Example

This result is understood by plotting the stability domain as a function of (f_1, f_2).

However, there are small, simultaneous variations that cause instability, e.g. $f_1 = 0.9$ and $f_2 = 1.1$.

We will next discuss an approach, multi-loop margins, to analyze such simultaneous variations.



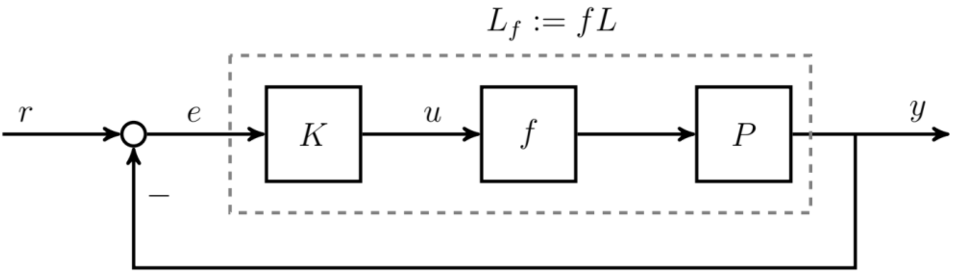
Review of SISO Symmetric Disk Margins

Reference: Week 1 + Seiler, Packard, Gahinet, An Introduction to Disk Margins, IEEE CSM, 2020.

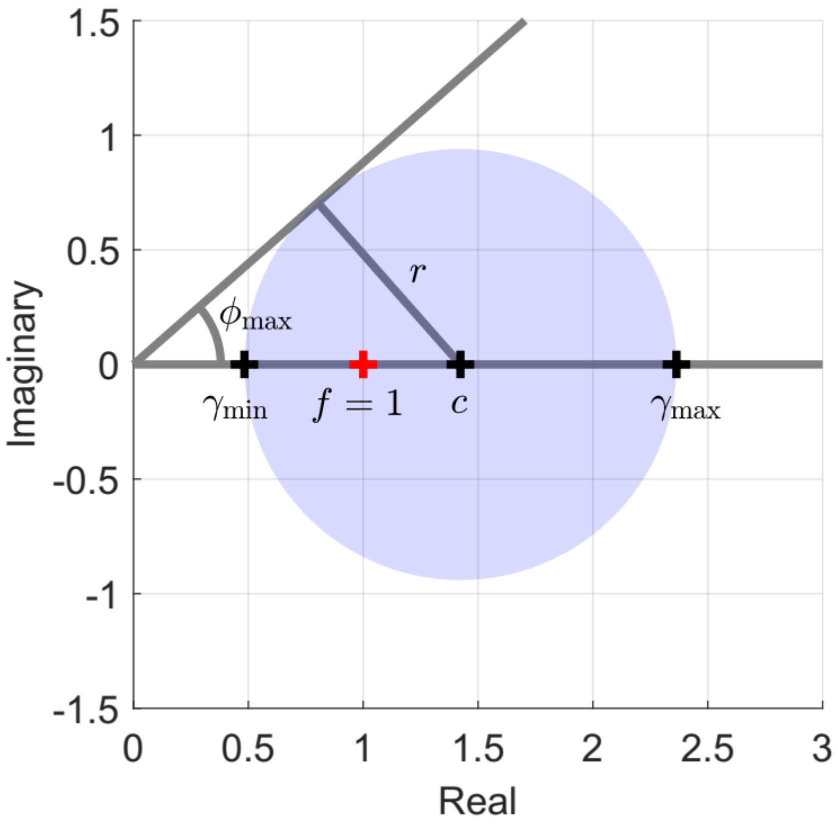
Modeling Gain and Phase Variations

Symmetric Disks

$$D(\alpha) = \left\{ \frac{1+\frac{\delta}{2}}{1-\frac{\delta}{2}} \; : \; \delta \in \mathbb{C} \;\&\; |\delta| < \alpha \right\}$$

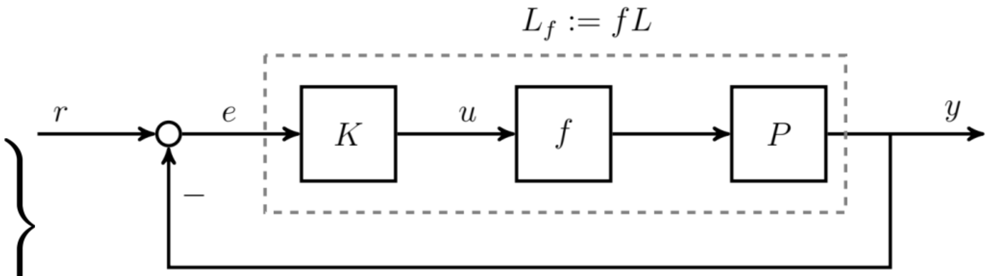


- Disk of gain and phase variations around *f=1.*



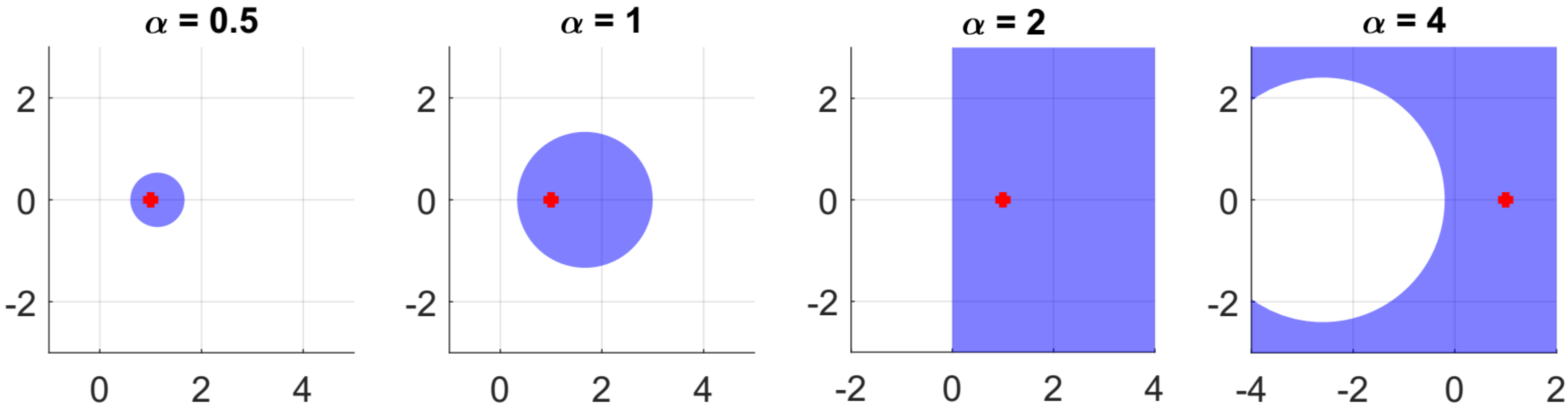
Modeling Gain and Phase Variations

Symmetric Disks



$$D(\alpha) = \left\{ \frac{1+\frac{\delta}{2}}{1-\frac{\delta}{2}} \; : \; \delta \in \mathbb{C} \;\&\; |\delta| < \alpha \right\}$$

- Disk of gain and phase variations around *f=1.*
- Disk grows with increasing α.



