kdFlex
Multi-Domain Rigid/Flexible Body Dynamics Toolkit

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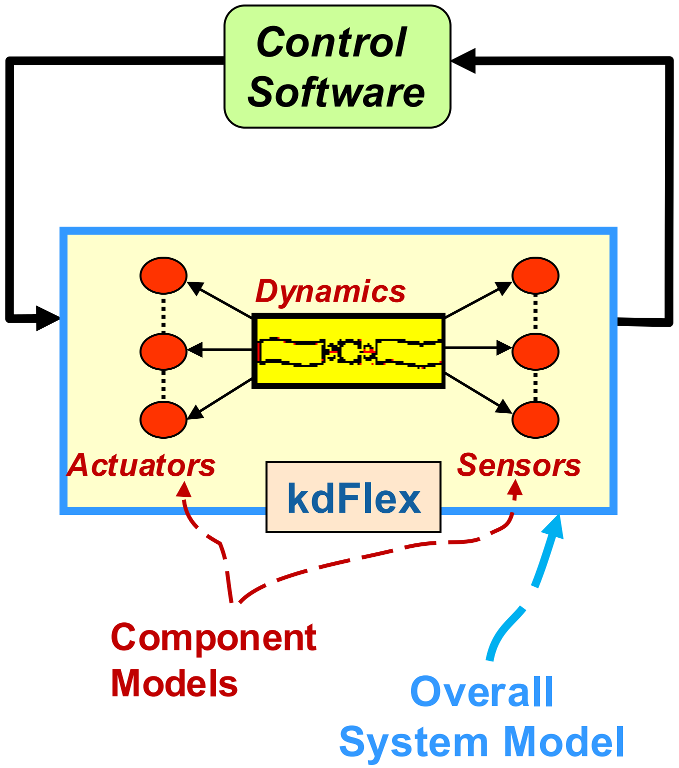
https://karanadyn.com

NESC Flight Mechanics TDT Briefing

Karana Dynamics Background

- **Karana Dynamics** has been founded by ex-team members from the **Dynamics and Real-Time Simulation (DARTS) Lab** at **NASA's Jet Propulsion Laboratory (JPL)**
  - **DARTS** is a high-performance dynamics simulation level tool (based on the **Spatial Operator Algebra (SOA)** multibody dynamics theory developed by Abhi and other researchers) for recursive minimal coordinate multibody dynamics to support high-speed, high-fidelity, large-scale, and mission-critical engineering applications
  - **DARTS** has been used for several deep space exploration missions to Jupiter, Saturn, Venus, etc., landers on Mars, first non-Earth helicopter on Mars, Mars rovers, and robotics technology development, and ARTEMIS I mission and is part of the DOLILU software for the forthcoming ARTEMIS II
- Abhi was a Senior Research Scientist at JPL for > 35 years, and founded and led the JPL DARTS Lab for over 25+ years
- **kdFlex** is the next of multiple generations of "DARTS"
  - developed from the ground up for broader access to the community
  - **kdFlex** beta release is available at http://karanadyn.com



JPL DARTS Simulation Toolkit

Closed-loop dynamics
Physics-based
Engineering sims
Key advanced middleware
Component based models
Workflow for reuse

Based on 'Spatial Operator Algebra (SOA)' methodology for multibody/robot dynamics

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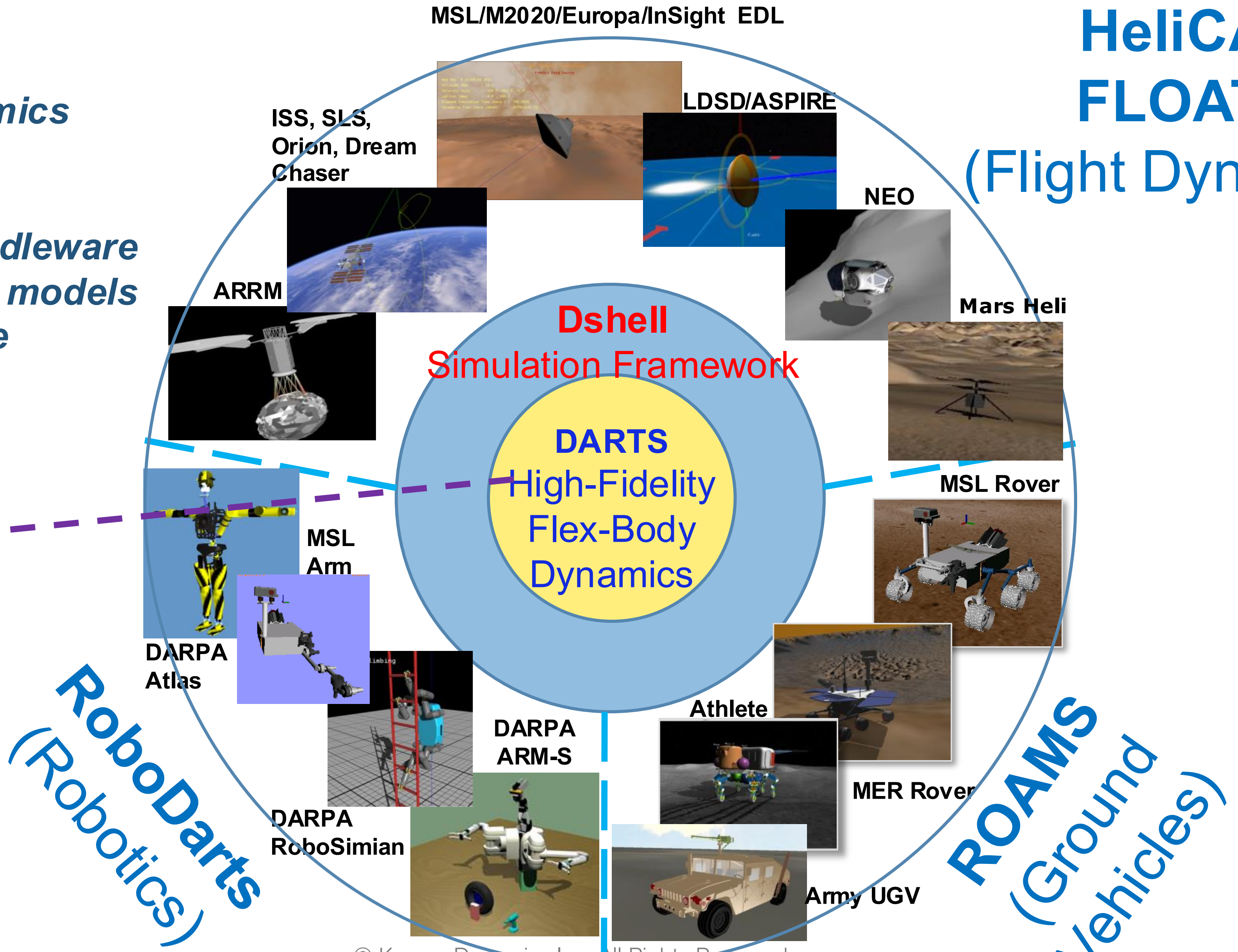
kdFlex

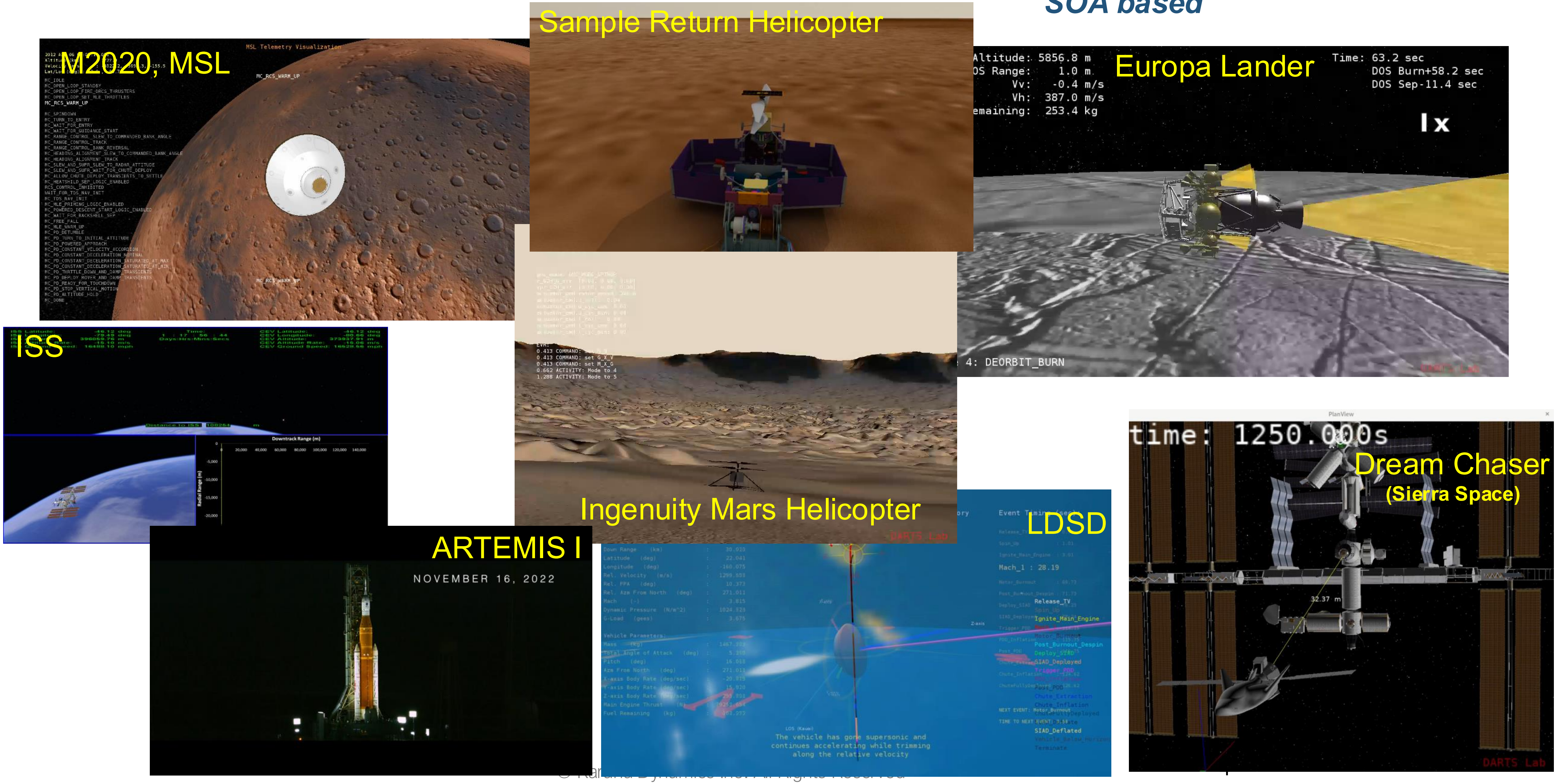


DSENDS
HeliCAT
FLOATS
(Flight Dynamics)

