Standard Check-Cases for Six-Degree-of-Freedom Flight Vehicle Simulations

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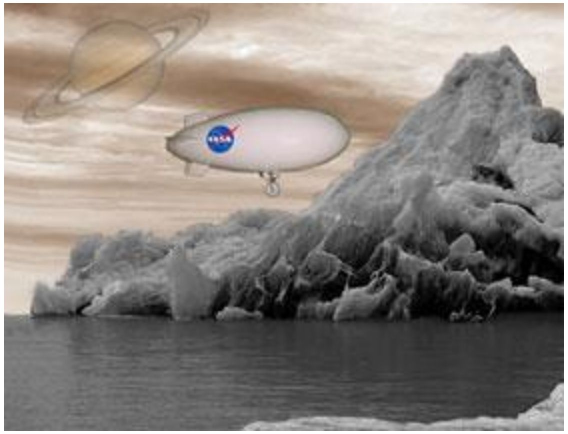
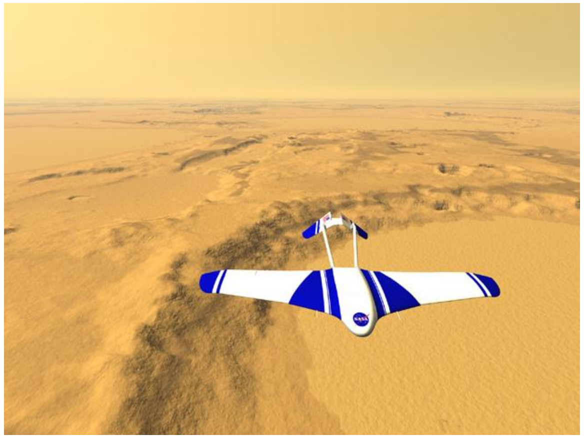
Problem Description

- The variety of NASA, industry and commercial flight simulation tools sometimes provide substantially different results when applied to the same flight problem.
- Some differences can be traced to errors in the implementation of equations of motion (kinematics and dynamics) and of the geodetic, gravitational, and atmosphere models.
- Currently, there are no standard benchmark check-cases for validation of flight simulation tools. This increases risk in relying on these tools for flight prediction and design in support of NASA's flight projects.





Participating Simulations

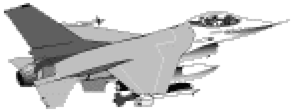
- Core - Dryden Flight Research Center
- JEOD - Johnson Space Center
- LaSRS++ - Langley Research Center
- MAVERIC - Marshall Space Flight Center
- POST-II - Langley Research Center
- VMSRTE - Ames Research Center
- JRBSim – Open-Source

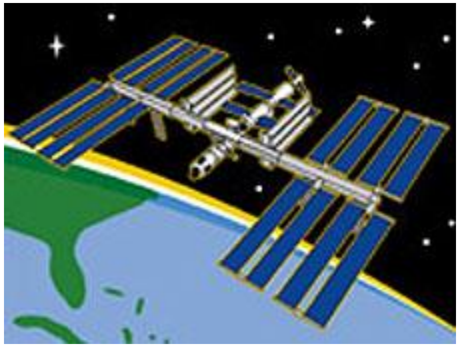
Approach

- Agree on set of atmospheric and exo-atmospheric scenarios
  - A scenario defines the vehicle model, vehicle initial conditions, vehicle maneuvers, the environment models (including geodesy, atmosphere and gravitation), and duration.
- Agree on output variables to compare and the time history recording exchange format (CSV)
- Develop unambiguous reference models
  - Reference models encoded in ANSI/AIAA S-119 format
- Generate resulting time-histories
- Compare resulting time-histories
- Refine results
  - Identify and eliminate disagreements on scenario configuration
  - Attempt to identify remaining differences in results
- Publish reference trajectory information
  - May be several ‘in-family’
  - Anonymize results so that a trajectory cannot be traced to a given simulation tool
  - Publically accessible at http://nescacademy.nasa.gov/flightsim

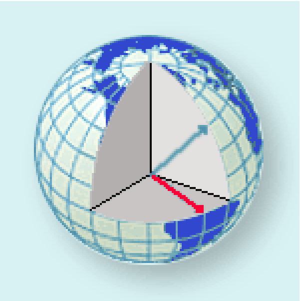
Reference models

- Vehicle models
  - Two Sphere models
    - One sphere in English units for atmospheric cases
    - One sphere in SI units for exo-atmospheric cases
  - Tumbling brick
  - Cylinder
  - Representative International Space Station mass
  - Single-engine fighter w/ simple control system
  - Two-stage rocket w/ constant thrust





- Environment models
  - 2 Geodesy (planetary) models (round and WGS-84)
  - 4 Gravitation models (constant, $1/R^2$, J2, and GEM-T1)
  - 3 Atmospheric models (constant, US 1976 and MET)
- Two Inertial Frames (True-of-Date and J2000 – IERS FK5)



Scenarios – Atmospheric

Scenario Matrix for EOM Validation Assessment project (Atmospheric)

2012-07-27

Scenario		Purpose	Gravitation	Geodesy	Atmosphere	Winds
1	Dropped sphere with no drag	Gravitation model, translational EOM	J2	WGS-84 rotating	US Std 1976	still air
2	Tumbling brick with no damping in vacuum	checks rotational EOM	J2	WGS-84 rotating	US Std 1976	still air
3	Tumbling brick with dynamic damping, no drag	checks inertial coupling effects	J2	WGS-84 rotating	US Std 1976	still air
4	Dropped sphere with constant CD, no wind	Simplest model	$1/R^2$	Round non-rotating	US Std 1976	still air
5	Dropped sphere with constant CD, no wind	Adds rotation	$1/R^2$	Round rotating	US Std 1976	still air
6	Dropped sphere with constant CD, no wind	Adds ellipsoid	J2	WGS-84 rotating	US Std 1976	still air
7	Dropped sphere with constant CD + steady wind	Adds wind effects	J2	WGS-84 rotating	US Std 1976	steady wind
8	Dropped sphere with constant CD + 2D wind shear	Adds 2D winds	J2	WGS-84 rotating	US Std 1976	f(alt)
9	Ballistically launched sphere eastward along equator	checks translational EOM	J2	WGS-84 rotating	US Std 1976	still air
10	Ballistically launched sphere northward along prime meridian from equator	checks Coriolis	J2	WGS-84 rotating	US Std 1976	still air
11	Simple linear aero model in trimmed flight across planet (subsonic)	checks aero-related equations, e.g. Mach, calibrated airspeed	J2	WGS-84 rotating	US Std 1976	still air
12	Simple linear aero model in trimmed flight across planet (supersonic)	checks aero-related equations, e.g. Mach, calibrated airspeed	J2	WGS-84 rotating	US Std 1976	still air
13	Maneuvering flight of 6DOF rigid aircraft with non-linear aerodynamics (subsonic)	checks multidimensional table lookups, alpha-dot, beta-dot, Mach etc.	J2	WGS-84 rotating	US Std 1976	still air
14	Maneuvering flight of 6DOF rigid aircraft with non-linear aerodynamics (supersonic)	checks multidimensional table lookups, alpha-dot, beta-dot, Mach etc.	J2	WGS-84 rotating	US Std 1976	still air
15	Circular flight around North pole	checks propagation near singularity, crossing dateline	J2	WGS-84 rotating	US Std 1976	still air
16	Circular flight around equator/dateline intersection	checks for proper sign and wind-up of heading, etc.	J2	WGS-84 rotating	US Std 1976	still air
17	Two-stage rocket to orbit	checks staging, high-altitude atmosphere table	J2	WGS-84 rotating	1976/MET	f(alt)