Modeling Gain and Phase Variations

Symmetric Disks

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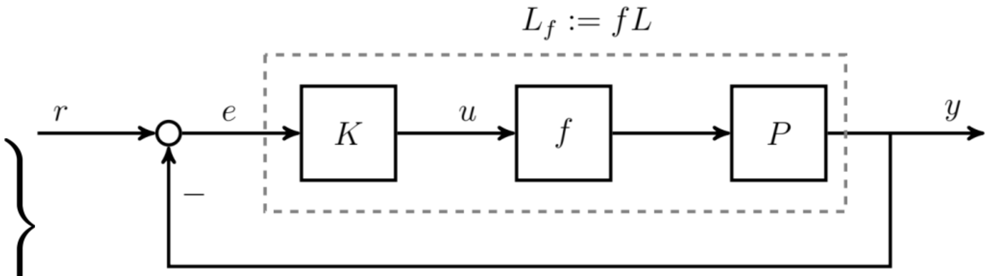
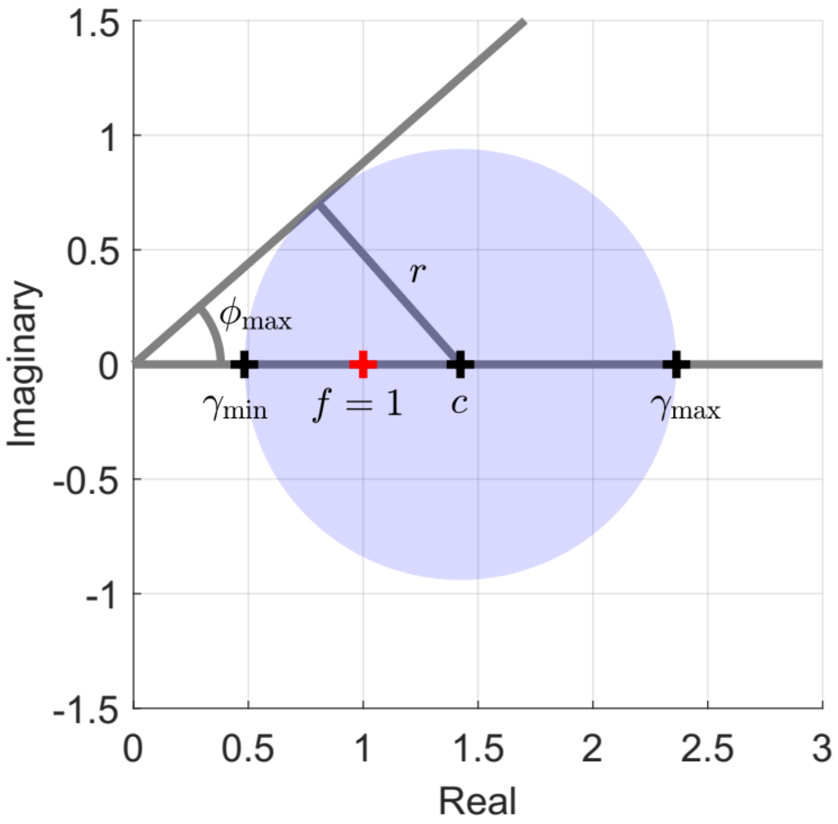


- Disk is symmetric in dB:

  $\gamma_{min}=1/\gamma_{max}$
- Ex: $\gamma_{max}=2$ *and* $\gamma_{min}=0.5$ corresponds to *+/- 6dB.*
- We can relate the symmetric disk to classical gain and phase variations.



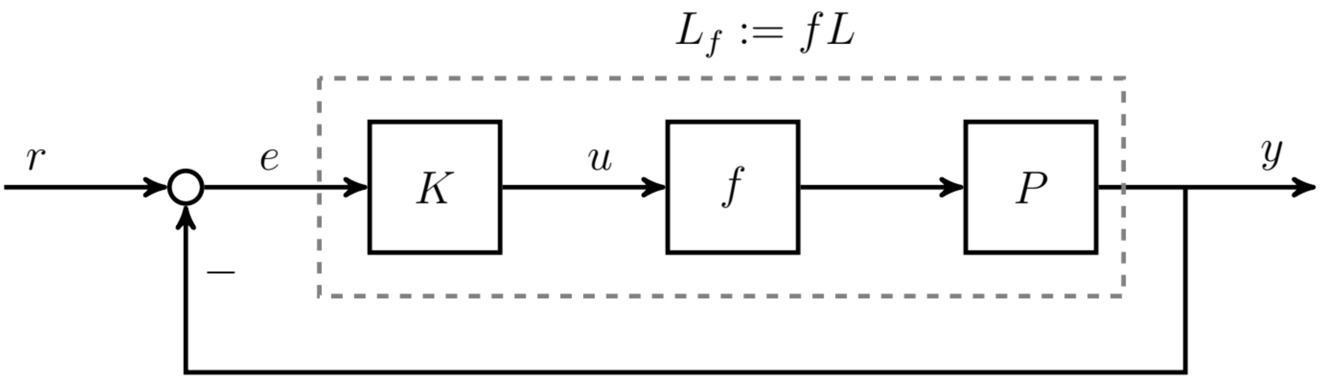
Symmetric Disk Margins

- **Definition:** The <u>symmetric disk margin</u> α_{max} is the largest value of α such that closed-loop with *is* stable for all complex perturbations $f \in D(\alpha)$.

- **Result:** Assume the feedback loop is nominally stable with *f=1*. The symmetric disk margin is:

$$\alpha_{max} = \frac{1}{||S-0.5||_\infty} = \frac{2}{||S-T||_\infty}$$

$L_f := fL$



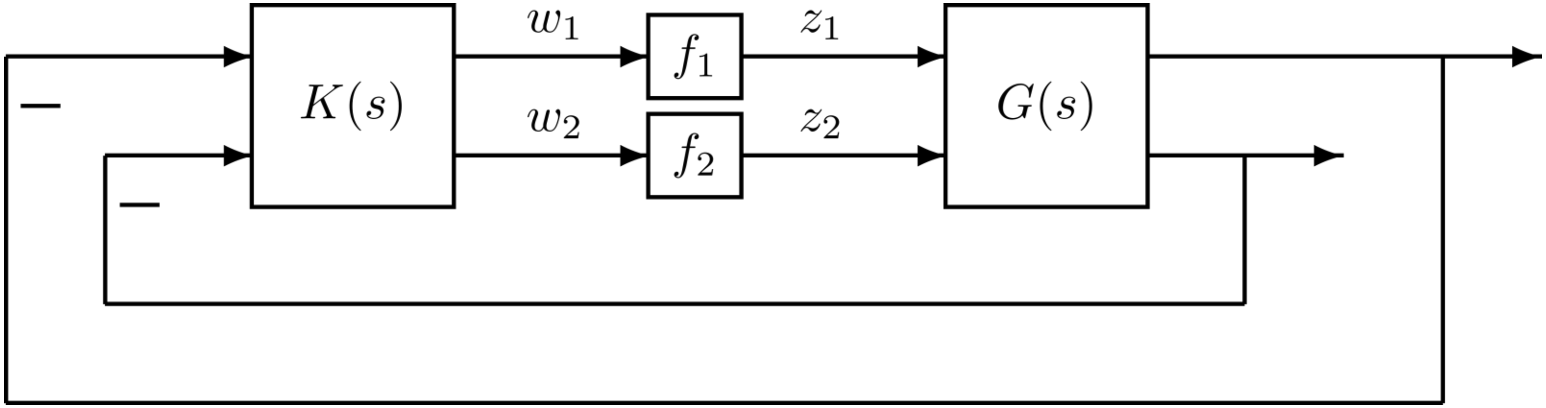
Multi-Loop Disk Margins

Reference: Seiler, Packard, Gahinet, An Introduction to Disk Margins, IEEE CSM, 2020.

Multi-loop Disk Margins

Multi-loop disk margins can be used to assess simultaneous perturbations on multiple channels.

The perturbations (f_1, f_2) are shown for the 2-by-2 feedback system below. However, the definitions hold for an arbitrary number of plant inputs.

