THE PSEUDO VELOCITY SHOCK RESPONSE SPECTRUM
Revision A

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Introduction

The Shock Response Spectrum (SRS) models the peak response of a single-degree-of-freedom (SDOF) system to a base acceleration, where the system's natural frequency is an independent variable. The SRS method is thoroughly covered in Reference 1. The purpose of this tutorial is to present some additional notes.

The absolute acceleration and the relative displacement of the SDOF system can be readily calculated.

The velocity, however, is more difficult to calculate accurately.[1]

The "pseudo velocity" is an approximation of the relative velocity.

The peak pseudo velocity is equal to the peak relative displacement multiplied by the natural frequency ω_n which has units of radians per second.

The peak pseudo acceleration is equal to the peak relative displacement multiplied by the natural frequency $\omega_n{}^2$.

The peak pseudo acceleration is thus equal to the peak pseudo velocity multiplied by the natural frequency ω_n. There may be little reason if any to calculate pseudo acceleration in practice, however, because the absolute acceleration can be calculated directly.

[1] Note that some theories claim that damage potential is more closely correlated with the velocity response than with other metrics, as shown in Reference 2 for example. One of the advantages of the pseudo velocity SRS is that it tends to produce a more uniform SRS than either acceleration or relative displacement.

Example

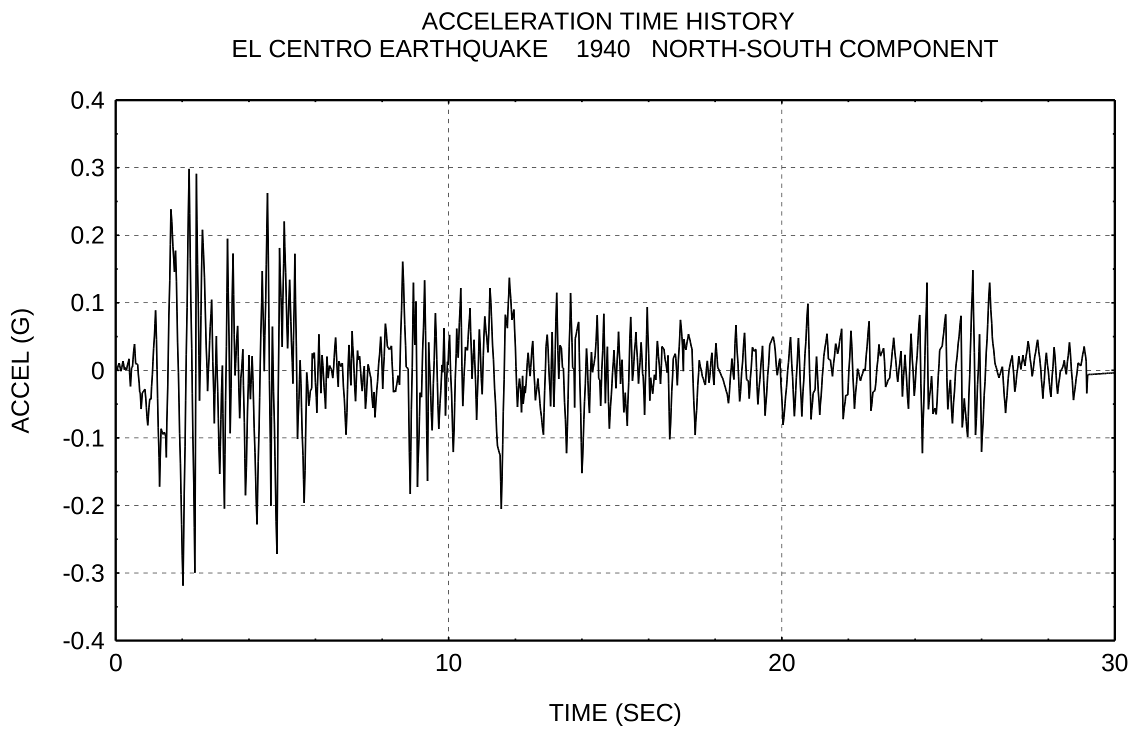


Figure 1.