Example - Atmospheric Scenario 1

Case	Dropped sphere with no drag			**A01**
Geodesy	WGS-84 rotating		**Duration**:	30 s
Gravitation	J2			
Atmosphere	US 1976 STD; no wind			
Vehicle	Dragless sphere			
Initial States	**Position**	**Velocity**	**Attitude**	**Rate**
Inertial[1][2]	(R, 0, 0)	(0, ω*R, 0)	(0, $-\pi/2$, 0)	(0, 0, 0)
Geocentric	(0, 0, R)	(0, 0, 0)	(0, 0, 0)	($-\omega$, 0, 0)
Geodetic	(0, 0, 30,000)	(0, 0, 0)	(0, 0, 0)	($-\omega$, 0, 0)
Ground ref pt	(0, 0, 0)	--	--	--
Ground relative	(0, 0, 30,000)	(0, 0, 0)	(0, 0, 0)	($-\omega$, 0, 0)
Notes	[1] normative (primary) reference frame; others for convenience [2] R = ER + 30,000 ft			

Scenarios – Exo-atmospheric

Scenario Matrix for EOM Validation Assessment project (Exo-atmospheric)

2012-07-27

	Scenario	Purpose	Gravitation	Atmosphere	3rd Body	Model
1	Earth Modeling Parameters	Environmental constants	N/A	N/A	N/A	N/A
2	Keplerian Propagation	Integration, RNP, orientation	$1/R^2$	N/A	N/A	ISS
3A	Spherical Harmonic Gravity: 4x4	4x4 Harmonic gravity model	4x4	N/A	N/A	ISS
3B	Spherical Harmonic Gravity: 8x8	8x8 Harmonic gravity model	8x8	N/A	N/A	ISS
4	Planetary Ephemeris	Third body gravitation	$1/R^2$	N/A	sun, moon	ISS
5A	Atmosphere: Min. Solar Activity	Free molecular flow	$1/R^2$	MET	N/A	ISS
5B	Atmosphere: Mean. Solar Activity	Free molecular flow	$1/R^2$	MET	N/A	ISS
5C	Atmosphere: Max. Solar Activity	Free molecular flow	$1/R^2$	MET	N/A	ISS
6A	Const Density Drag	Response to constant force	$1/R^2$	const	N/A	sphere
6B	Aero Drag with Dyn. Atmos.	Response to dynamic drag	$1/R^2$	MET	N/A	sphere
6C	Plane Change Maneuver	Response to propulsion firing	$1/R^2$	N/A	N/A	cylinder
6D	Earth Departure Maneuver	Response to propulsion firing	$1/R^2$	N/A	N/A	cylinder
7A	Combined Translational Test: 4x4 Gravity	Translation response	4x4	N/A	sun, moon	sphere
7B	Combined Translational Test: 8x8 Gravity	Translation response	8x8	N/A	sun, moon	sphere
7C	Combined Translational Test: 4x4 Gravity w/ Drag	Translation response	4x4	MET	sun, moon	sphere
7D	Combined Translational Test: 8x8 Gravity w/ Drag	Translation response	8x8	MET	sun, moon	sphere
8A	Rotation Test: No rotation rate	Integration methods for rotation	$1/R^2$	N/A	N/A	ISS
8B	Rotation Test: Initial rotation rate	Integration methods for rotation	$1/R^2$	N/A	N/A	ISS
9A	Torque w/ no initial rotation	Rotational response	$1/R^2$	N/A	N/A	ISS
9B	Torque w/ initial rotation	Rotational Response	$1/R^2$	N/A	N/A	ISS
9C	Torque + Force w/ no initial rotation rate	Rotational Response	$1/R^2$	N/A	N/A	ISS
9D	Torque + Force w/ initial rotation rate	Rotational Response	$1/R^2$	N/A	N/A	ISS
10A	Gravity Gradient: circular orbit, no initial rotation rate	Gravity gradient modeling	$1/R^2$	N/A	N/A	cylinder
10B	Gravity Gradient: circular orbit, initial rotation rate	Gravity gradient modeling	$1/R^2$	N/A	N/A	cylinder
10C	Gravity Gradient: elliptical orbit, no initial rotation rate	Gravity gradient modeling	$1/R^2$	N/A	N/A	cylinder
10D	Gravity Gradient: elliptical orbit, initial rotation rate	Gravity gradient modeling	$1/R^2$	N/A	N/A	cylinder
FULL	Integrated 6-DOF Orbital Motion	Combined effects response	8x8	MET	sun, moon	ISS

Example – Exo-atmospheric Scenario

- Simulation
  - Simulation Duration: 28,800 seconds
  - Data Collection Rate: 60 seconds
- Vehicle
  - Orbital State: Typical ISS Orbit
  - Mass Properties: Representative ISS Mass
- Dynamics
  - Rotational Propagation: No
    - Initial Rotation Rate: LVLH
  - External Torques: No
  - External Forces: No
- Environmental Models
  - Gravity Model: On
    - Order: Spherical
    - Planetary Ephemeris: Off
    - Sun/Moon Perturbations: Off
  - Gravity Gradient Torque: Off
  - Atmospheric Model: On
    - F10.7: 128.8
    - Geomagnetic Index: 15.7
  - Aerodynamic Drag Model: Off
    - Coefficient of Drag: N/A
    - Cross-sectional Area: N/A

Comparison Data

- Results stored in comma-separated value (CSV)
  - Emerged as the common format that all teams could supply
- States/values to be stored are:
  - Elapsed simulation time
  - Vehicle rigid-body states (angular & linear velocities and positions)
    - Relative to inertial frame
    - Relative to geodetic frame (atmospheric)
    - Relative to orbit (exo-atmospheric)
  - Output variables – gravitation, aerodynamic forces and moments, atmospheric properties (density, temperature, pressure)
  - Storage frequency is scenario dependent
  - Precision to 10 significant digits, minimum

SELECT RESULTS

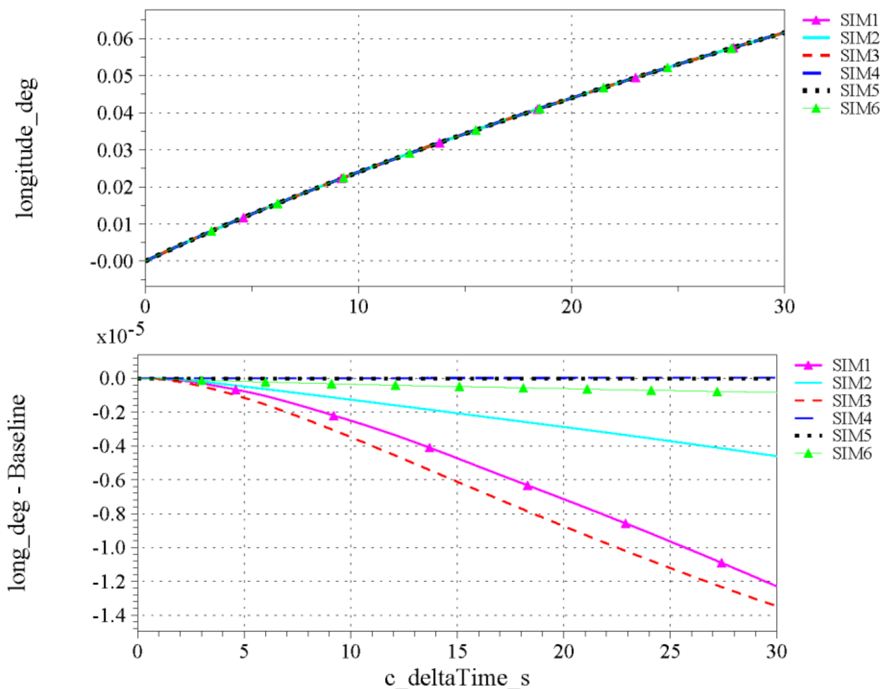
Results are anonymized using designations SIM 1 through SIM 6 for the atmospheric cases and SIM A through SIM D for the orbital cases. Designation order does not match order of tools on slide 4.