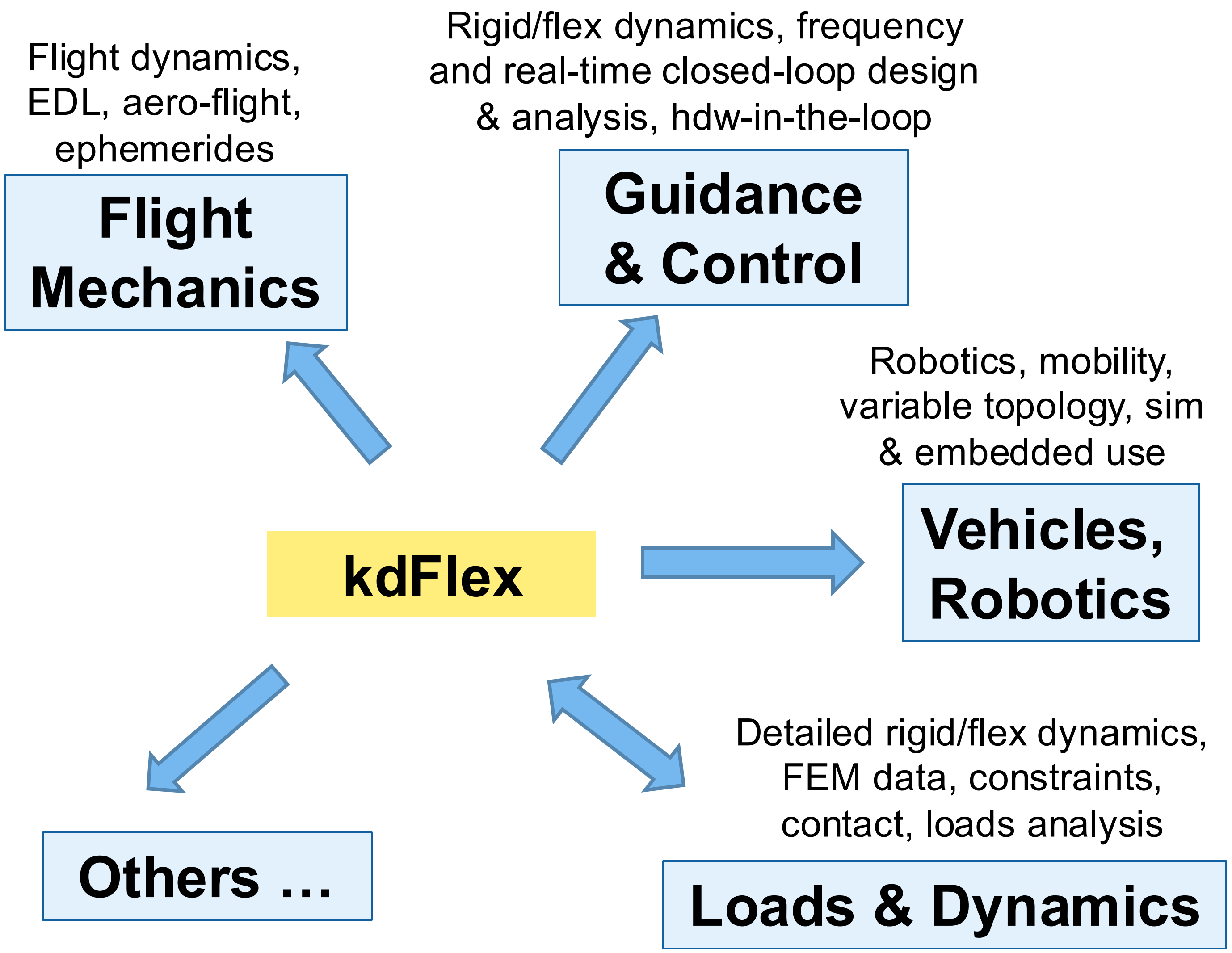
- MSR (SRL, NAV, SRH, EES)
- Lunar Terrain Vehicle
- Sierra Dream Chaser
- ARTEMIS I & II
- CADRE
- EELS
- Endurance
- Chopper
- M2020, Ingenuity
- ARRM, Europa Lander
- LDSD, ASPIRE
- MSL, InSight, Phoenix
- ...
- SIM
- SRTM
- Cassini

Flight Dynamics Simulations (JPL DARTS software)

SOA based

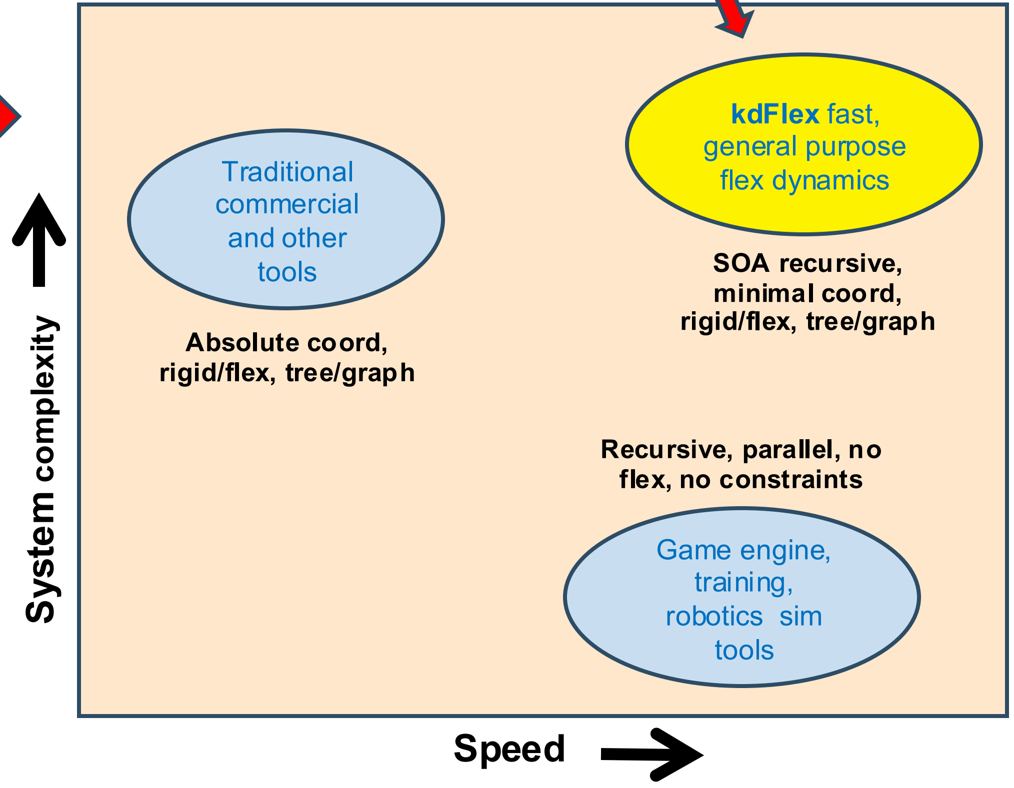


kdFlex Multi-Domain Dynamics Toolkit



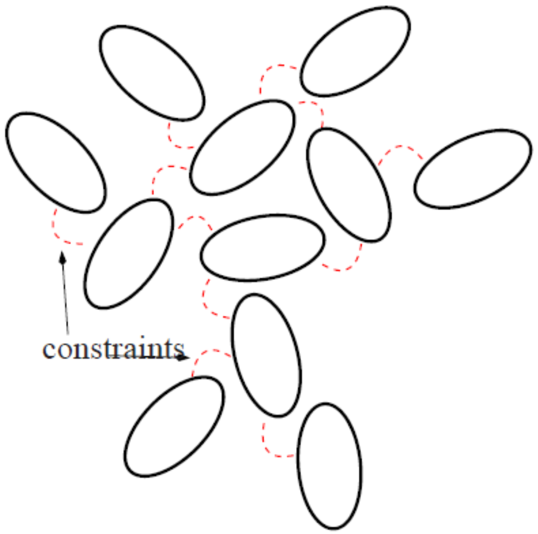
kdFlex is an **SOA** (Spatial Operator Algebra) based next-gen dynamics engine that avoids the unholy choice between speed and fidelity

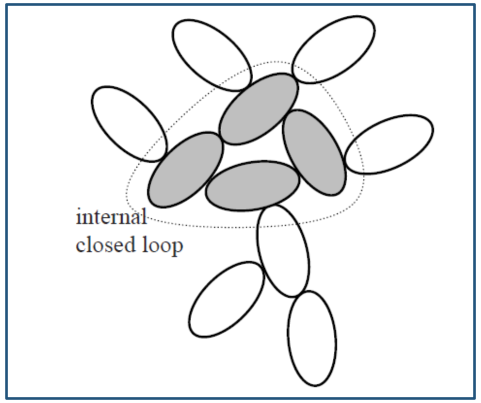
- broad-based computational methodology and fast "O(N)" algorithms

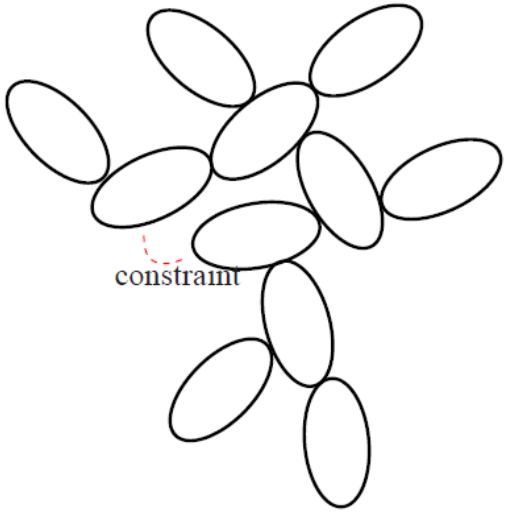


kdFlex - a one-stop shop for cross-cutting, high-performance dynamics solutions.