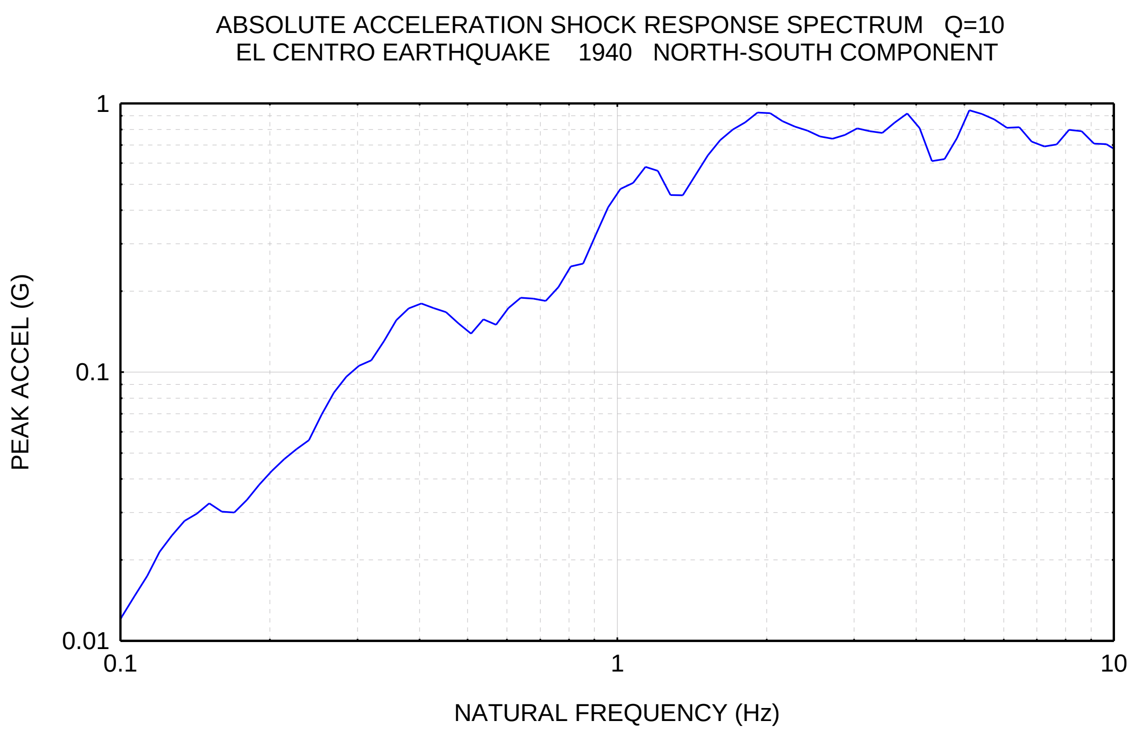


Figure 2.