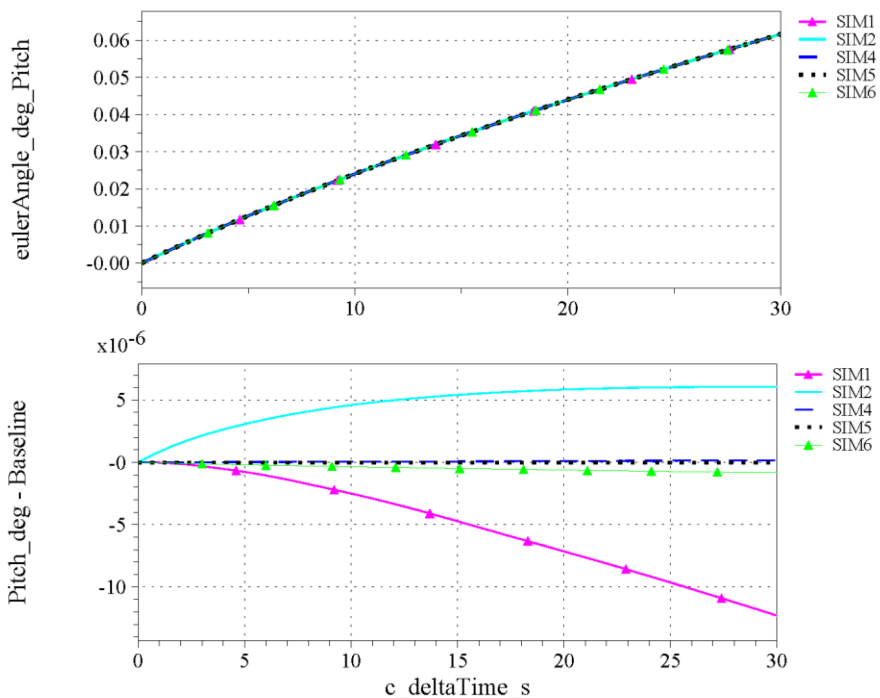
Atmospheric Scenario 9 (1/3)

Case	Sphere launched ballistically eastward along equator			**A09**
Geodesy	WGS-84 rotating		**Duration**:	30 s
Gravitation	J2			
Atmosphere	US 1976 STD; no wind			
Vehicle	Sphere with constant C_D			
Initial States	**Position**	**Velocity**[1][2]	**Attitude**	**Rate**
Inertial[2]	(R, 0, 0)	(V_o, ω*R+V_o, 0)	(–π/2, 0, π/2)	(-ω, 0, 0)
Geocentric	(0, 0, R)	(0, V_o, -V_o)	(0, 0, 90)	(0, 0, 0)
Geodetic	(0, 0, 0)	(0, V_o, -V_o)	(0, 0, 90)	(0, 0, 0)
Ground ref pt	(0, 0, 0)	--	--	--
Ground relative	(0, 0, 0)	(0, V_o, -V_o)	(0, 0, 90)	(0, 0, 0)
Notes	[1] R = Earth equatorial radius [2] V_o = 1,000 ft/s (45 degree initial vertical trajectory)			

Atmospheric Scenario 9 (2/3)

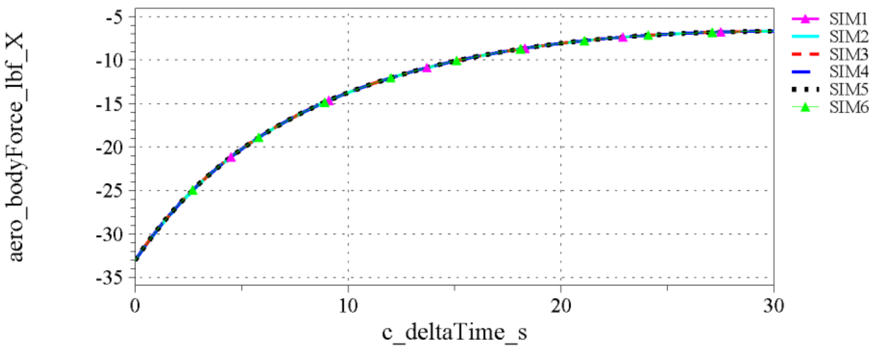
- Due to East trajectory with initial zero rotation rate relative to Earth, pitch angle grows with longitude
- Longitude difference represents separation of up to 5 ft
  - See next slide for explanation
- Pitch angle difference equal to longitude difference except SIM 2
  - SIM 2 has additional difference in integration error for Euler angles