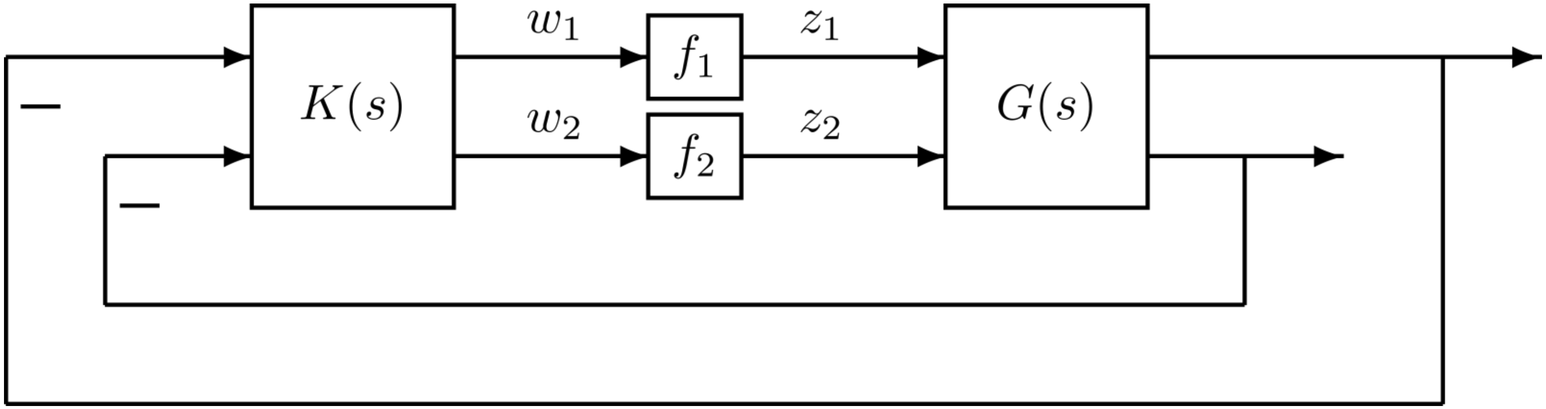


Multi-loop Disk Margins

Notation: If $\alpha \in [0,2]$, then $Disk(\alpha)$ is the "symmetric" disk of (complex) perturbations:

$$D(\alpha) = \left\{ \frac{1+\frac{\delta}{2}}{1-\frac{\delta}{2}} \; : \; \delta \in \mathbb{C} \;\&\; |\delta| < \alpha \right\}$$

Note: If $\alpha = 2$ then the symmetric disk is the entire right half of the complex plane, i.e. $\alpha \in Disk(0, \infty) \leftrightarrow Re(\alpha) \geq 0$.

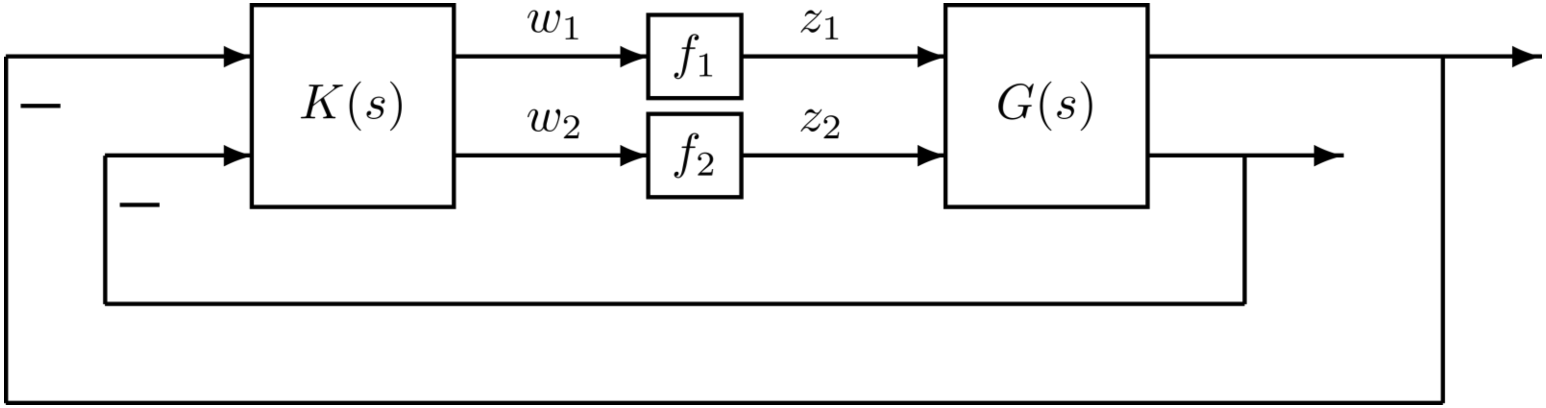


Multi-loop Disk Margins

Notation: If $\alpha \in [0,2]$, then $Disk(\alpha)$ is the "symmetric" disk of (complex) perturbations:

$$D(\alpha) = \left\{ \frac{1+\frac{\delta}{2}}{1-\frac{\delta}{2}} \ : \ \delta \in \mathbb{C} \ \& \ |\delta| < \alpha \right\}$$

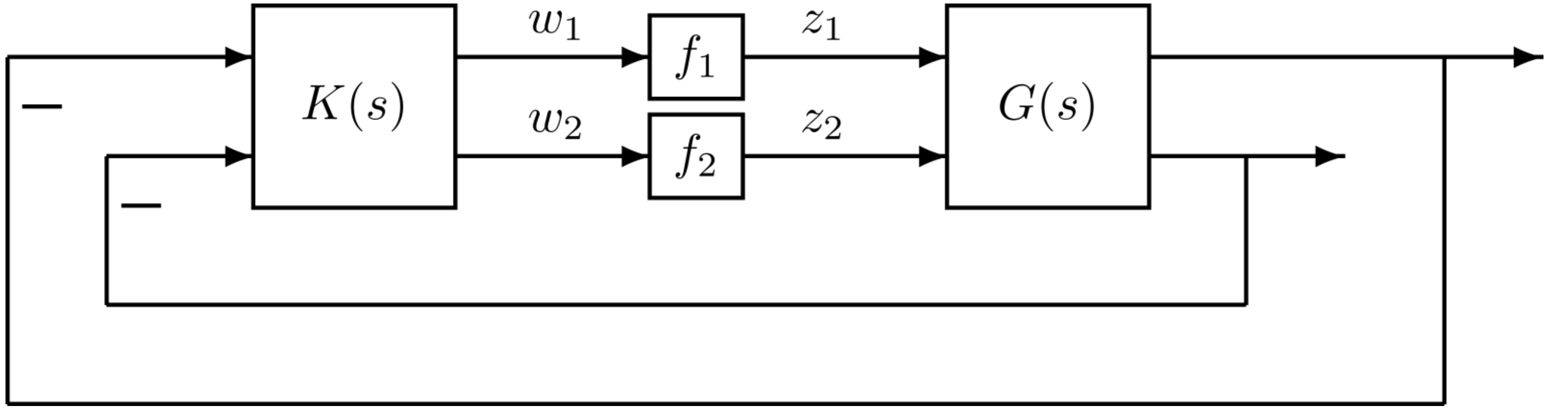
Definition: The multi-loop disk margin at the plant input is the largest $\alpha_{max} \in [0,2]$ such that the feedback loop is stable for all (f_1, f_2) in the symmetric disk.