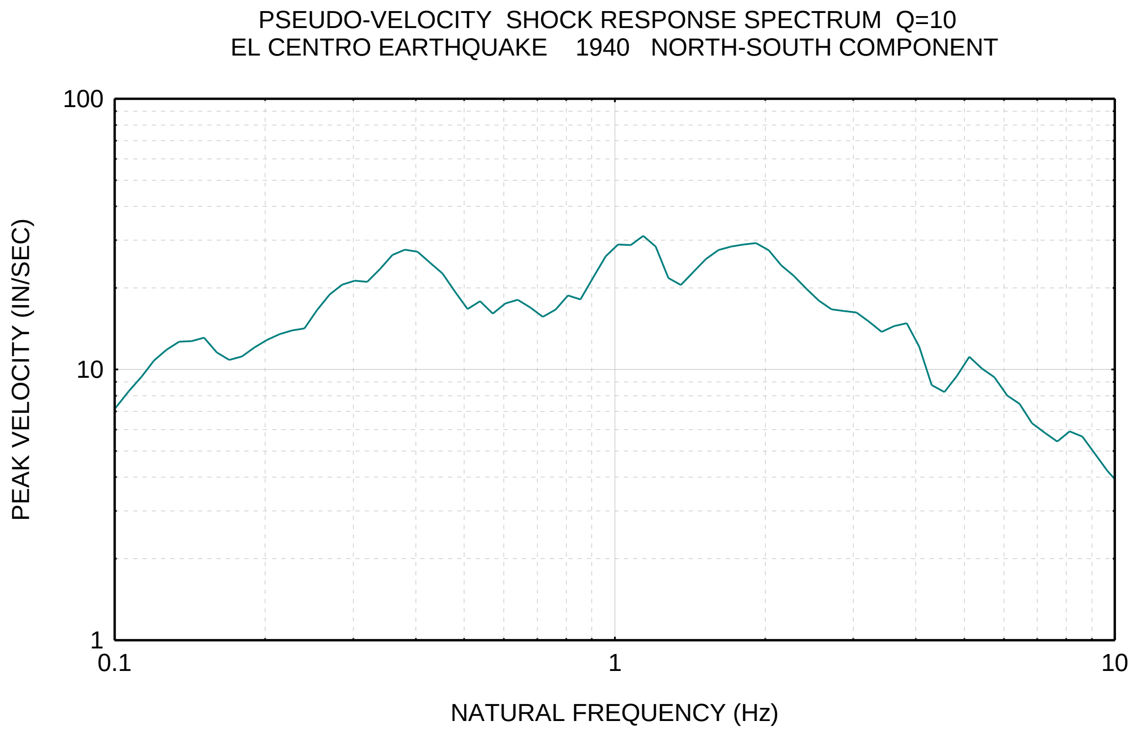


Figure 3.

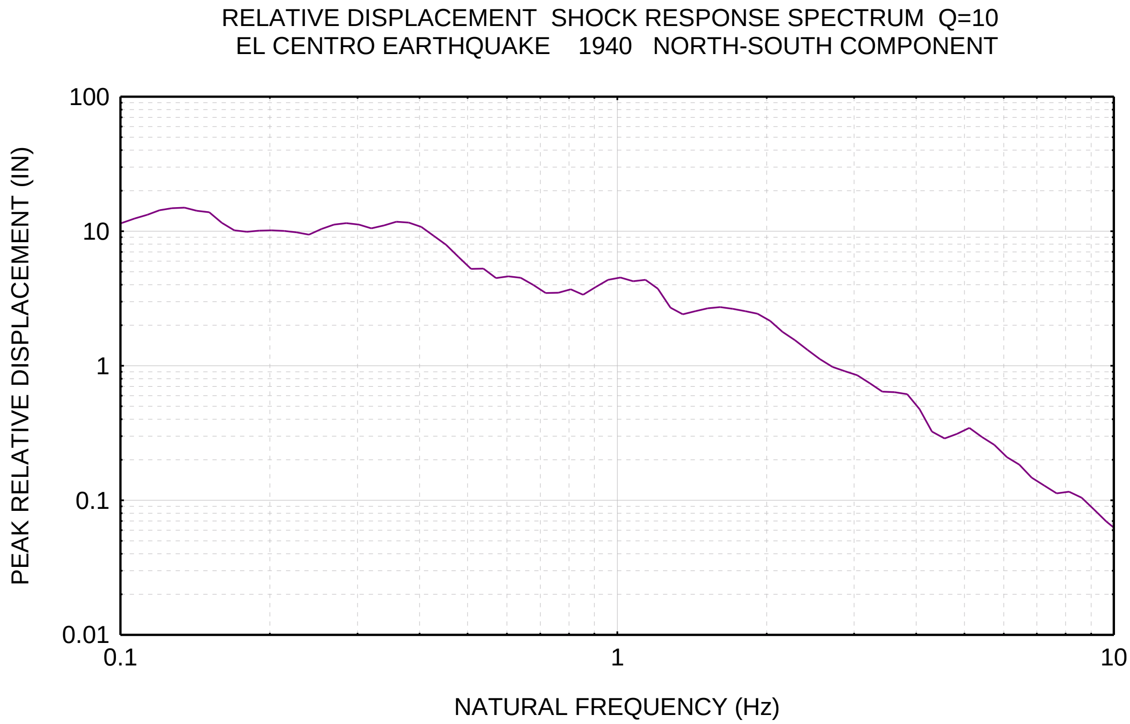


Figure 4.

Now consider a system with a natural frequency of 1 Hz and an amplification factor of Q=10.