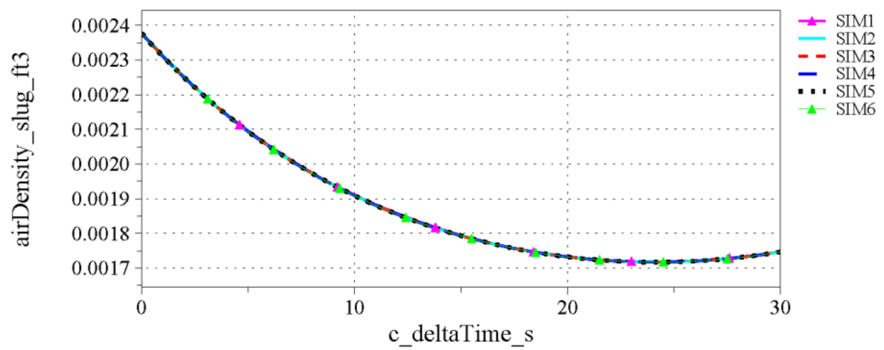


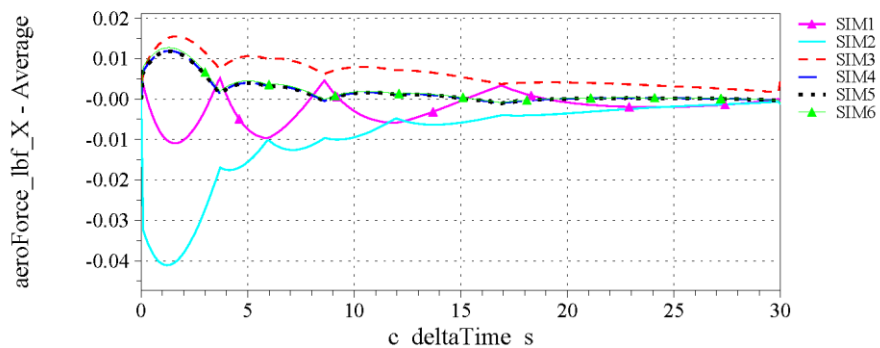


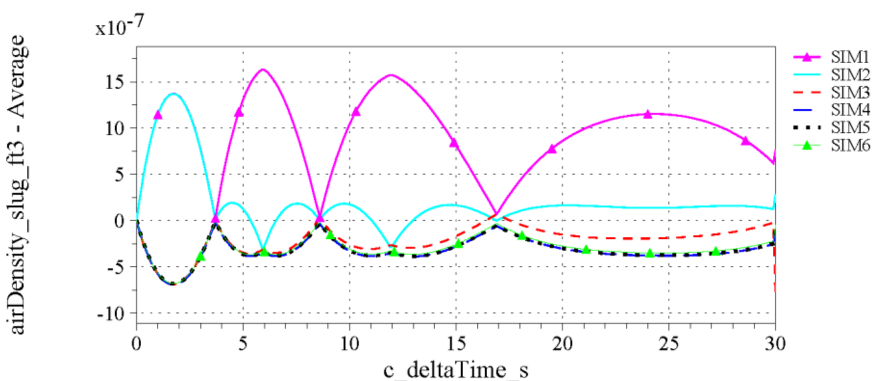
Atmospheric Scenario 9 (3/3)

- Aerodynamic drag differences are driver of translational difference in SIM1 and SIM2
- Differences in atmospheric density contribute to drag differences
  - SIM 1 and SIM 2 both use a table lookup for 1976 Atmosphere Model
  - Density is a non-linear function in the 1976 Atmosphere Model
- SIM 2 exhibits recording lag of one frame for drag
- SIM 3 drag difference due to velocity and altitude difference
- Drag model feedback assumed to amplify trajectory integration error or other unidentified EOM difference for SIM 3 and, to a lesser extent, SIM 1









Atmospheric Scenario 11 (1/3)

Case	Subsonic winged flight – trimmed straight & level			**A11**
Geodesy	WGS-84 rotating	**Duration**:	30 s	
Gravitation	J2			
Atmosphere	US 1976 STD; no wind			
Vehicle	Simplified F-16 model; stability augmentation off			
Initial States	**Position**	**Velocity**[2]	**Attitude**	**Rate**
Inertial	[4195599.3, -16425651.4, 12242837.8] ft	[1527.1, 632.8, 323.5] ft/s	[3]	[3]
ECEF	[4195599.3, -16425651.4, 12242837.8] ft	[329.3, 326.9, 323.5] ft/s	[3]	[3]
Geodetic	10,000 ft over FFA (10,013 MSL)	[V_o, V_o, 0]	[3]	[3]
Ground ref pt	FFA[1]	--	--	--
Ground relative	[0, 0, 10000] ft	[393.0, 406.9, 0] ft/s	[3]	[3]
Notes	[1] FFA is [36°01.09' N, 75°40.28' W, 13 ft] with 1° W variation [2] V_o = 400 ft/s with a track angle of 45° true or 46° magnetic; V_{total} = 565.7 ft/s. [3] Attitude and angular rate will depend on the trim solution for the simulation. The geodetic angular rate should be at or near zero.			

Atmospheric Scenario 11 (2/3)

- Trim algorithm determines initial attitude and rotational rate
- Simulations had different targets for rotational rate
  - SIM 2 targeted zero inertial rotational rate
  - SIM 4 targeted zero roll and yaw rate relative to the Earth with pitch rate set to maintain pitch angle as vehicle flew over surface of the Earth
  - SIM 5 targeted a rotational rate in all three axes intended to maintain all three Euler angles as vehicle flew over curved surface of the Earth
- SIM 2 has initial roll angle because J2 gravitation vector is aligned to geodetic normal rather than geocentric radial
  - Roll angle is nearly equal to difference between geodetic and geocentric latitude
  - Trim is aligning vehicle lift vector with resultant direction of gravity

	Roll Rate	Pitch Rate	Yaw Rate
SIM 2	0.00000	0.00000	0.00000
SIM 4	0.00250	-0.00395	-0.00234
SIM 5	0.00253	-0.00394	-0.00314

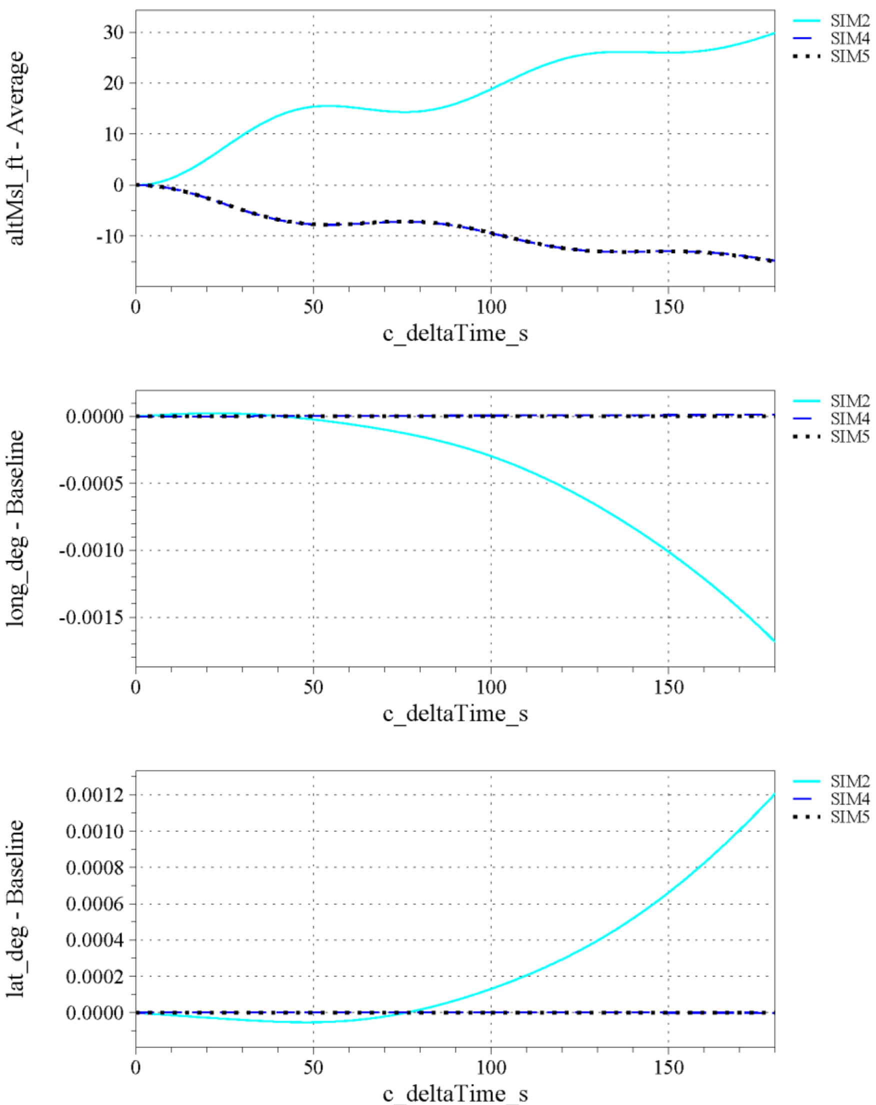
Inertial Rotational Rate at T = 0 (deg/s)