Multi-loop Disk Margins

The multi-loop disk margin at the input is a single number $\alpha_{max} \in [0,2]$ such that:

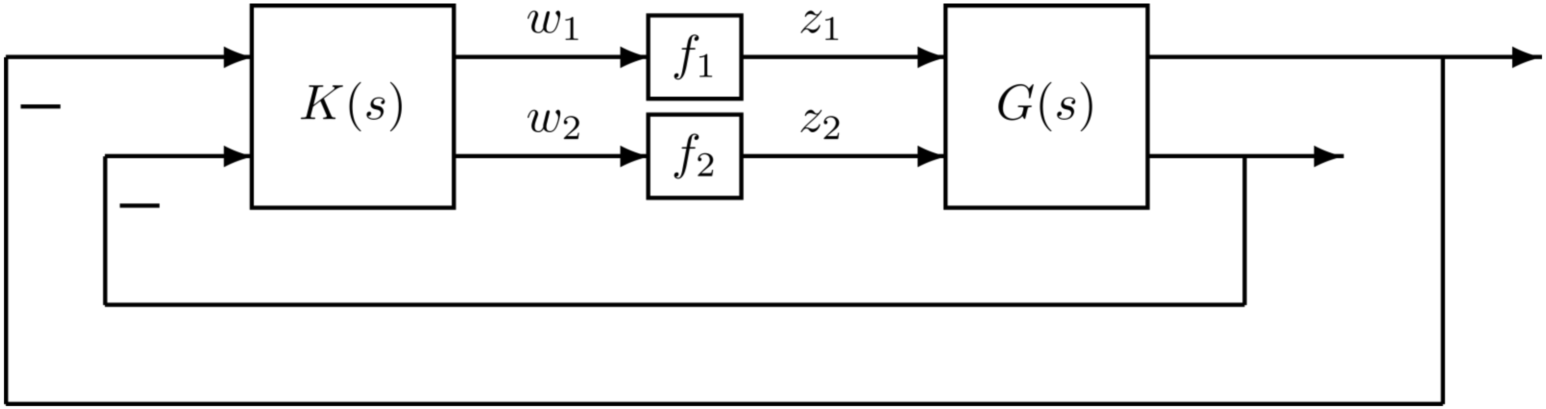
- The feedback system is stable for all (f_1, f_2) strictly inside the symmetric disk $Disk(\alpha_{max})$. The perturbations can vary independently, i.e. (f_1, f_2) are not necessarily equal.



Multi-loop Disk Margins

The multi-loop disk margin at the input is a single number $\alpha_{max} \in [0,2]$ such that:

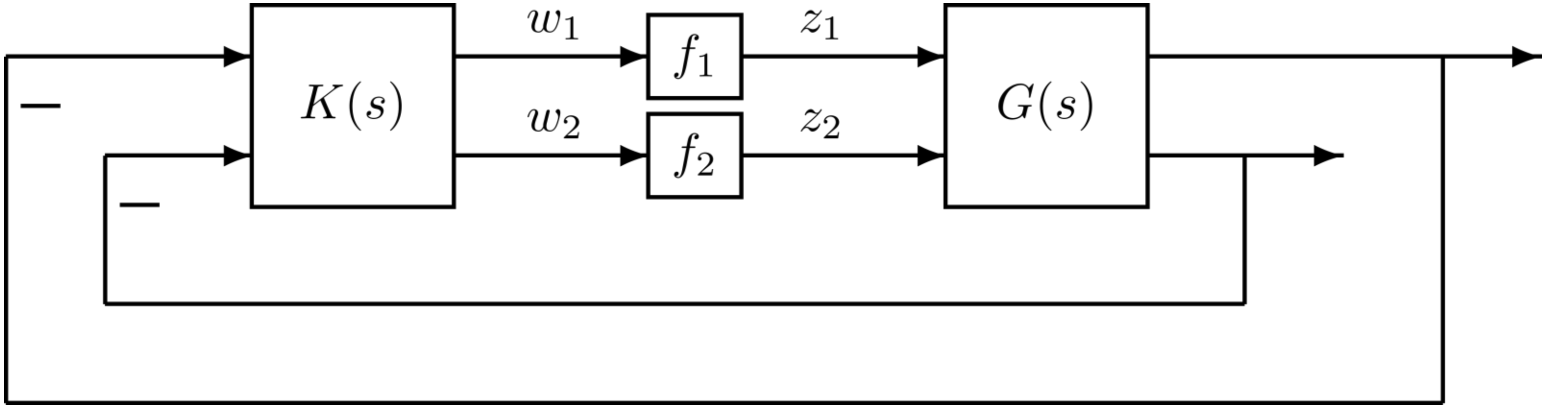
- The feedback system is stable for all (f_1, f_2) strictly inside the symmetric disk $Disk(\alpha_{max})$. The perturbations can vary independently, i.e. (f_1, f_2) are not necessarily equal.
- If $\alpha_{max} < 1$ then there are complex numbers (f_1, f_2) on the edge of the symmetric disk that cause instability. There are equivalent LTI systems $(F_1(s), F_2(s))$ that remain in the disk and cause instability. These can be studied further in high fidelity, nonlinear sims.



Multi-loop Disk Margins

A typical rule of thumb is to require a multi-loop input disk margin with $\alpha_{max} \geq \frac{1}{3}$. This implies:

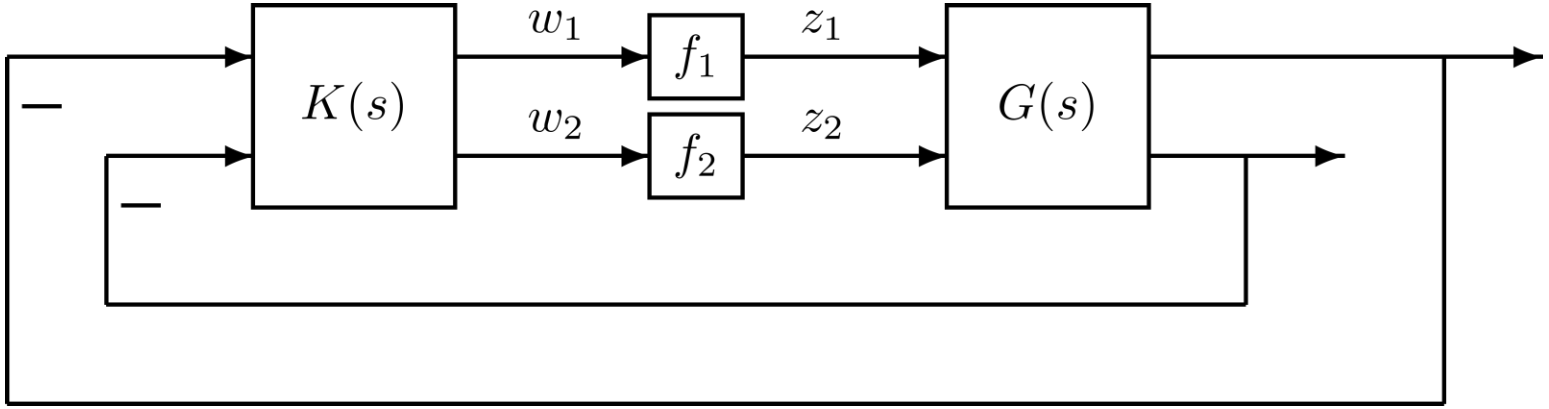
- Classical gain variations simultaneously and independently at each input of [0.5,2] = ±6 dB.
- Classical phase variations simultaneously and independently at each input of $\pm 37^o$.



Multi-loop Disk Margins

A typical rule of thumb is to require a multi-loop input disk margin with $\alpha_{max} \geq \frac{1}{3} \Rightarrow$ ±6 dB / $\pm 37^o$ of gain/phase margins.

The multi-loop margins tend to decrease as the system has more inputs/outputs because the multi-loop uncertainty can couple together. Thus, $\alpha_{max} \geq \frac{1}{3}$ is often not achievable large MIMO systems. The required value for the multi-loop margin is determined, in such cases, based on domain expertise.