Its peak response to the El Centro time history in Figure 1 would be:

Absolute Acceleration	0.46 G
Psuedo Velocity	29 in/sec
Relative Displacement	4.5 inch

Note that

$$(4.5\,\text{inch})\left(2\pi\left(1\text{Hz}\right)\right) \approx 29\,\text{in}/\text{sec}$$

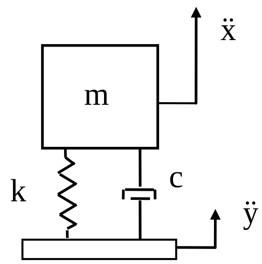
<u>References</u>

1. T. Irvine, An Introduction to the Shock Response Spectrum Rev P, Vibrationdata, 2002.

2. Gaberson and Chalmers, Modal Velocity as a Criterion of Shock Severity, Shock and Vibration Bulletin, Naval Research Lab, December 1969.

APPENDIX A

Equation of Motion for a SDOF System Subjected to Base Excitation



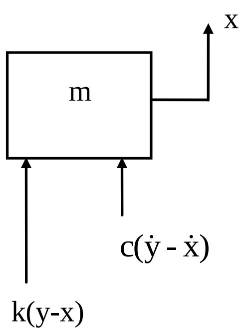
m is the mass

c is the viscous damping coefficient

k is the stiffness

x is the absolute displacement of the mass

y is the base input displacement



The equation of motion for a single-degree-of-freedom system subjected to base excitation is

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky \tag{A-1}$$

Define the relative displacement z.

$$z = x - y \tag{A-2}$$

$$x = z + y \tag{A-3}$$

Substitute equation (A-3) into (A-1).

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{y} \tag{A-4}$$

$$m(\ddot{z} + \ddot{y}) + c(\dot{z} + \dot{y}) + k(z + y) = c\dot{y} + ky \tag{A-5}$$

$$m\ddot{z} + m\ddot{y} + c\dot{z} + c\dot{y} + kz + ky = c\dot{y} + ky \tag{A-6}$$

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \tag{A-7}$$

$$\ddot{z} + (c/m)\,\dot{z} + (k/m)\,z = -\ddot{y} \tag{A-8}$$

By convention,

$$(c/m) = 2\xi\omega_n \tag{A-9}$$

$$(k/m) = \omega_n{}^2 \tag{A-10}$$

Substitute equations (A-9) & (A-10) into (A-8).

$$\ddot{z} + 2\xi\,\omega_n\dot{z} + \omega_n{}^2 z = -\ddot{y} \tag{A-11}$$

Recall

$$\ddot{z} = \ddot{x} - \ddot{y} \tag{A-12}$$

$$\ddot{x} - \ddot{y} + 2\xi\,\omega_n\dot{z} + \omega_n{}^2 z = -\ddot{y} \tag{A-13}$$

$$\ddot{x} + 2\xi\,\omega_n\dot{z} + \omega_n{}^2 z = 0 \tag{A-14}$$

Now assume that the damping term is approximately zero.

$$\ddot{x} + \omega_n{}^2 z \approx 0 \tag{A-15}$$

The absolute acceleration is thus approximately equal to the relative displacement multiplied by $\omega_n{}^2$. Note that the polarity sign is irrelevant.

$$\ddot{x} \approx -\omega_n{}^2 z \tag{A-16}$$

<u>True Relative Velocity</u>

The following derivation is intended for the time domain.

$$\ddot{x} + 2\xi\omega_n \dot{z} + \omega_n{}^2 z = 0 \tag{A-17}$$

$$2\xi\omega_n \dot{z} = -\omega_n{}^2 z - \ddot{x} \tag{A-18}$$

$$\dot{z} = -\frac{\omega_n{}^2}{2\xi\omega_n} z - \frac{1}{2\xi\omega_n}\ddot{x} \tag{A-19}$$

$$\dot{z} = -\frac{\omega_n}{2\xi} z - \frac{1}{2\xi\omega_n}\ddot{x} \tag{A-20}$$

$$\dot{z} = -\frac{1}{2\xi}\left[\omega_n z + \frac{1}{\omega_n}\ddot{x}\right] \tag{A-21}$$