	Phi	Theta	Psi
SIM 2	-0.17183	2.643331	45
SIM 4	0.00000	2.63873	45
SIM 5	0.00000	2.63893	45

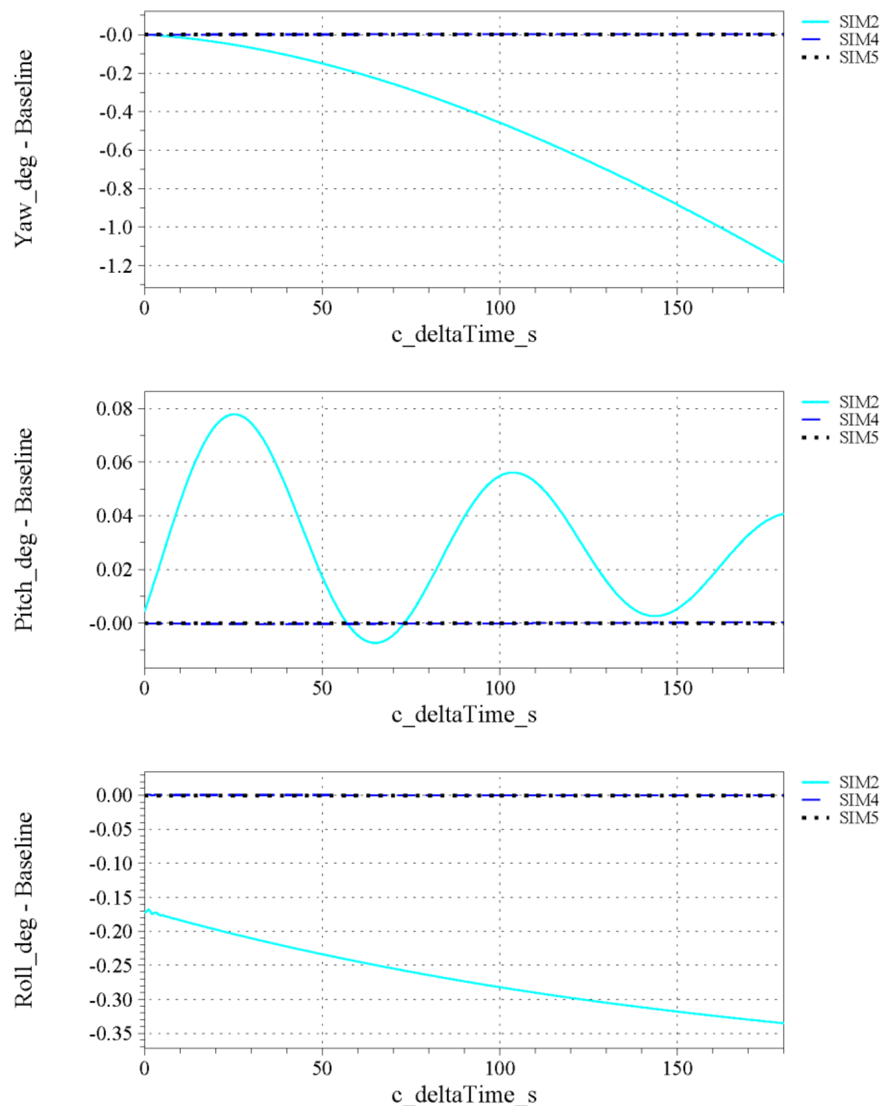
Euler Angles at T = 0 (deg)

Atmospheric Scenario 11 (3/3)

Geodetic Altitude, Longitude, Latitude Difference Plots



Yaw Angle (psi), Pitch Angle (theta), Roll Angle (phi) Difference Plots



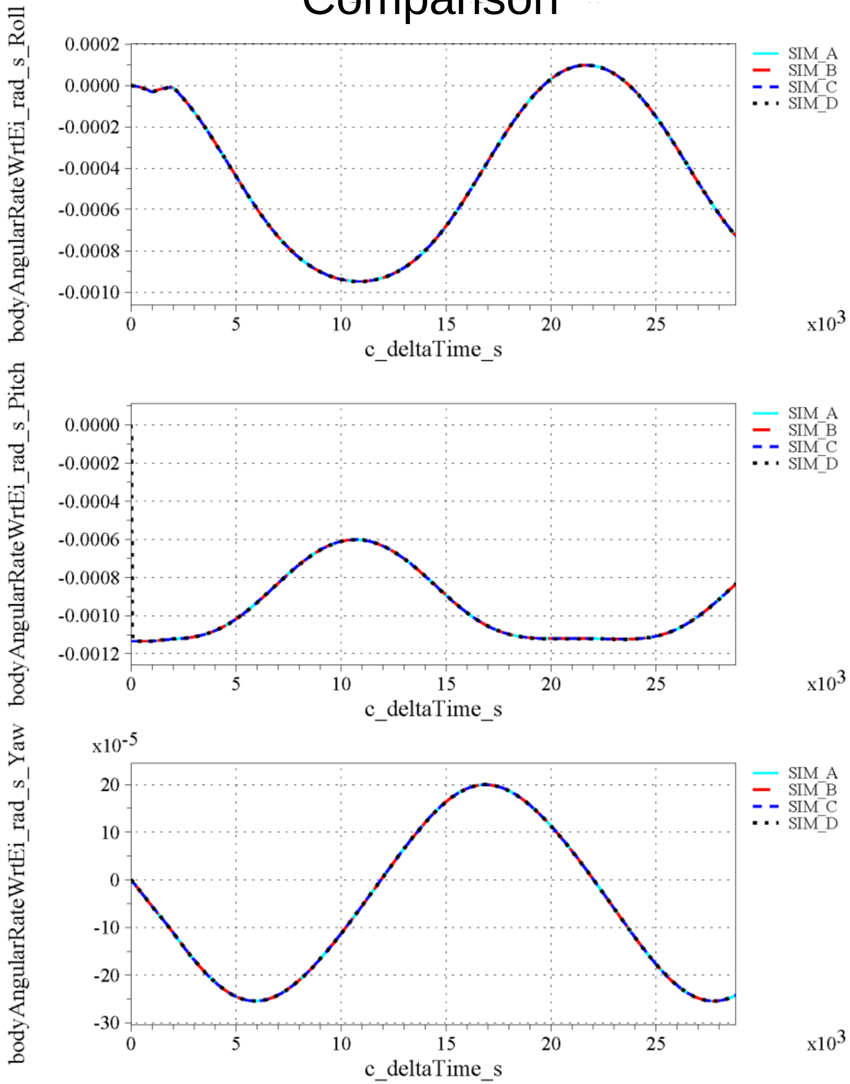
Orbital Scenario 9D (1/5)