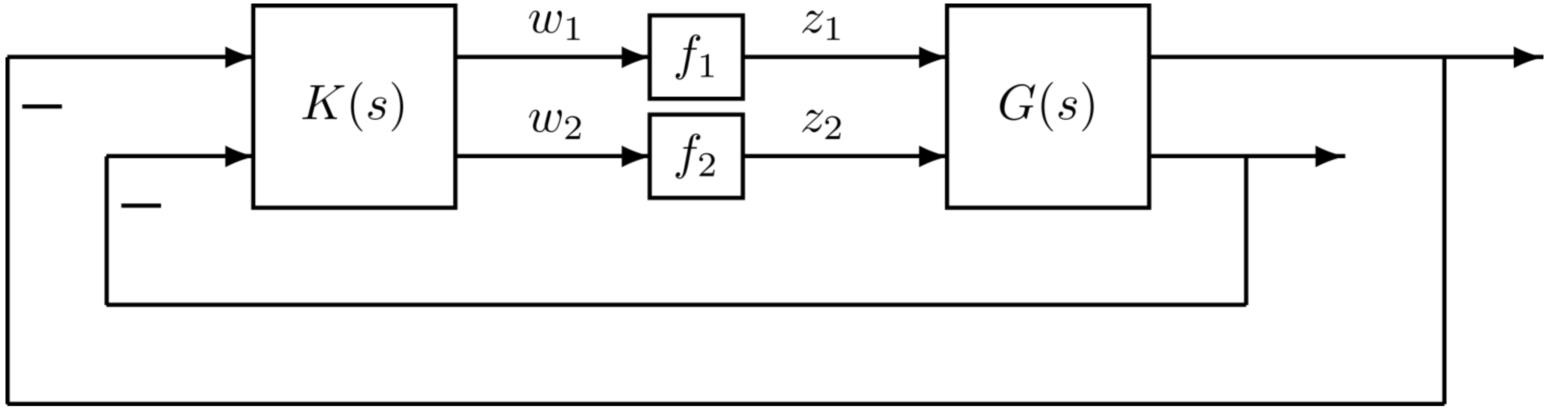


Multi-loop Disk Margins

We can compute multi-loop margins with:

A. perturbations on all input channels (as on previous slides),
B. perturbations on all output channels, or
C. perturbations on all input and output channels.

The margins can be significantly different in each case.



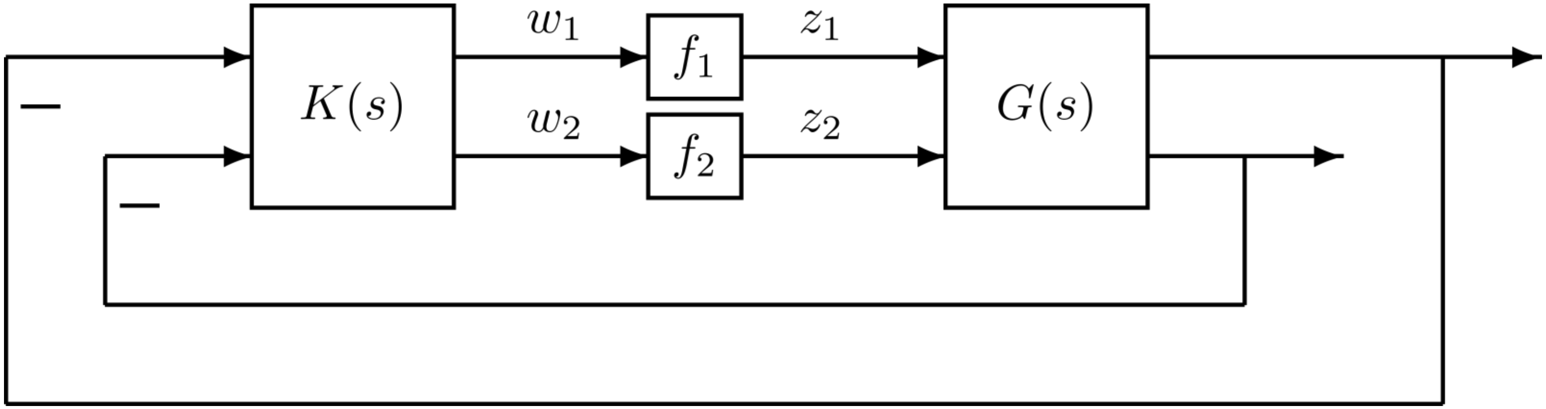
Multi-loop Disk Margins

We'll discuss the numerical algorithms later. The theory is part of a more general framework known as the structured singular value.

Matlab Syntax:

```
[DMI,MMI] = diskmargin(K*G);
[DMO,MMO] = diskmargin(G*K);
MMIO = diskmargin(G,K);
```

DMI are loop-at-a-time disk margins and MMI are multi-loop disk margins at the plant input. Similar for DMO/MMO. MMIO are multi-loop disk margins at the plant input and output.



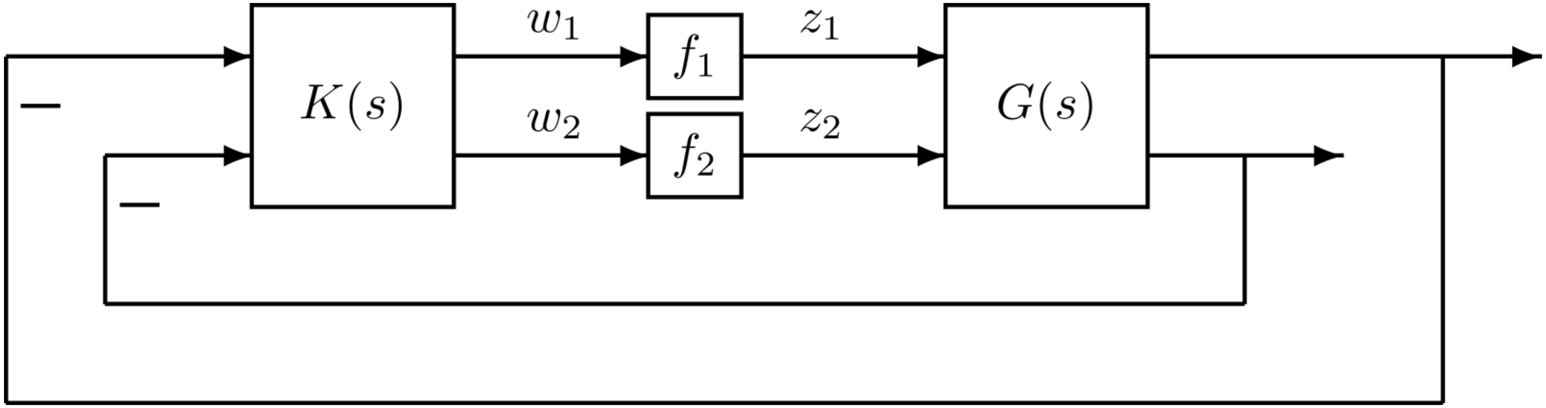
Spinning Satellite Example

Plant and Controller:

$$G(s) := \frac{1}{s^2 + a^2} \begin{bmatrix} s - a^2 & a(s+1) \\ -a(s+1) & s - a^2 \end{bmatrix} \text{ and } K(s) := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

with *a = 10.*

Recall the feedback system has large loop-at-a-time margins but is sensitive to simultaneous margins in both loops.



Spinning Satellite Example

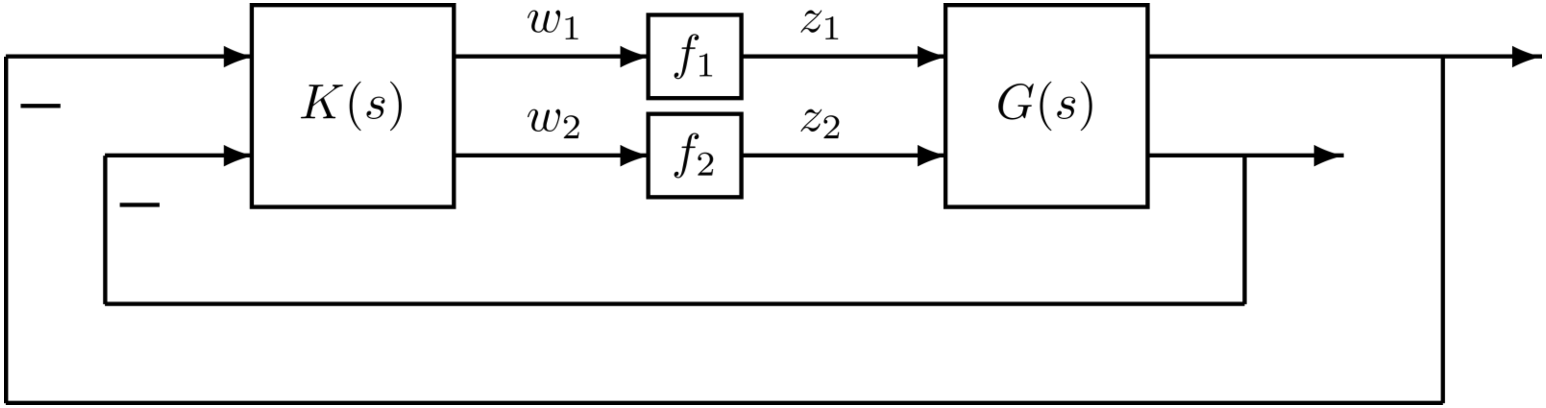
Plant and Controller:

$$G(s) := \frac{1}{s^2 + a^2} \begin{bmatrix} s - a^2 & a(s+1) \\ -a(s+1) & s - a^2 \end{bmatrix} \text{ and } K(s) := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

with *a = 10.*

Recall the feedback system has large loop-at-a-time margins but is sensitive to simultaneous margins in both loops.