Multibody Dynamics Formulation Options







Absolute/Redundant coordinates

- All bodies are treated as independent
- Bilateral constraints used for all hinges, DAE form
- Large no. of dofs, simple to set up but O(N^3)
- Constant, block diagonal mass matrix

Minimal/Relative coordinates

- Minimal hinge coordinates, ODE form
- Smaller no. of dofs and O(N) methods
- Dense, configuration dependent mass matrix

Equations of Motion (EoM)

$$\mathcal{M}(\theta)\ddot{\theta} + \mathcal{C}(\theta,\dot{\theta}) + G_c^*(\theta,t)\lambda = \mathcal{T}$$

Algebraic constraints term for redundant coordinates only →

$$G_c(\theta,t)\dot{\theta} = \mathfrak{U}(t)$$

Spatial Operator Algebra (SOA)

Spatial Operator Algebra (SOA) – analytical Mass Matrix Inversion

$$\mathcal{M}(\theta) \triangleq H\phi\mathbf{M}\phi^* H^*$$

Analytical Newton-Euler factorization of the mass matrix

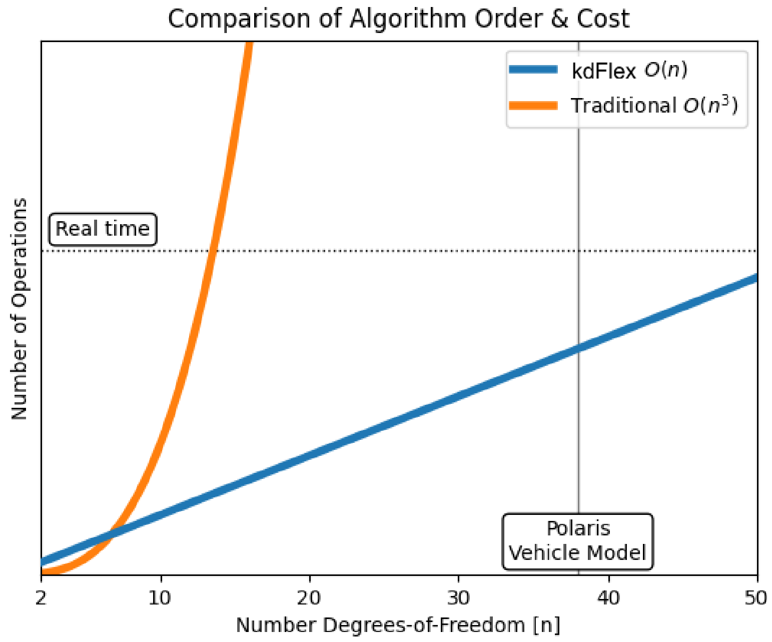
$$\mathcal{M} = [\mathbf{I} + H\phi\mathcal{K}]\mathcal{D}[\mathbf{I} + H\phi\mathcal{K}]^*$$

Analytical Innovations factorization of the mass matrix

$$[\mathbf{I} + H\phi\mathcal{K}]^{-1} = [\mathbf{I} - H\psi\mathcal{K}]$$

$$\mathcal{M}^{-1} = [\mathbf{I} - H\psi\mathcal{K}]^* \mathcal{D}^{-1} [\mathbf{I} - H\psi\mathcal{K}]$$

Analytical operator inversion of the mass matrix!



Solving the EoM

$$\mathcal{M}^{-1}\mathcal{T} \Longrightarrow [\mathbf{I} - H\psi\mathcal{K}]^* \mathcal{D}^{-1} [\mathbf{I} - H\psi\mathcal{K}]\mathcal{T} \Longrightarrow$$

O(N) recursions!

Does not require computing the mass matrix or its inversion (all O(N^3) steps)!

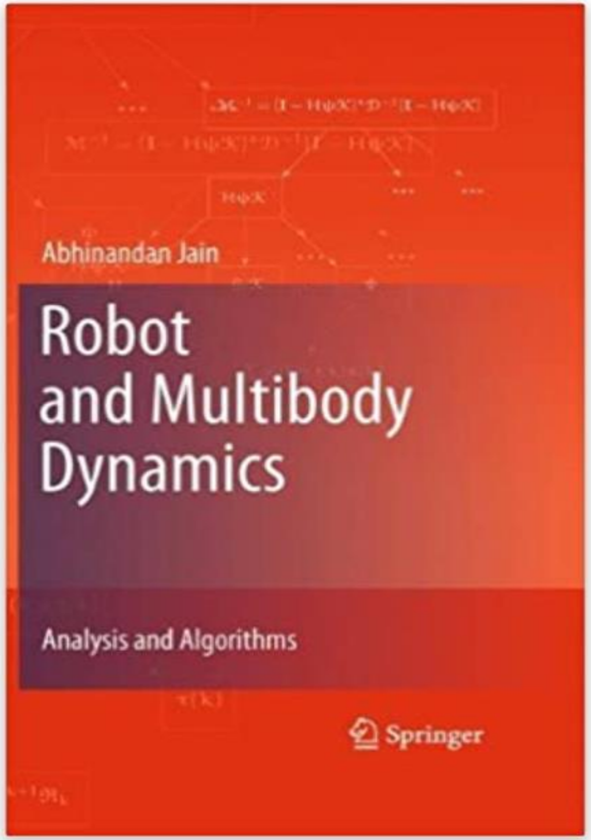
For a Deeper Dive into SOA ...

Discussion group *for SOA theory and methodology. More info at* https://portal.karanadyn.com/publications

SOA Discussion Group: Session slides and videos

Session	Date	Topic	Slides	Video	Updated
1	March 30, 2020	**SOA Introduction**	Slides	Video	
2	April 6, 2020	**1. Spatial Notation I**	Slides	Video	
3	April 13, 2020	**1. Spatial Notation II**	Slides	Video	May 9, 2020
4	April 20, 2020	**2. Single Rigid Body**	Slides	Video	
5	April 27, 2020	**3. Serial Chain Kinematics I**	Slides	Video	
6	May 4, 2020	**3. Serial Chain Kinematics II**	Slides	Video	May 4, 2020
7	May 11, 2020	**4. Serial Chain Dynamics I**	Slides	Video	
8	May 18, 2020	**4. Serial Chain Dynamics II**	Slides	Video	May 18, 2020
9	May 25, 2020	**5. Articulated Body Inertias**	Slides	Video	
10	June 1, 2020	**6. Mass Matrix Inversion**	Slides	Video	
11	June 8, 2020	**7. Recursive Forward Dynamics I**	Slides	Video	
12	June 15, 2020	**7. Recursive Forward Dynamics II**	Slides	Video	
13	June 22, 2020	**8. Graph Theory Structure I**	Slides	Video	
14	June 29, 2020	**8. Graph Theory Structure II**	Slides	Video	June 29, 2020
15	July 6, 2020	**8. Graph Theory Structure III**	Slides	Video	July 6, 2020
16	July 13, 2020	**8. Graph Theory Structure IV**	Slides	Video	July 18, 2020
17	July 20, 2020	**9. Multibody Graphs I**	Slides	Video	
18	July 27, 2020	**9. Multibody Graphs II**	Slides	Video	July 27, 2020
19	August 3, 2020	**10. Closed-chain dynamics**	Slides	Video	August 10, 2020
20	August 10, 2020	**11. Constraint Embedding**	Slides	Video	August 17, 2020
21	August 17, 2020	**12. Flexible Bodies**	Slides		

SOA theory book

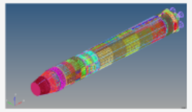


Publications: https://portal.karanadyn.com/publications

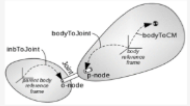
kdFlex Flexible Body Dynamics for Structures and Controls

Design Iteration

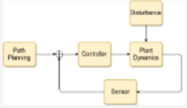
FEMBridge ***kdFlex***



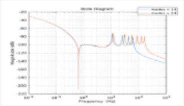
Structural Dynamics



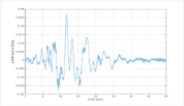
Multibody Dynamics



GN&C



Linear Analysis



Nonlinear Time Domain

NASTRAN FEM data for flex body model

Flexible multibody dynamics modeling

- Linear superposition
- Rigid bodies with springs
- Nonlinear flex bodies with small deformation

Flex body simulation for closed-loop control analyses

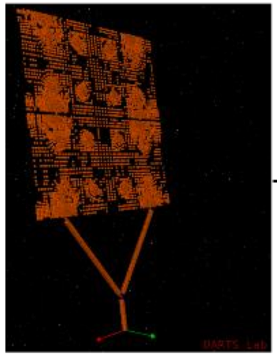
FEMBridge-kdFlex approach for a system

FEM

FEMBridge (component bodies)

kdFlex integrated multibody model

NASTRAN Bulk Data

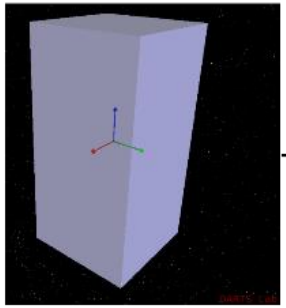


OP2 → Flex Data Extraction → HDF5

- Modal trunctation
- Fixed/free interface selection
- Global mass/stiffness/modes
- Craig Bampton mass/stiffness/modes
- Normalized Craig-Bampton modes
- Modal integrals
- Transform model into desired coordinate system

FEMBridge facilitates local processing of FEM data – steps that are often left to the structures team that generates the FEM data.

NASTRAN Bulk Data



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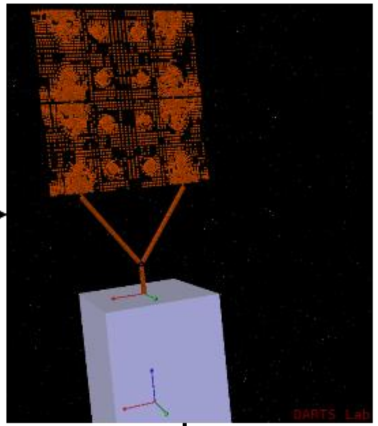
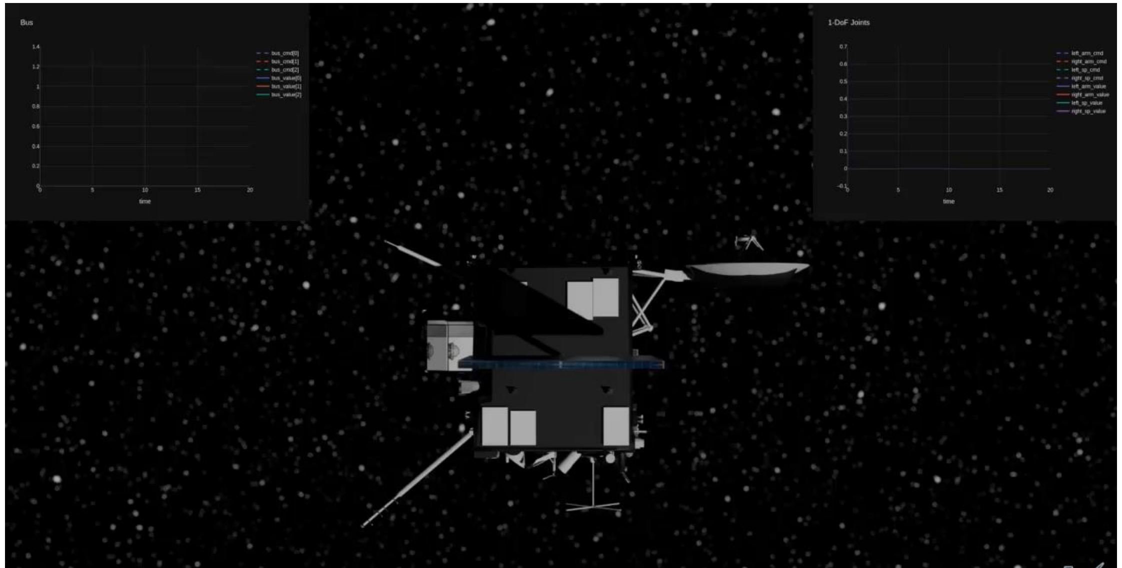