- Simulation
  - Simulation Duration: 28,800 seconds
  - Data Collection Rate: 60 seconds
- Vehicle
  - Orbital State: Near Circular Orbit
    - Time = 2007/11/20 00:00:00 UTC
    - r_x = −4315967.74 m
    - r_y = 960356.20 m
    - r_z = 5167269.53 m
    - v_x = 129.091037 m/s
    - v_y = −7491.513855 m/s
    - v_z = 1452.515654 m/s
  - Mass Properties: Representative ISS Mass
- Dynamics
  - Rotational Propagation: Yes
    - Initial Rotation Rate: LVLH
  - External Torques: 10Nm for 1000 s at 1000 s about body X axis
  - External Forces: 10 N for 1000s at 1000s along body X axis
- Environmental Models
  - Gravity Model: On
    - Order: Spherical
    - Planetary Ephemeris: Off
    - Sun/Moon Perturbations: Off
  - Gravity Gradient Torque: Off
  - Atmospheric Model: On
    - F10.7: 128.8
    - Geomagnetic Index: 15.7
  - Aerodynamic Drag Model: Off

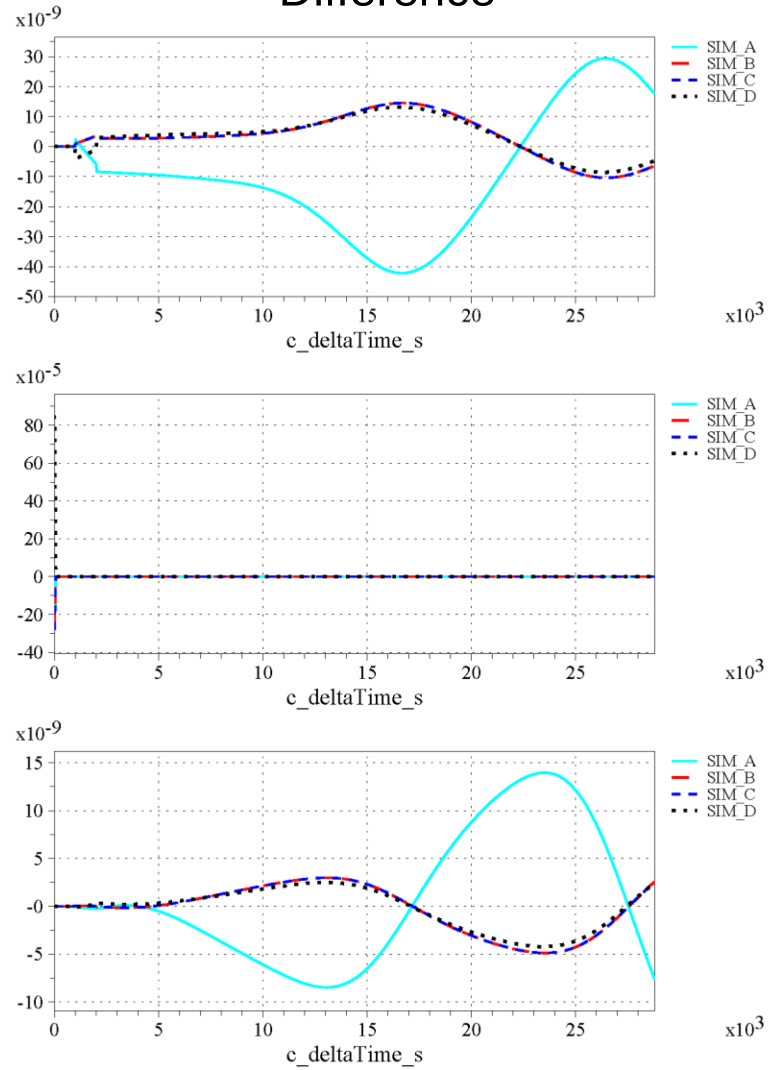
Orbital Scenario 9D (2/5)

Inertial Rotational Rate

Comparison



Difference



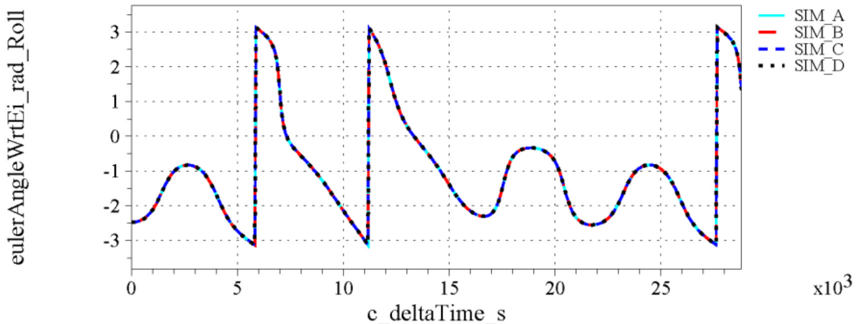
Orbital Scenario 9D (3/5)

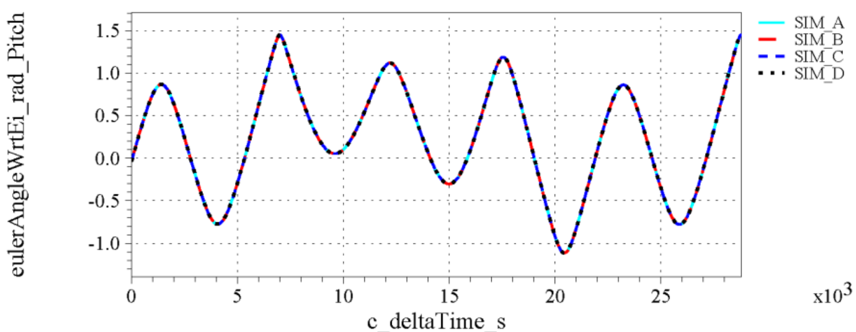
- Force and torque are modeled as a square pulse
  - The integration error of a numerical technique can increase at the discontinuous leading and trailing edges of the square pulse.
- The angular rate difference chart shows the sudden change in integration error at the leading and trailing edge of the torque
  - Angular rate differences exhibit growth at leading edge of torque
  - SIM D rotation rate rejoins SIM B and C at trailing edge but interim difference has a lasting affect on attitude
  - SIM A exhibits difference in rotational rate from leading to trailing edge; this drives a permanent difference in rotational rate and attitude
- Translational and rotational dynamics are coupled since thrust is applied along a fixed body axis during torque
  - SIM D orbit after combined thrust-torque pulse differs just enough to separate SIM D from other simulations by 2 meters at end of run
  - SIM A manages to achieve same orbit as SIM B & C despite differences in rotation rate during thrust-torque pulse

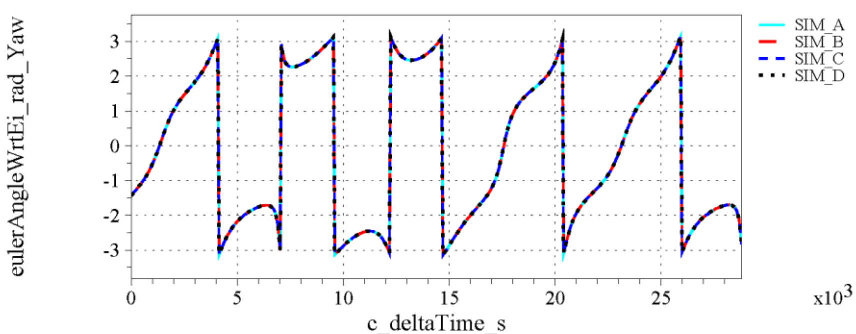
Orbital Scenario 9D (4/5)