Multi-loop margin at plant input is $\alpha_{max} = 0.0997$. The plant can only tolerate simultaneous and independent variations inside the symmetric disk with diameter *[0.905, 1.105]*.



Spinning Satellite Example

```
% Plant
a=10; A=[0 a; -a 0]; B=eye(2); C=[1 a; -a 1]; D=0;
G = ss(A,B,C,D);
% Controller
K = eye(2);
% Loop-at-a-time (DMI) and multi-loop (DMI)
% disk margins at input
[DMI,MMI] = diskmargin(K*G);
MMI =
   GainMargin: [0.9051 1.1049]
   PhaseMargin: [-5.7060 5.7060]
   DiskMargin: 0.0997
   LowerBound: 0.0997
   UpperBound: 0.0999
   Frequency: 1.0000e-04
   WorstPerturbation: [2×2 ss]
```

Small multi-loop margins indicate that the system is sensitive to simultaneous perturbations at both plant inputs.

Spinning Satellite Example

```
% Plant
a=10; A=[0 a; -a 0]; B=eye(2); C=[1 a; -a 1]; D=0;
G = ss(A,B,C,D);
% Controller
K = eye(2);
% Loop-at-a-time (DMI) and multi-loop (DMI)
% disk margins at input
[DMI,MMI] = diskmargin(K*G);
MMI =
  GainMargin: [0.9051 1.1049]
  PhaseMargin: [-5.7060 5.7060]
  DiskMargin: 0.0997
  LowerBound: 0.0997
  UpperBound: 0.0999
  Frequency: 1.0000e-04
  WorstPerturbation: [2×2 ss]
```

Multi-loop margins are difficult to compute exactly, in general (for reasons that we will discuss later.) Thus we compute upper and lower bounds but these are often close to each other.

Spinning Satellite Example

```
% Plant
a=10; A=[0 a; -a 0]; B=eye(2); C=[1 a; -a 1]; D=0;
G = ss(A,B,C,D);
% Controller
K = eye(2);
% Loop-at-a-time (DMI) and multi-loop (DMI)
% disk margins at input
[DMI,MMI] = diskmargin(K*G);
MMI =
   GainMargin: [0.9051 1.1049]
   PhaseMargin: [-5.7060 5.7060]
   DiskMargin: 0.0997
   LowerBound: 0.0997
   UpperBound: 0.0999
   Frequency: 1.0000e-04
   WorstPerturbation: [2×2 ss]
```

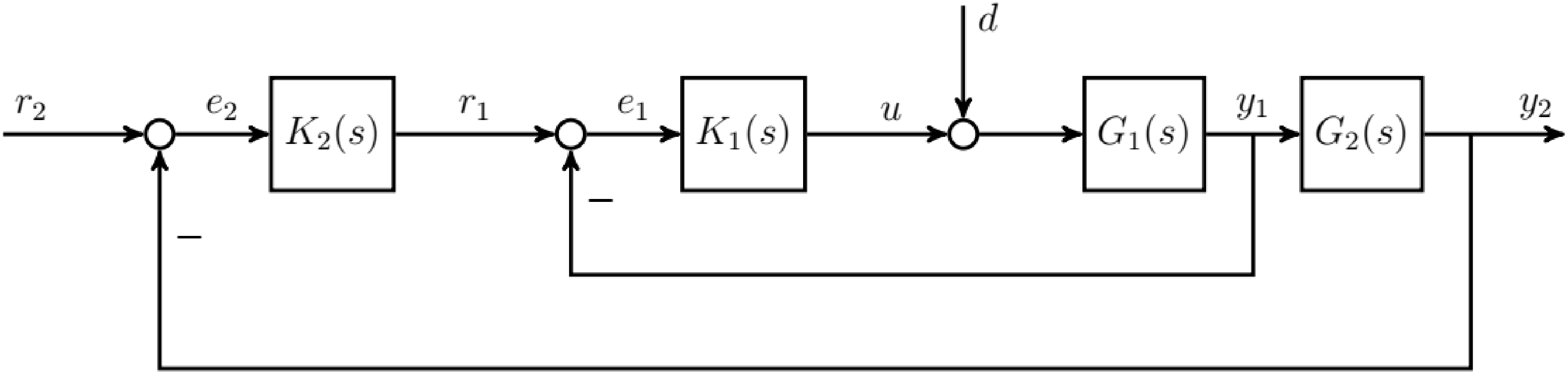
`.WorstPerturbation` gives the LTI perturbation $diag([F_1(s), F_2(s)])$ that causes instability with closed-loop poles on the axis at `.Frequency`.

Margins For Cascaded Loops

Consider the cascaded-loop below.

The main design principle in cascaded systems is that the inner feedback loop should be fast relative to the outer loop.

We can apply the multi-loop margin framework to illustrate that the margins degrade when this principle is violated.

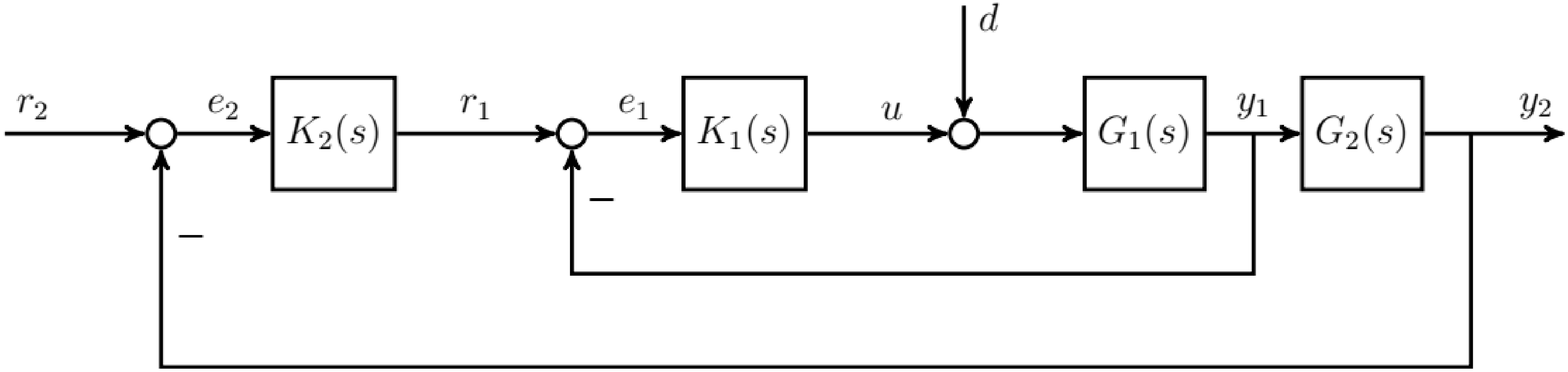


Margins For Cascaded Loops

Plant:

$$G(s) = \underbrace{\frac{\omega_{n2}^2}{s^2 + 2\zeta_2\omega_{n2}\, s + \omega_{n2}^2}}_{:=G_2(s)} \cdot \underbrace{\frac{\omega_{n1}^2}{s^2 + 2\zeta_1\omega_{n1}\, s + \omega_{n1}^2}}_{:=G_1(s)}$$