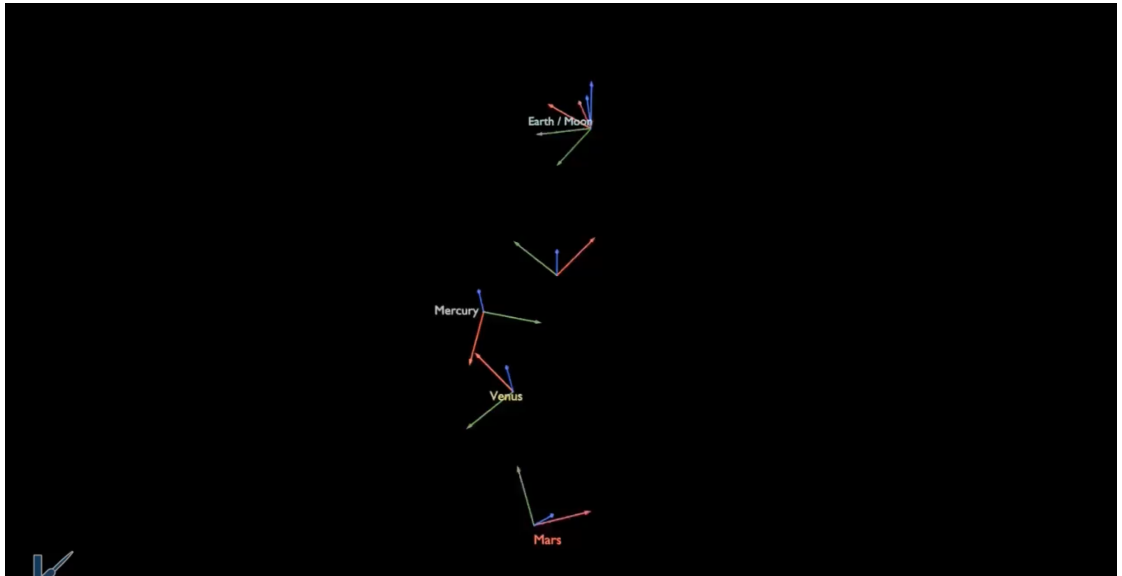
Spot checks with full-system modes. Iterate as needed.

Key kdFlex features

- Minimal coordinates **rigid/flex** multibody dynamics - unique
- Fast **O(N)** recursive algorithms based on SOA methodology
- Serial, tree, and **closed-chain** multibody topologies
- Small deformation **flexible body** dynamics
- **FEMBridge** tool to process FEM bulk data into **kdFlex** body data
- No code generation involved; model file input with units & quantities
- **ODE** (not DAE) solvers, no constraint error management
- Allows articulation at hinges; built-in/custom hinge types
- Built-in system **equilibration** and **linearization** capability
- G&C **frequency domain analysis**; **real-time closed loop** sims
- Allows run-time changes to system configuration
- Supports integration with **external device etc models**
- Supports mass property changes (eg. fuel consumption)
- Support for flex body **stress recovery** through post processing
- Sim services: **web-based 3D graphics**; data logging etc
- **Python/C++**, object oriented; Platforms: Linux, Windows with WSL 2



Closed-loop, G&C in the loop flexible body spacecraft simulation



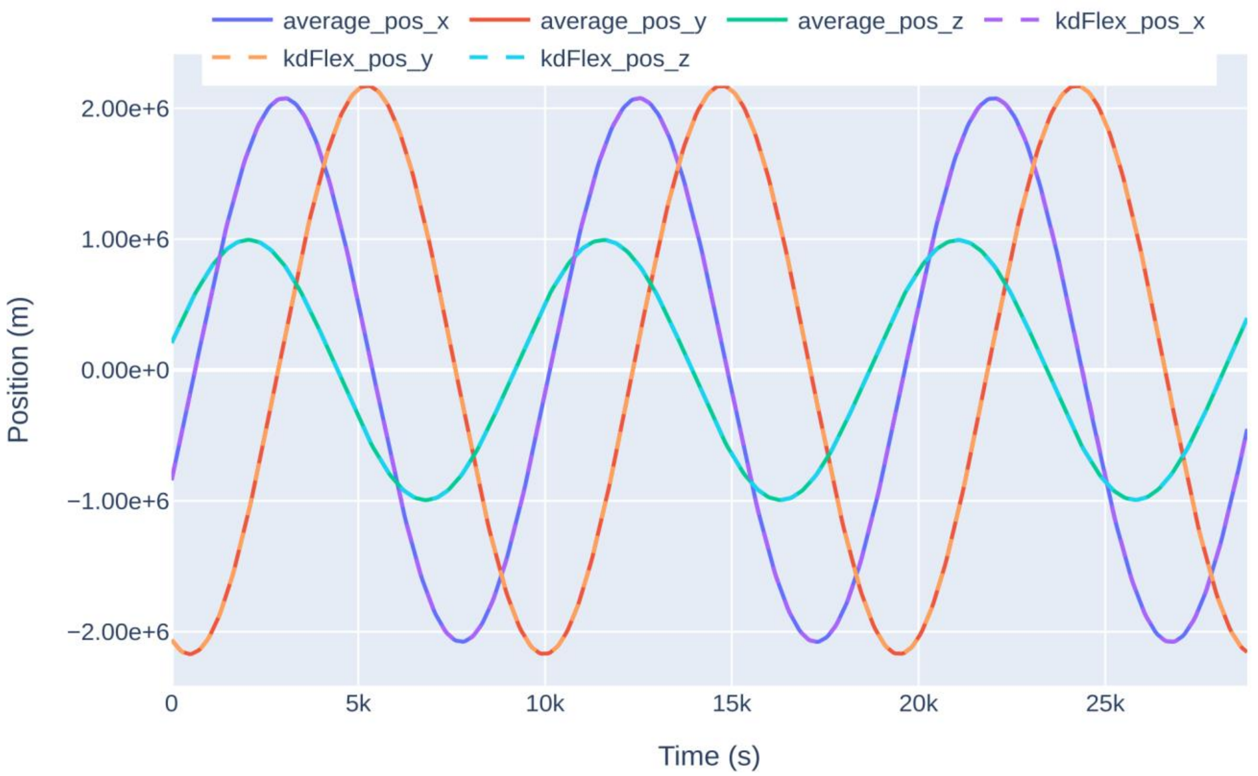
Ephemerides based celestarium sim

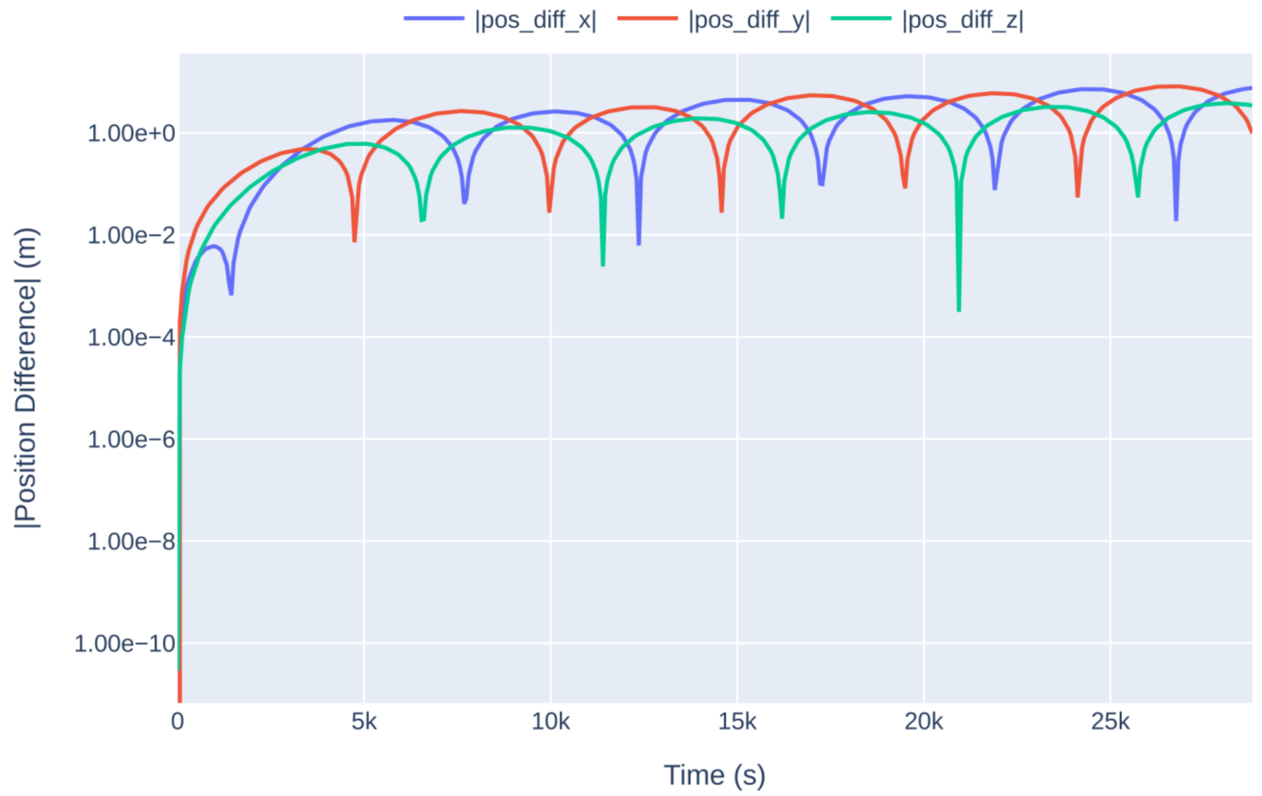
Additional Flight Dynamics and G&C Relevant Features

- **FEMBridge** for generating FEM based reduced order models using modal truncation
- Normalized spherical harmonics gravity modeling
- Gravity gradient support
- Non-inertial dynamics for PCR (planet-centric rotating frame)
- N-body gravity models
- Apparent mass models for parachute dynamics
- Support for deployables
- Separation/docking events support
- Ephemerides support
- Frames layer for relative pose, velocity, and acceleration queries
- Variable mass support
- Servo- and aero-elastic modeling
- Nonlinear, closed-loop real-time simulation performance
- Frequency domain analysis support
- Support for multi-rate models
- Large suite of adaptive, variable-time step integrators
- Nano-second precision time representation
- Support for units and quantities in input decks, auto-detection of uninitialized parameters
- Multiple format input decks

NESC Lunar Check Case 5 (https://nescacademy.nasa.gov/flightsim)

- NESC lunar check case 5 is a high-altitude circular orbit of the Apollo spacecraft around the moon. It uses an 8x8 spherical harmonic gravity model for the moon and N-body point-mass perturbaton gravity models for the Earth and Sun.
- The plots below show the position of the spacecraft vs. time for **kdFlex** compared with the average of eight other simulations for the same case. The left figure shows the position of kdFlex and the position of the average. The right figure shows the absolute value of the difference between the position in **kdFlex** and the position of the average.