Inertial Euler Angles

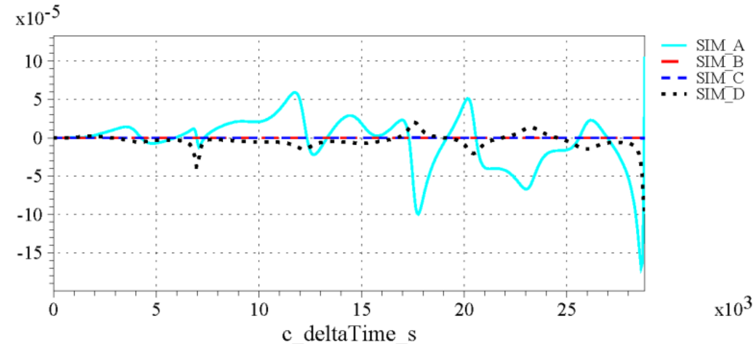
Comparison

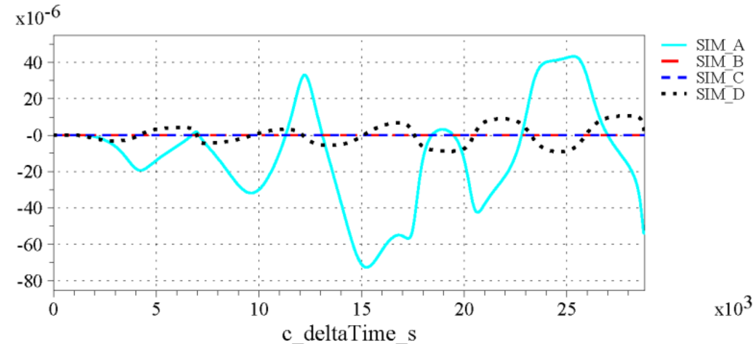


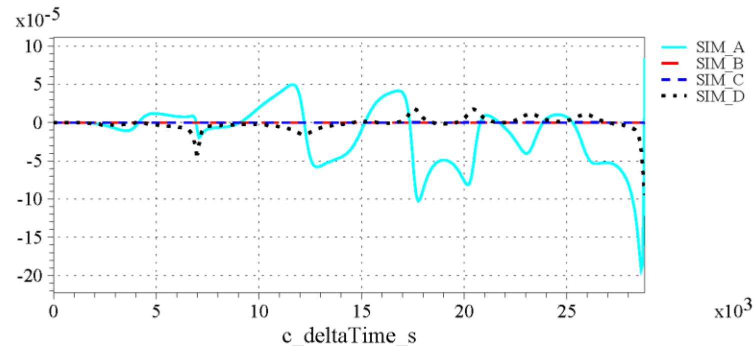




Difference



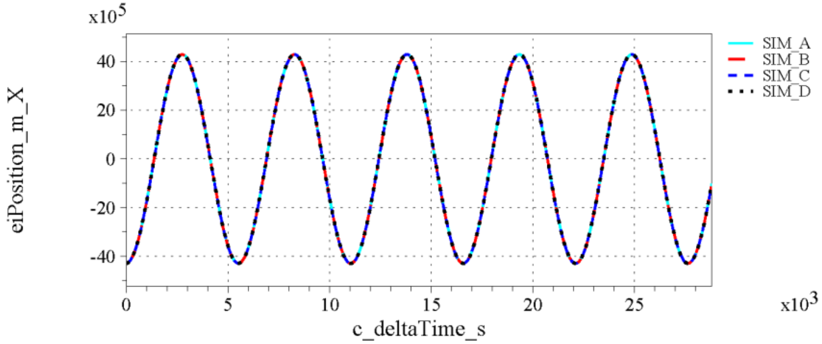


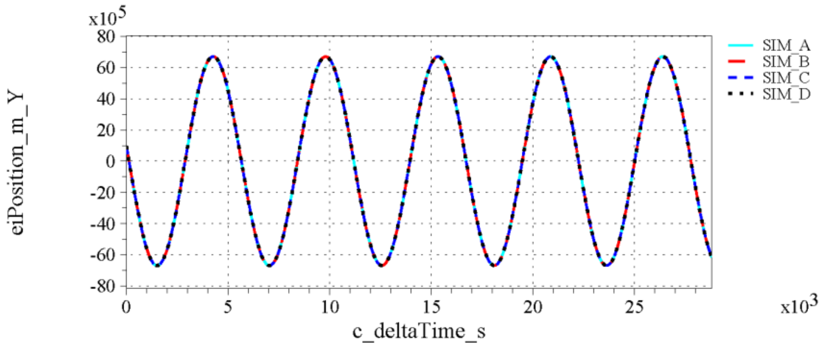


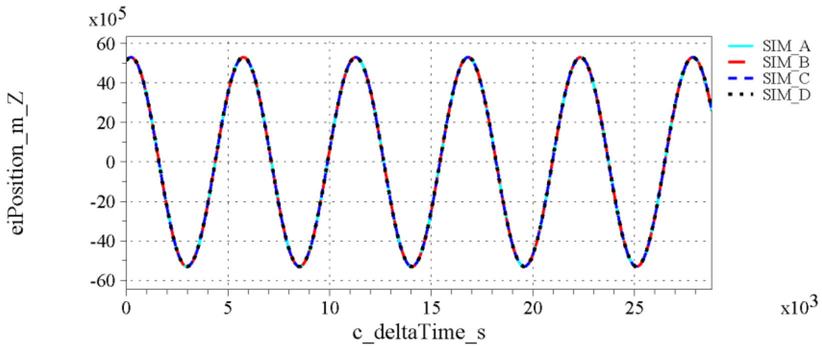
Orbital Scenario 9D (5/5)

Inertial Position

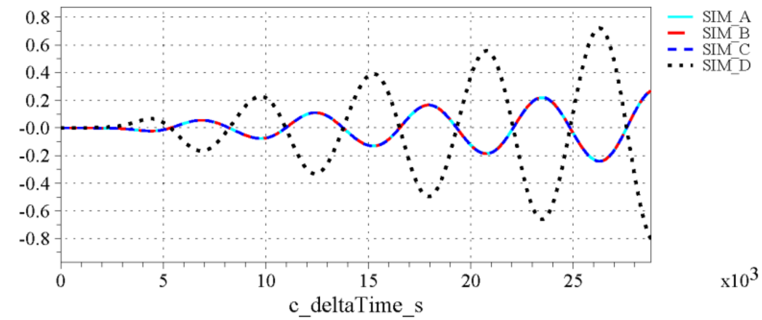
Comparison

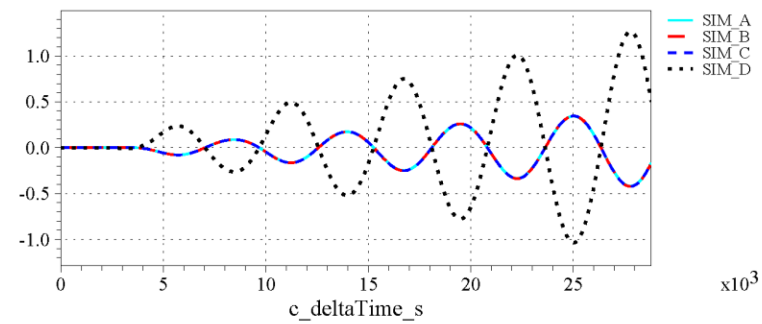


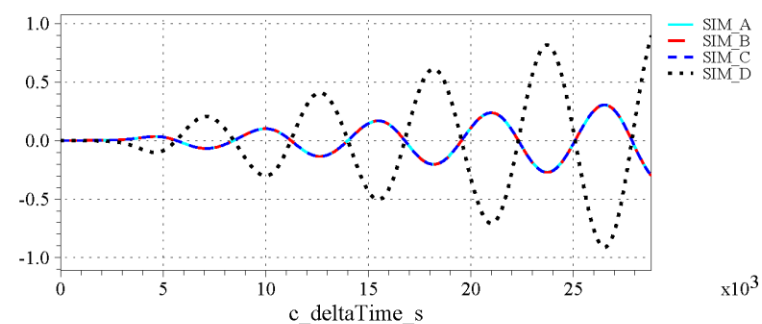




Difference







Conclusions (1/2)