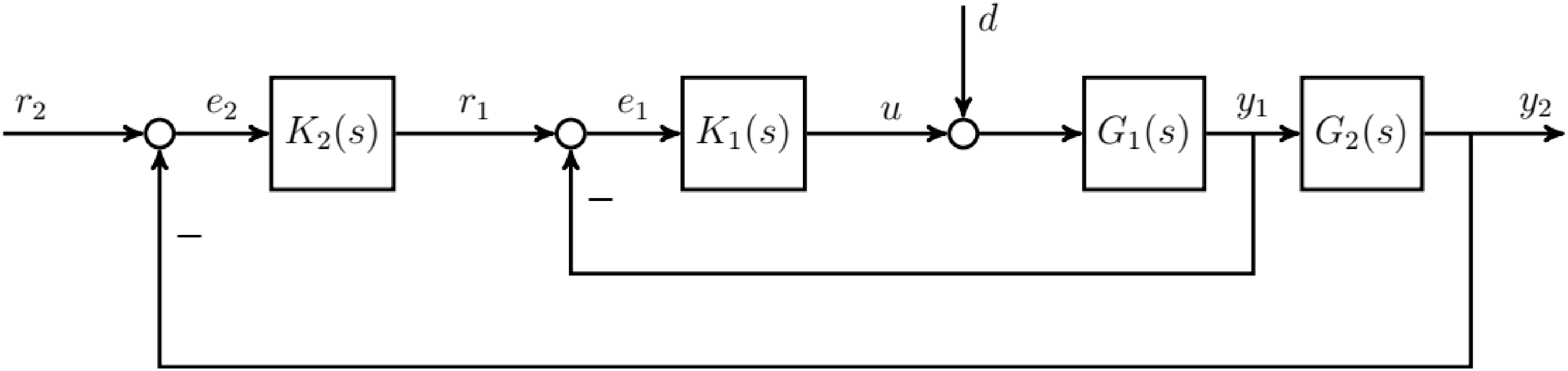
where $\omega_{n1} = 20$ rad/s, $\omega_{n2} = 5$ rad/s, and $\zeta_1 = \zeta_2 = 0.7$.



Margins For Cascaded Loops

1. Design inner-loop PID with loop bandwidth with $\omega_{L1} = 30$rad/s.
Inner-loop margins with outer-loop open:

Disk margin = 1.23 → disk phase margin = 63^{o}.



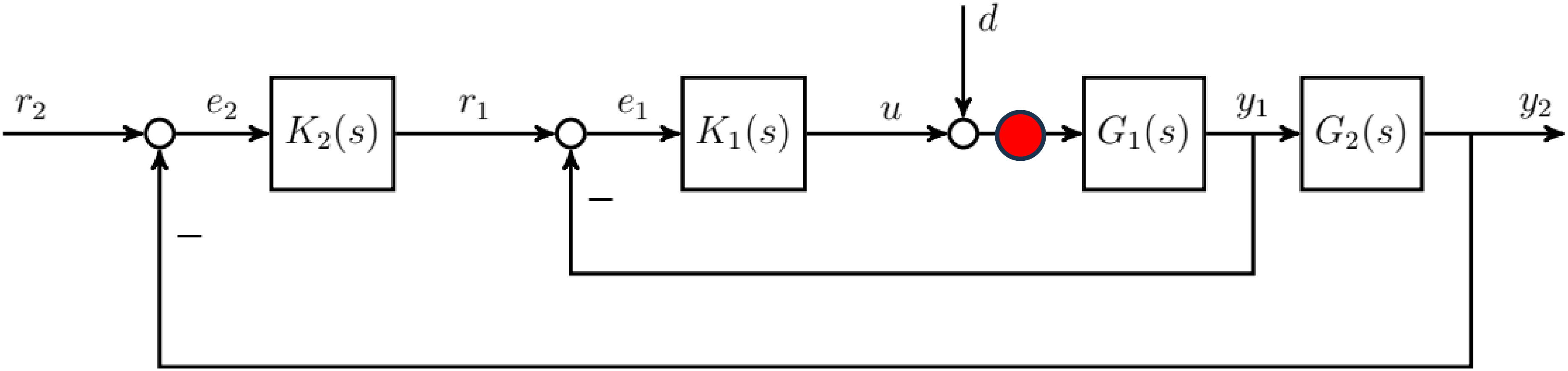
Margins For Cascaded Loops

1. Design inner-loop PID with loop bandwidth with $\omega_{L1} = 30$rad/s. Inner-loop margins with outer-loop open:

Disk margin = 1.23 → disk phase margin = 63°.

2. Design outer-loop PID with loop bandwidth with $\omega_{L2} = 5$rad/s. Margins with both inner and outer loop closed:

a. Input to G_1: Disk margin = 1.23 → disk phase margin = 63°.



Margins For Cascaded Loops

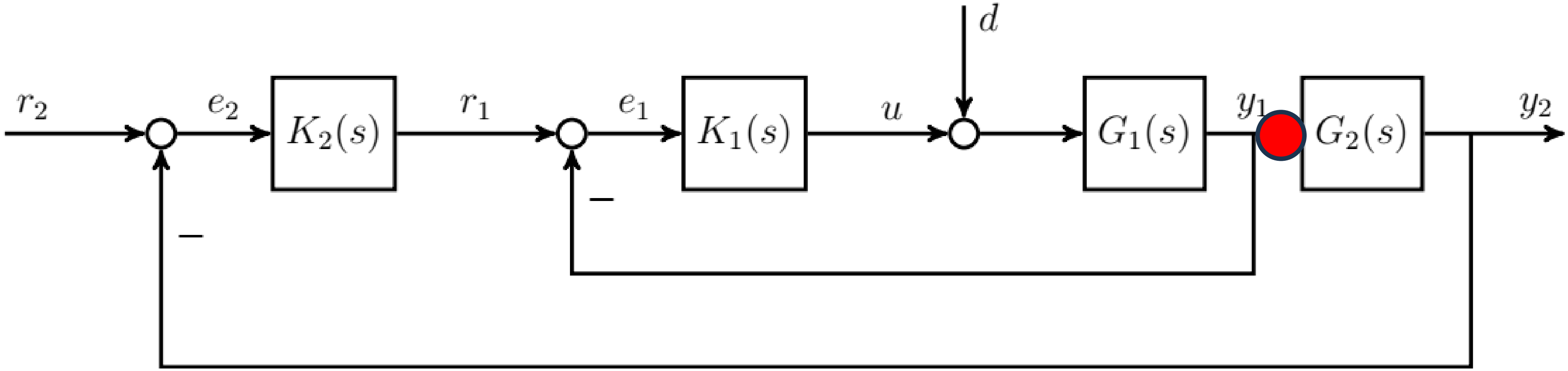
1. Design inner-loop PID with loop bandwidth with $\omega_{L1} = 30$rad/s. Inner-loop margins with outer-loop open:

Disk margin = 1.23 → disk phase margin = 63°.

2. Design outer-loop PID with loop bandwidth with $\omega_{L2} = 5$rad/s. Margins with both inner and outer loop closed:

a. Input to G_1: Disk margin = 1.23 → disk phase margin = 63°.

b. Input to G_2: Disk margin = 1.25 → disk phase margin = 64°.