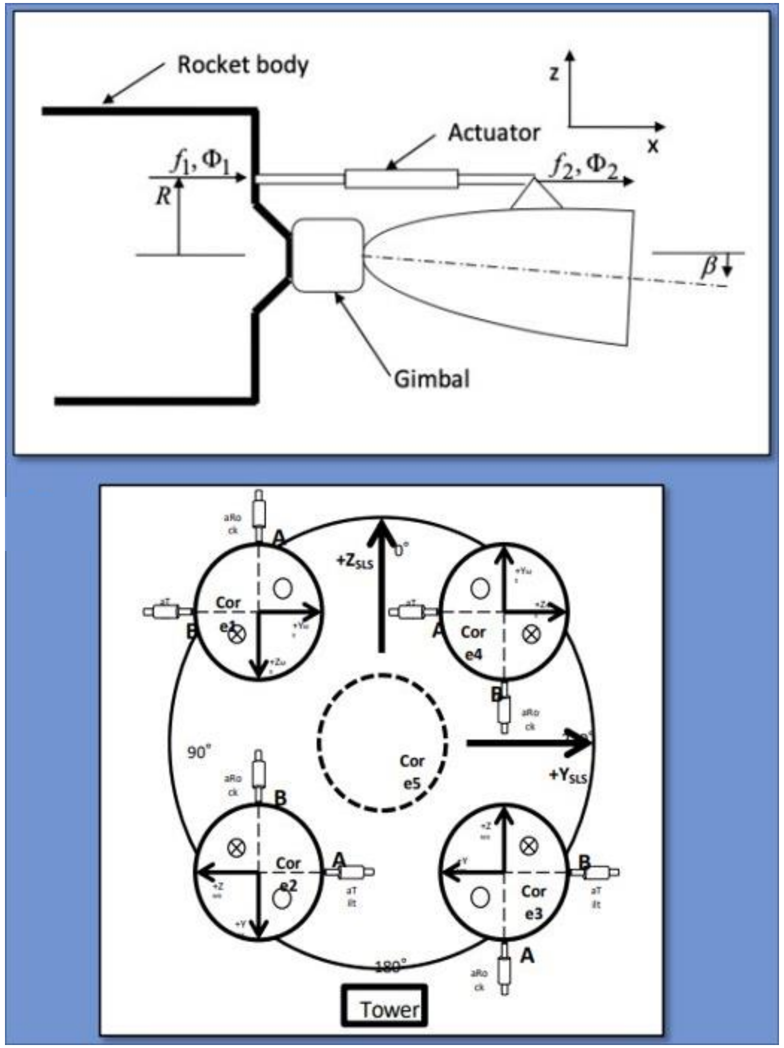


Parachute Apparent Mass Dynamics (NESC/JPL/Karana/LaRC/Dragonfly)

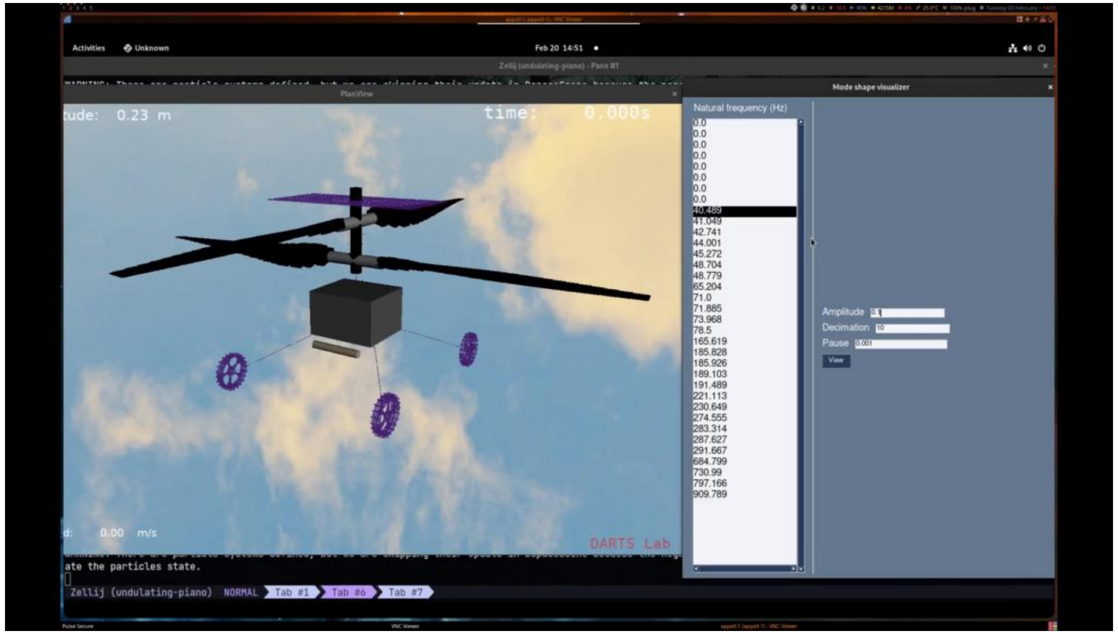
- Recent NESC assessment for evaluating LaRC **apparent mass** modeling approach for DragonFly (led by Heather)
  - Apparent mass dynamics are important parachute dynamics at Titan which has 5x thicker atmosphere than Earth
  - Apparent mass dynamics involves acceleration dependent forces – potentially requiring <u>implicit</u> methods
- Developed a novel theoretical reformulation for apparent mass dynamics to enable the incorporating it into standard dynamics codes – using simpler <u>explicit</u> approaches
- Initially implemented in **kdFlex**, and then ported to **DARTS/DSENDS**

Example kdFlex flexible body problems

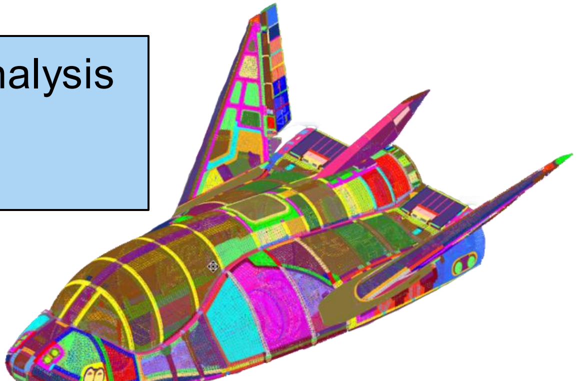
Servo-elastic analysis for launch vehicle thrust vector control



Sample Return Helicopter had a narrow design space required us to have an iterative loop with mechanical design to update masses, stiffnesses, etc. mechanical properties for G&C needs.

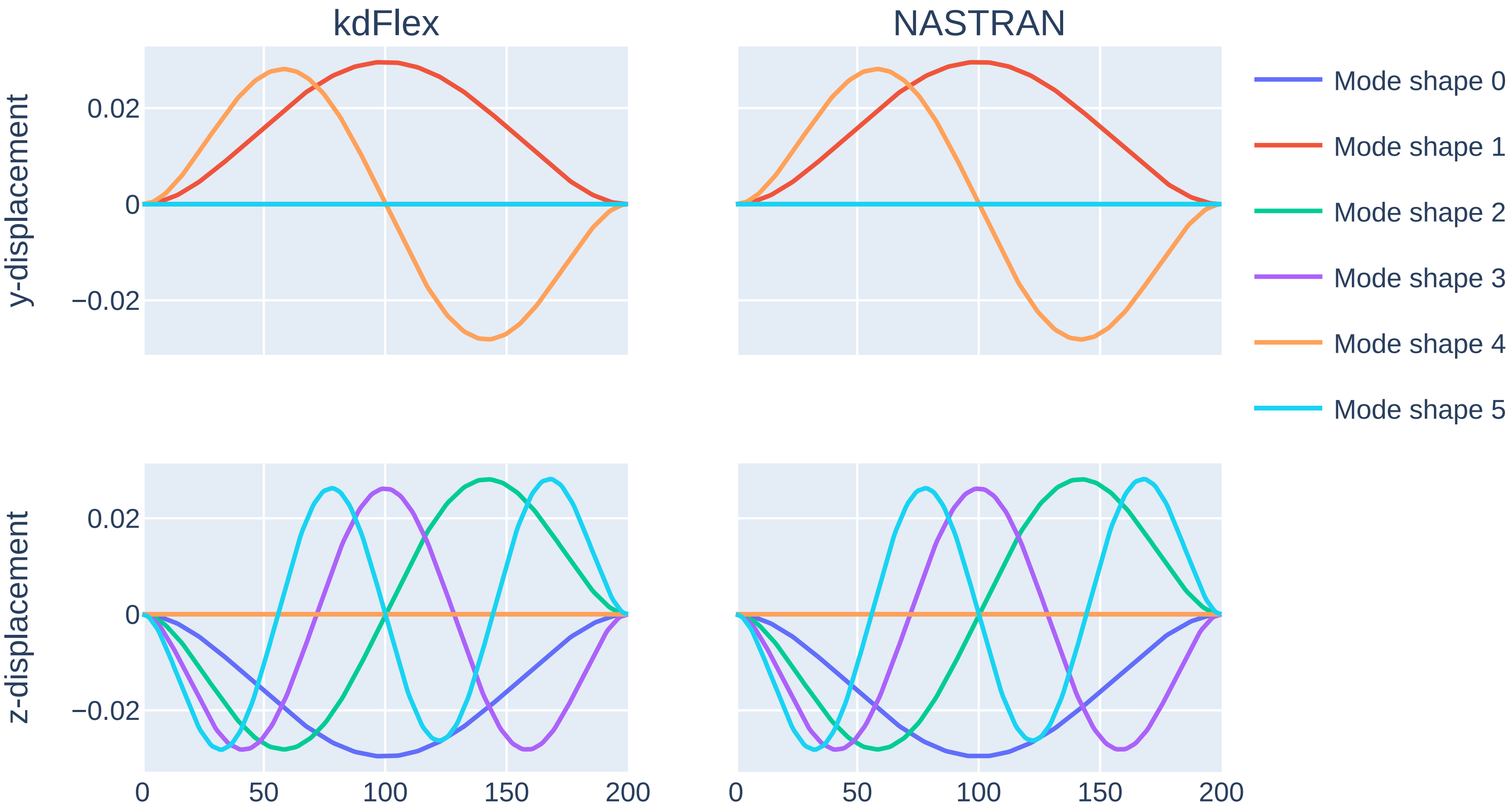


Aero-elastic dynamics analysis for earth re-entry vehicle for G&C system.



Recovering system-level modes with kdFlex

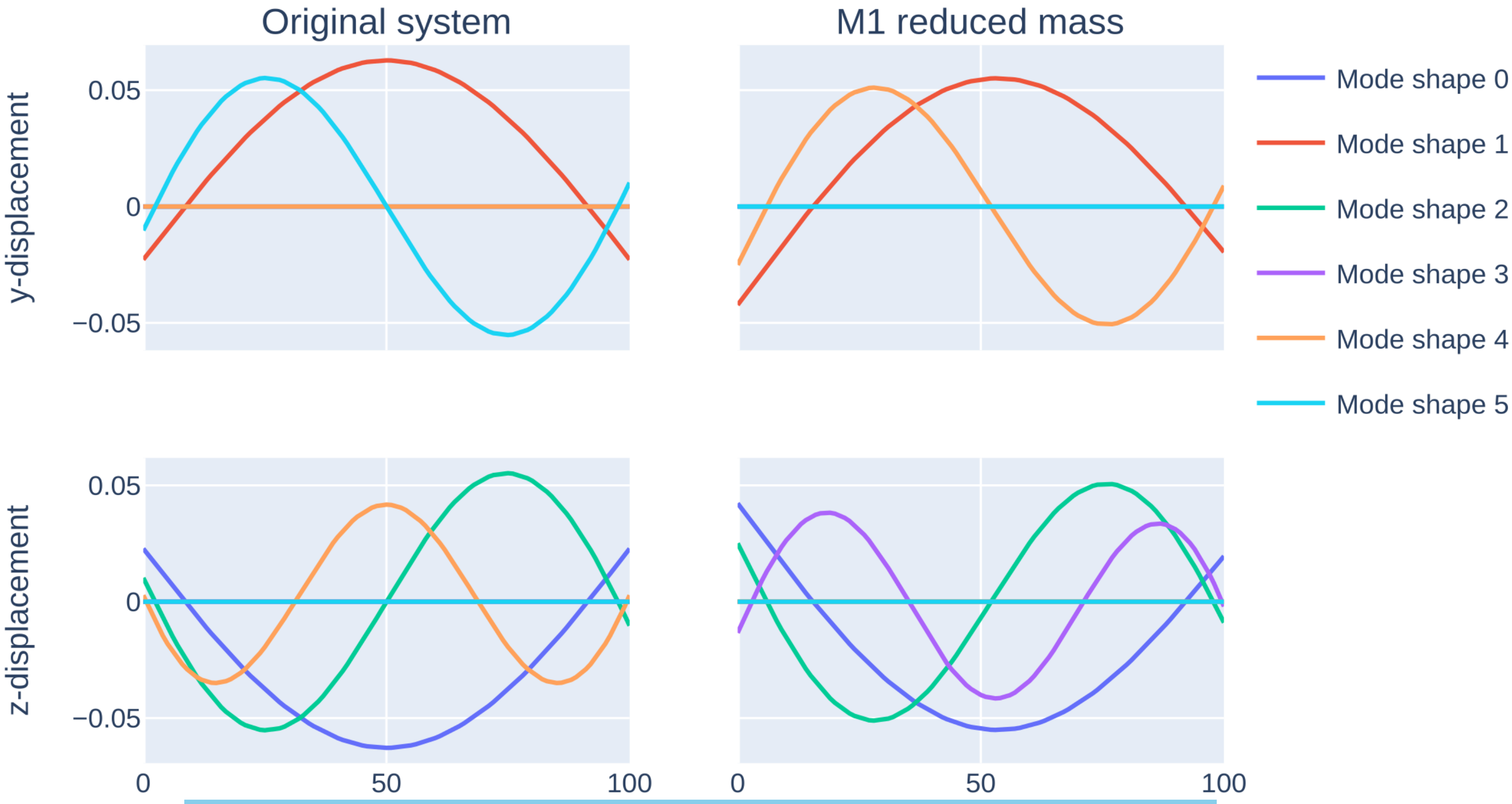
- Component mode synthesis with multibody formulation supports system-level mode recovery with large system changes
  - Large articulation
  - Mass property changes
  - Closed chain
  - Body detachment/attachment
- Does not require new FEM bulk data for such changes



kdFlex (left) and NASTRAN (right) system-level modes for a fixed-fixed double beam—two single beams connected via a locked joint.

System Level Modes: Mass Reduction & Separation

- Consider the barbell system shown on the top right. The red mass is M1 and the blue mass is M2.
- The image shows the y- and z-displacements for the first 6 mode shapes.
- The original system includes the full masses of both M1 and M2.
- The M1 mass and inertia are subsequently to 1/3 of their original values.
- Such mass reduction might arise from an like fuel consumption.
- **kdFlex** is able to capture how such mass changes affect the system mode shapes, even from lumped mass separation (e.g. heat shield separation)