Margins For Cascaded Loops

1. Design inner-loop PID with loop bandwidth with $\omega_{L1} = 30$rad/s. Inner-loop margins with outer-loop open:

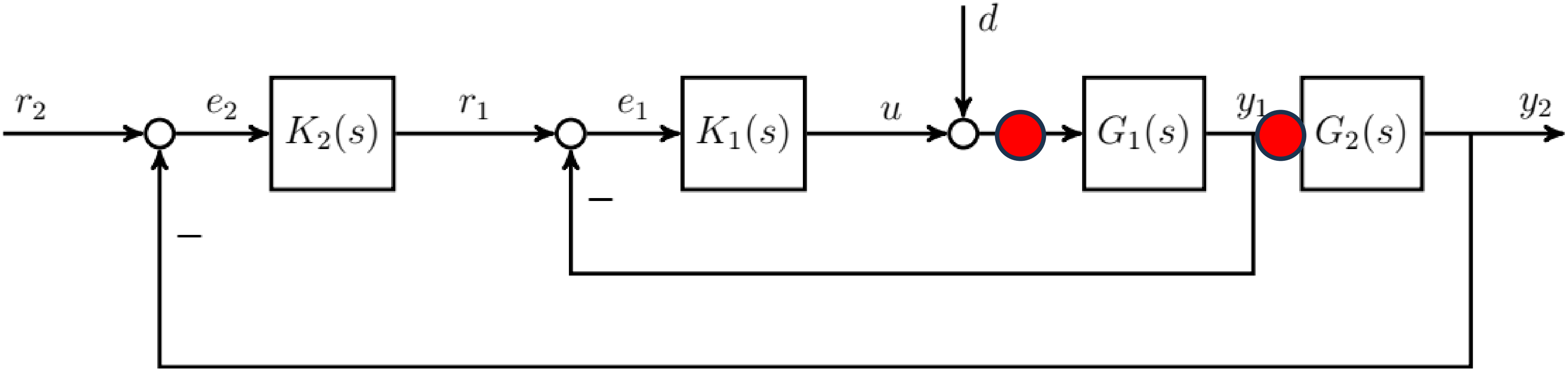
Disk margin = 1.23 → disk phase margin = 63°.

2. Design outer-loop PID with loop bandwidth with $\omega_{L2} = 5$rad/s. Margins with both inner and outer loop closed:

a. Input to G_1: Disk margin = 1.23 → disk phase margin = 63°.

b. Input to G_2: Disk margin = 1.25 → disk phase margin = 64°.

c. Input to G_1 and G_2: Disk margin = 0.84 → disk phase margin = 46°.



Margins For Cascaded Loops

1. Design inner-loop PID with loop bandwidth with $\omega_{L1} = 30$rad/s. Inner-loop margins with outer-loop open:

Disk margin = 1.23 → disk phase margin = 63°.

2. Design outer-loop PID with loop bandwidth with $\omega_{L2} = 5$rad/s. Margins with both inner and outer loop closed:

a. Input to G_1: Disk margin = 1.23 → disk phase margin = 63°.

b. Input to G_2: Disk margin = 1.25 → disk phase margin = 64°.

c. Input to G_1 and G_2: Disk margin = 0.84 → disk phase margin = 46°.

Large bandwidth separation between loops so cascade margins are sufficient. There is noticeable degradation if uncertain occurs at both G_1 and G_2 (e.g. actuator uncertainty and aircraft uncertainty in a flight control system.)

Margins For Cascaded Loops

1. Design inner-loop PID with loop bandwidth with $\omega_{L1} = 30$rad/s. Inner-loop margins with outer-loop open:

Disk margin = 1.23 → disk phase margin = 63°.

2. Design outer-loop PID with loop bandwidth with $\omega_{L2} = 15$rad/s. Margins with both inner and outer loop closed:

a. Input to G_1: Disk margin = 0.80 → disk phase margin = 43°.

b. Input to G_2: Disk margin = 0.61 → disk phase margin = 34°.

c. Input to G_1 and G_2: Disk margin = 0.37 → disk phase margin = 21°.

Margins significantly degrade with the two loop bandwidths are close. Again, there is noticeable degradation if uncertain occurs at both G_1 and G_2.

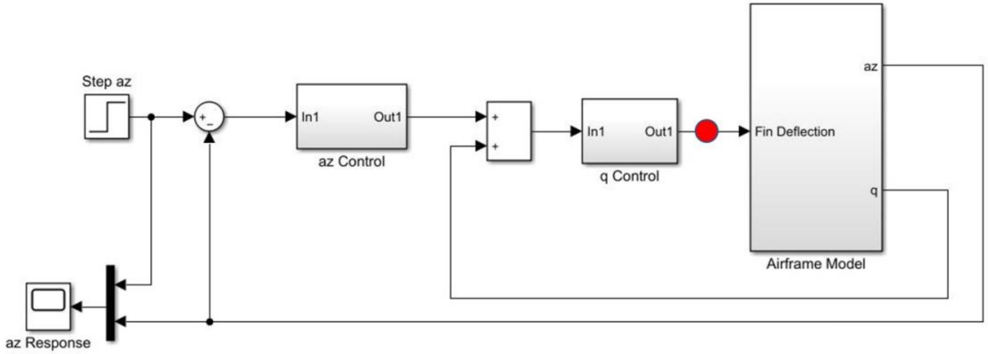
General Multi-Loop Margins

We can generalize multi-loop margins to allow for symmetric disk uncertainty injected at arbitrary points in a block diagram.

Example: Stability Margins of a Simulink Model,

https://www.mathworks.com/help/robust/ug/stabilitymargin-of-a-simulink-model.html

- One perturbation is inserted at the plant input.
- Two perturbations in the aero subsystem. These model the discrepancy in the modeled and actual aerodynamics.



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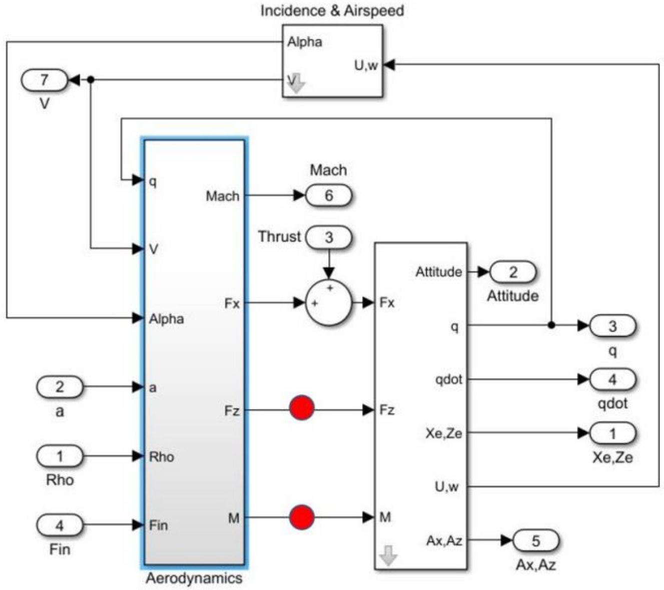

General Multi-Loop Margins

The model is nonlinear but we can linearize at a given operating point and assess the margins of the linearized dynamics.

$$\alpha_{max} = 0.428 \Rightarrow \text{Symmetric disk diameter is } [0.65, 1.54].$$

```
% Open simulink model from Matlab example
open_system('airframemarginEx.slx')
% Specify analysis point at plant input
aPoints(1) = linio('airframemarginEx/q Control‵, 1,
     'looptransfer');
% Specify analysis points inside aerodynamic model
blk = ['airframemarginEx/Airframe Model/' ...
     'Aerodynamics & Equations of Motion/Aerodynamics'];
aPoints(2) = linio(blk,3,'looptransfer');
aPoints(3) = linio(blk,4,'looptransfer');
% Linearize and compute disk margins at three analysis points
L3 = linearize('airframemarginEx',aPoints);
[DM3,MM3] = diskmargin(-L3)
```

Conclusions

- Any input/output closed-loop TF is one of the “gang of six”:
$$S_I(s), S_O(s), T_I(s), T_O(s), K(s)S_O(s), G(s)S_I(s)$$
- Loop-at-a-time margins for MIMO system assess robustness with uncertainty in one channel. This analysis can be overly optimistic.
- Multi-loop disk margins assess robustness to uncertainty introduced simultaneously in multiple channels.

We will discuss the theory that is required to compute multi-loop disk margins in upcoming lectures.