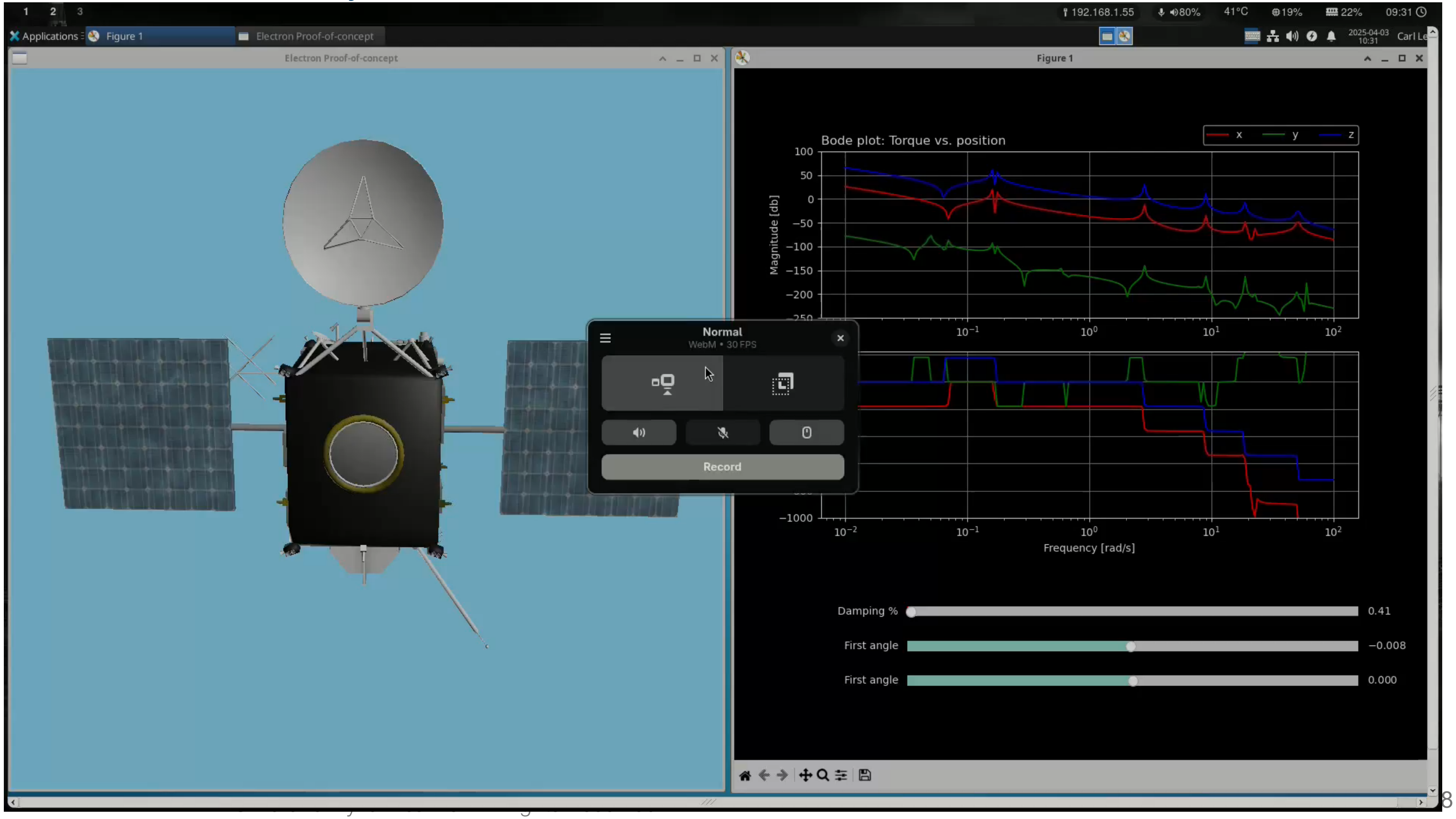


Frequency Domain Control Analysis with Model Variation

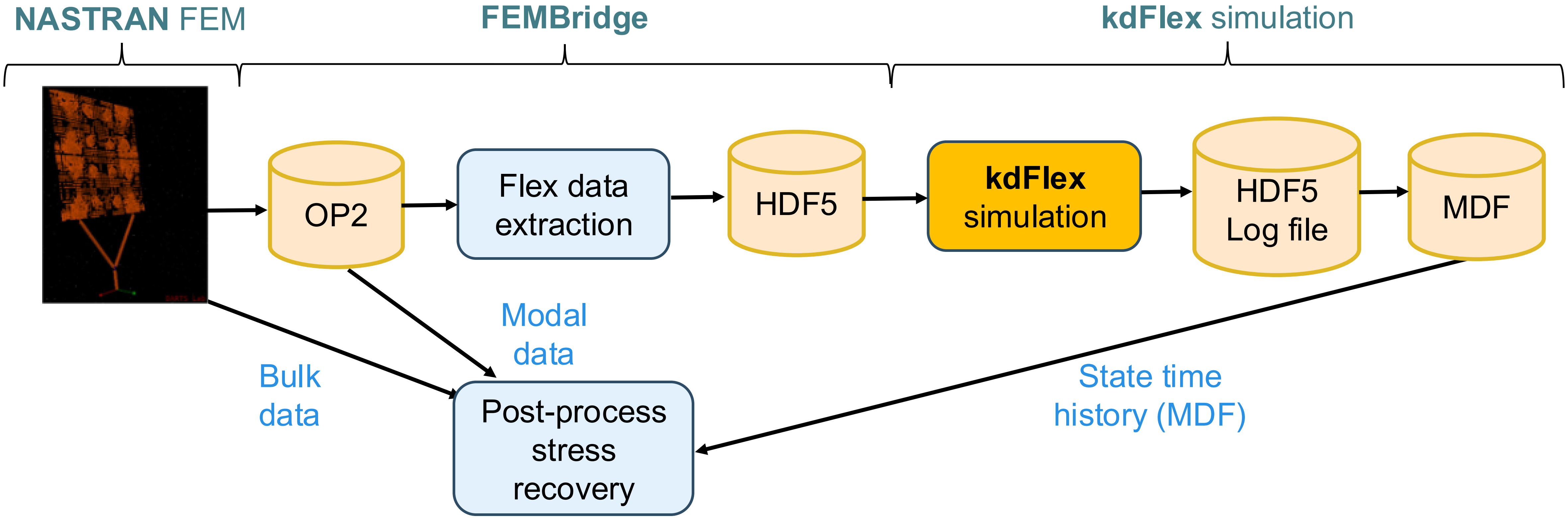
Control system design is typically done using linear state space models. **kdFlex** includes ability to equilibrate and generate linear state space models about any set point for control system analysis and design. This is a very useful capability for servo- and aero-elastic analyses.

Example:

- Spacecraft model with 10 rigid and 20 flexible DoFs
- Changing solar panel angles and damping changes nonlinear system modes and frequencies
- **kdFlex** is being used to interactively generate Bode plots to compute the system frequencies



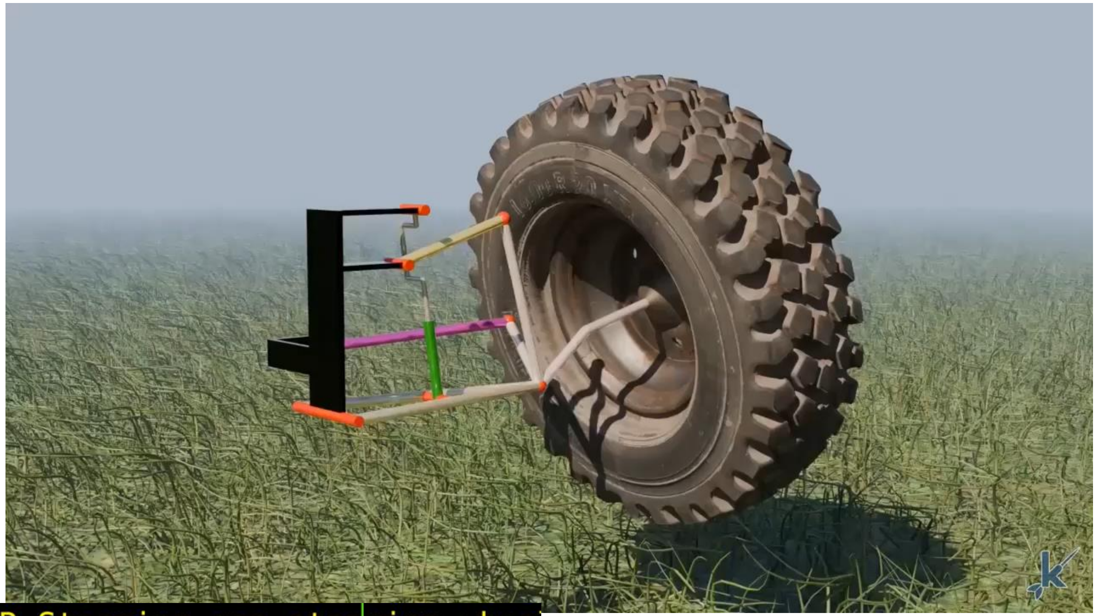
FEMBridge based Stress Recovery



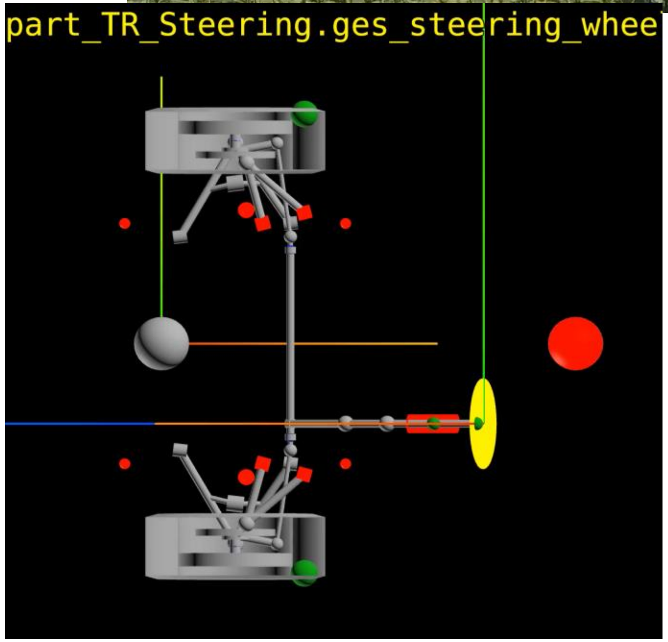
- Initial pre-processing uses NASTRAN to process FEM bulk data for kdFlex flexible body data
- During the simulation, extra logging is added to record the modal coordinates, velocities, and accelerations over time
- This data is used to write an MDF file
- The MDF file, FEM, and OP2 file are used as inputs to run NASTRAN for post-processing to recover stress, strain etc. for the flexible bodies

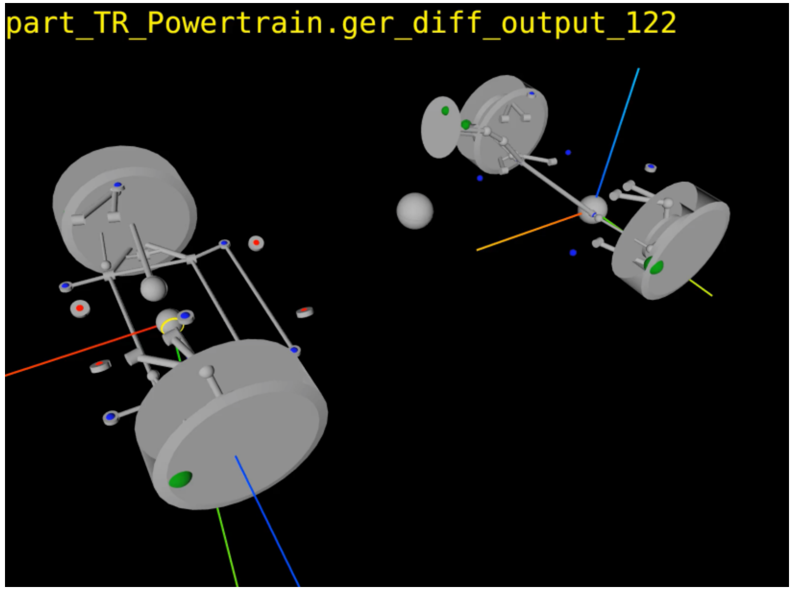
Loads and Dynamics, Stress Recovery

- Closed-chain, multibody suspension system with a single flexible body
- The time-varying stresses are recovered via post-analysis using **FEMBridge**
- Allows stress recovery with realistic G&C scenarios

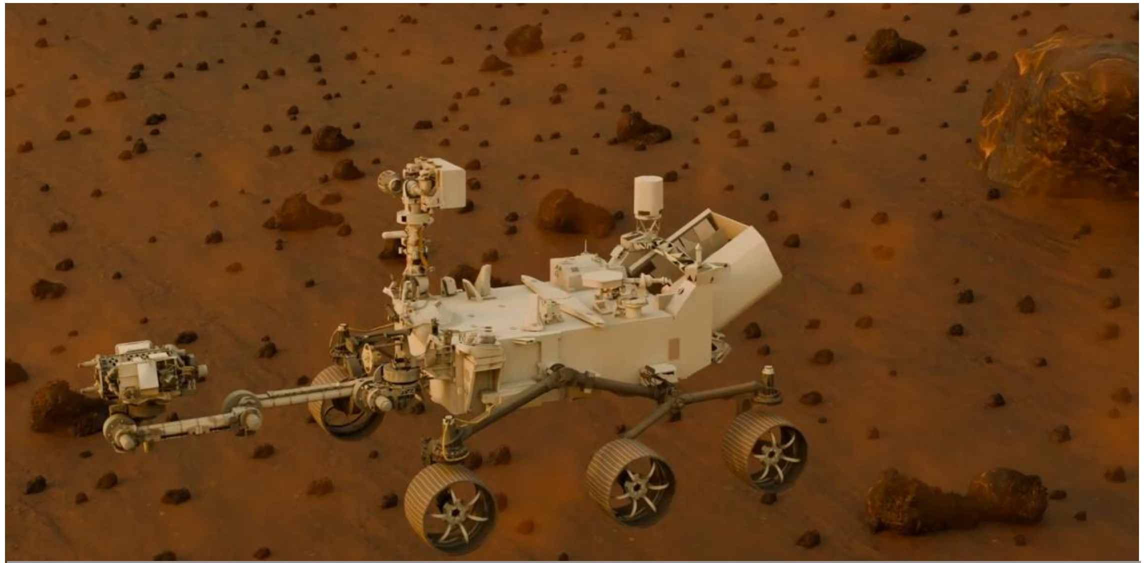


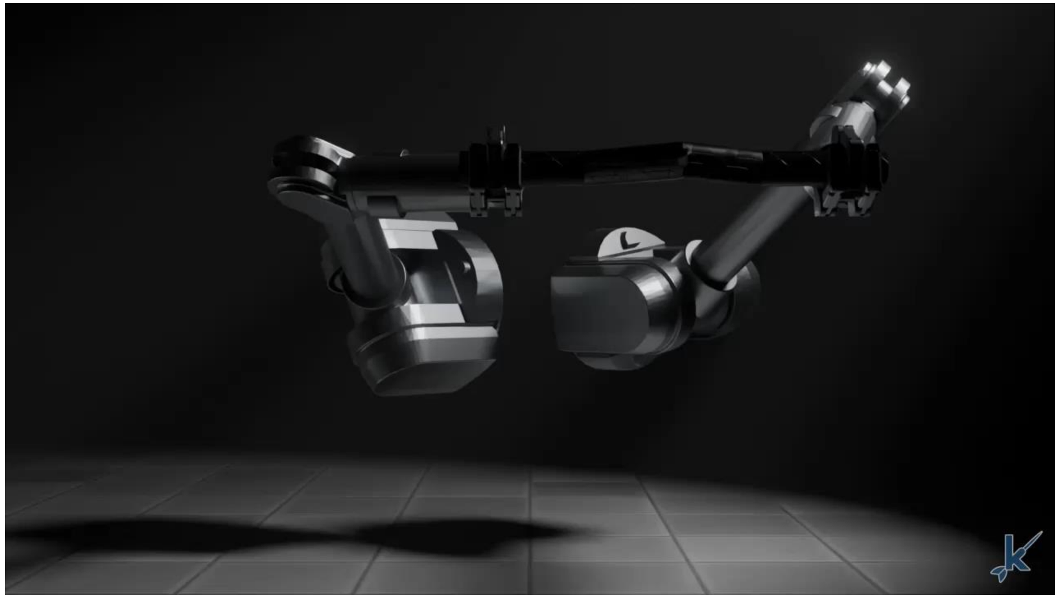
- Example imported sample vehicle with **66** bodies
- Original model has **396** dofs, **104** constraints with O(N^3) cost algorithms
- **kdFlex** model has **81** dofs, **41** constraints with O(N) cost algorithms

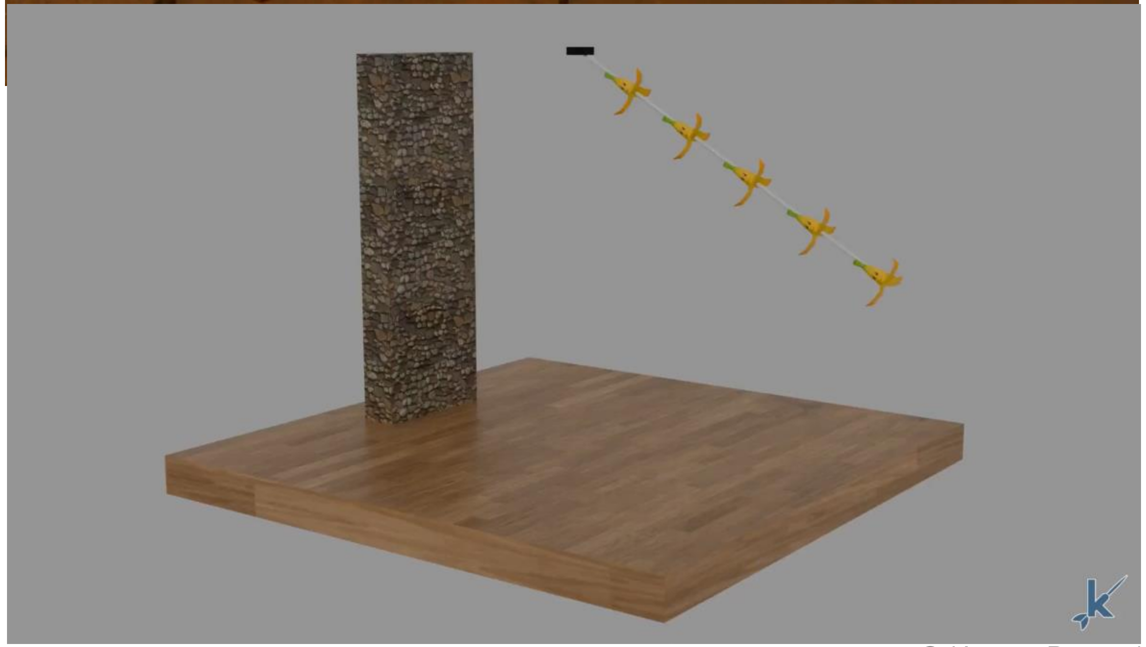


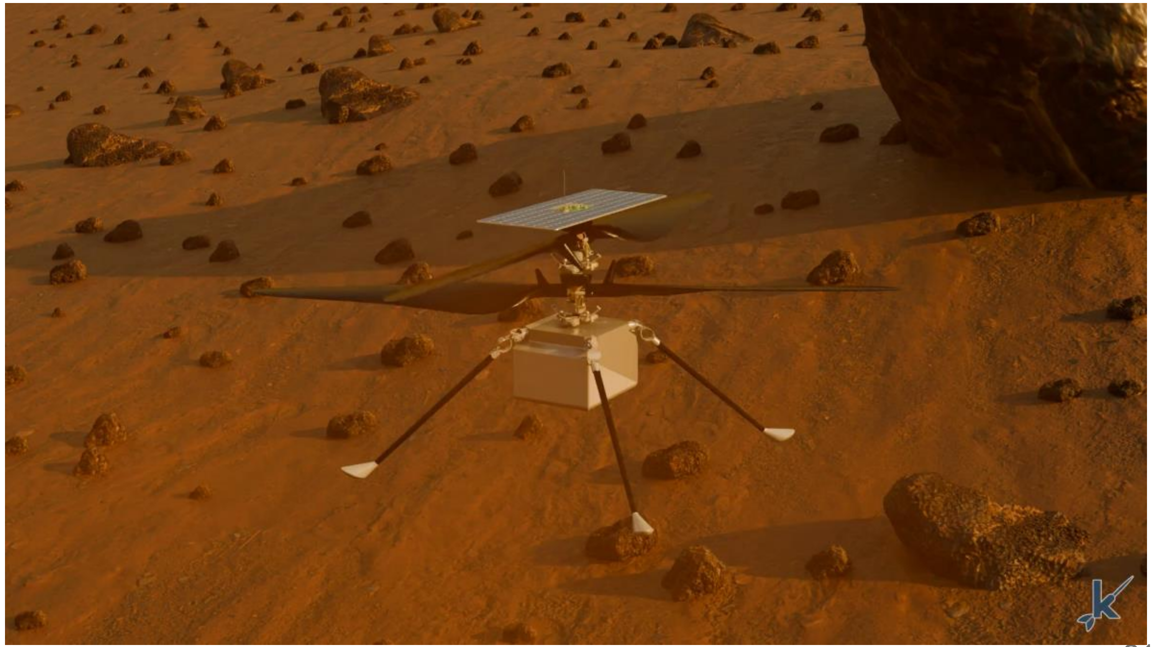


Example robotics/autonomy simulations









kdFlex Multi-Domain Dynamics Toolkit

kdFlex is an **SOA** (Spatial Operator Algebra) based next-gen dynamics engine that avoids the unholy choice between speed and fidelity
- broad-based computational methodology and fast "O(N)" algorithms

Flight dynamics, EDL, aero-flight, ephemerides

Flight Mechanics

Rigid/flex dynamics, frequency and real-time closed-loop design & analysis, hdw-in-the-loop

Guidance & Control

kdFlex

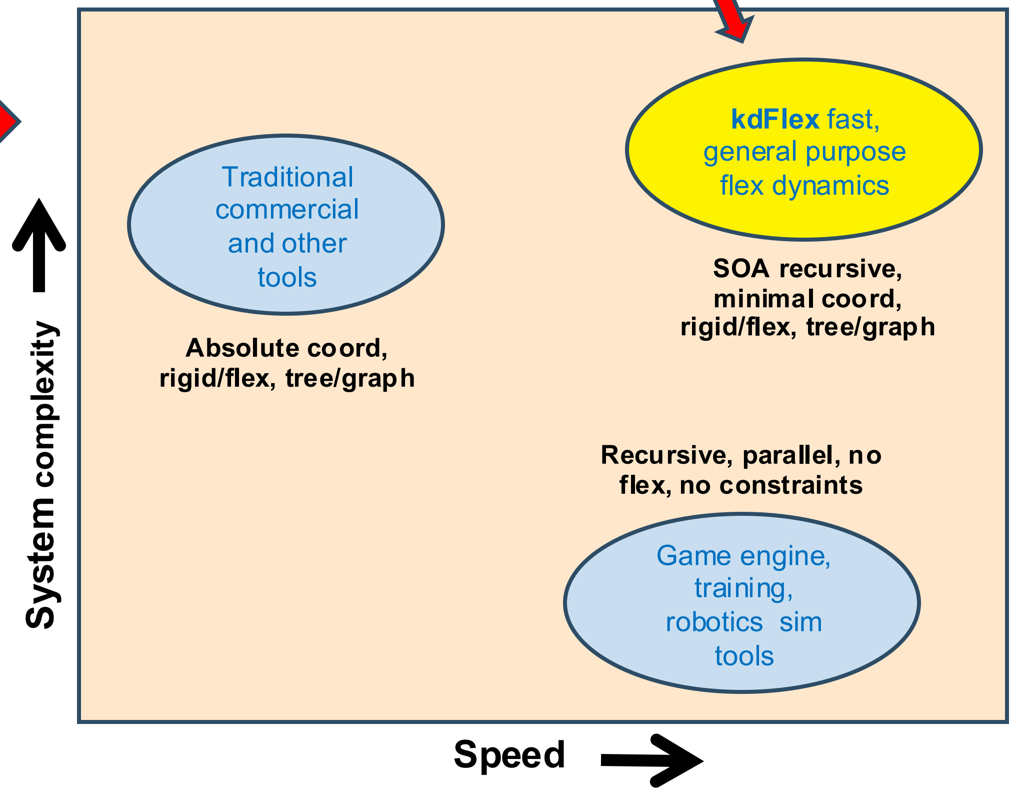
Robotics, mobility, variable topology, sim & embedded use

Vehicles, Robotics

Others …

Detailed rigid/flex dynamics, FEM data, constraints, contact, loads analysis

Loads & Dynamics



kdFlex - a one-stop shop for cross-cutting, high-performance dynamics solutions.

Summary

- **kdFlex** includes **SOA** general purpose rigid+flexible multibody dynamics and includes all relevant terms in EoM
  - Does away with need for specialized dynamics model development
- **kdFlex** provides support for servo-elastic analysis
  - Built-in capability to include other models, e.g., nonlinear friction
  - Build-in capability to linearize and support frequency-domain analysis
- Convenient **FEMBridge** capability for modal analysis and iteratively exploring multiple reduced-order modeling options for G&C
  - Avoids need for generation of multiple NASTRAN bulk data files
  - Supports stress recovery – can help with loads analysis with realistic G&C scenarios
- **kdFlex** also supports real-time closed-loop, time-domain FSW simulations
- Advanced
  - Support for system-mode recovery with large changes: articulation, fuel fills, with topology changes
  - The **kdFlex** general-purpose capability can support dynamics modeling needs across multiple programs

- Evaluation version of **kdFlex** is available at http://www.karanadyn.com !
- .. as are tutorials and a chatbot for support.
- Please check it out – feedback - and questions – are most welcome!