- 25+ rounds of comparison and refinement across the seven simulation tools were necessary to achieve the level of matching presented
- Remaining differences in results have been traced to:
  - Lingering differences in scenario configuration or in simulation parameters (e.g. physical constants, unit conversions)
  - Differences in integration error
  - tabular versus equation-based atmosphere models
  - Differing versions for the MET atmosphere model
  - Heritage simplifications derived from a flat or spherical Earth assumption
  - Differing targets for trim algorithms
  - Differing results from gravitation model including differences in direction of J2 gravitation vector
- Other Lessons Learned
  - Modeling even simple vehicles posed challenges. Teams introduced differences in model implementation, scenario configuration, and modeling parameters.
  - Precise specification of scenarios would be assisted by a standard for specifying the state vector of a 6-DOF flight simulation.

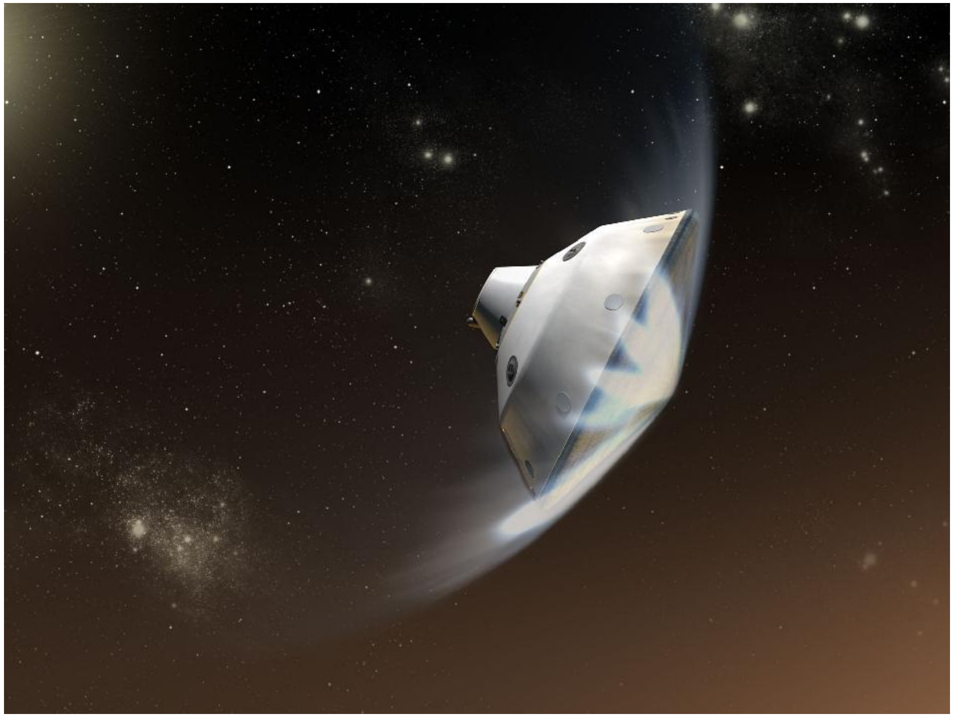
Conclusions (2/2)

- Other Lessons Learned (cont.)
  - In the atmospheric cases, each tool obtained a fair match with at least two other tools. Correlation was closest among tools targeting similar problem domains.
  - Trajectories in the orbital cases matched well, but minor differences remain.
  - Atmospheric trajectories did not match as well as orbital trajectories. Atmospheric flight is highly non-linear, and more tools participated in the atmospheric cases.
  - The effort to develop a validation data set for multiple test cases across numerous simulation tools was estimated at one year and took two and half.
  - Teams employed different definitions for similar sounding variable names in the recorded data. This caused miscommunication in early comparisons. Simulation comparisons would benefit from employing ANSI/AIAA S-119-2011 for unambiguous definition of recorded variables.
  - Every team made at least one improvement to their simulation tool as a result of running the check cases.
  - NASA succeeded in producing a validation data set for 6-DOF flight simulations using the check cases presented. Additional scenarios and results would improve the value of the data set. Future needs include supersonic maneuvering flight and atmospheric re-entry scenarios.

Accessing the Check Cases

- The check case descriptions and data from the initial participating simulations is publically available at URL http://nescacademy.nasa.gov/flightsim/
- Check cases descriptions are detailed in Volume II of NASA/TM-2015-218675
- Trajectory data from the initial participants is provided as zipped CSV files

NESC Academy Webcast



Questions?

Full Data Set Available at http://nescacademy.nasa.gov/flightsim

BACKUP SLIDES

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Reference models – Geodesy

- Round
  - Constant radius sphere with same surface area as WGS-84
    $R_2 = 6.3710071809 \times 10^6$ m (2.0902255199×10^7 ft)
- WGS-84
  - Ellipsoidal Earth with a semi-major (equatorial) radius, flattening parameter, coefficient of eccentricity, and average sidereal rotation
  - WGS-84 defining parameters[3]

equatorial radius	a	6,378,137 m	20,925,600 ft
flattening parameter	$1/f$	298.257223563	
Gravitational constant	GM	$3.986004418 \times 10^{14}$ m^3/s^2	$1.407644311 \times 10^{16}$ ft^3/s^2
Angular rotation rate	ω	7.292115×10^{-5} rad/s	4.178073×10^{-3} deg/s

  - Derived parameters

    $e^2 = 6.69437999014 \times 10^{-3}$ (derived from f) “first eccentricity squared”

Reference models – Coordinate Frames (1/2)

- Earth-Centered Earth-Fixed (ECEF)
  - **X** axis points from the center of the Earth to the intersection of equator and prime meridian.
  - **Z** axis points from the center of the Earth to the geographic North Pole.
  - **Y** = **Z** x **X**
- Two Earth-Centered Inertial (ECI) frames
  - True-of-Date
    - ECI and ECEF axes are aligned at simulation start.
    - Used in atmospheric cases.
  - J2000
    - Modeled on the mean equator and mean equinox of the epoch at noon on 1 Jan 2000 Terrestrial Time.
    - Formally defined with respect to extra-galactic quasar sources (FK5 frame).
    - The IERS publishes code and data to transform J2000 to ECEF
    - Used in exo-atmospheric cases.

Reference models – Coordinate Frames (2/2)

- Ellipsoidal planet adds challenge to calculate position in several coordinate frames: inertial, geocentric, geodetic
  - Conversion between inertial and geocentric is closed form
  - Conversion from geodetic to geocentric coordinates is closed form
  - Conversion from geocentric to geodetic coordinates often uses an iterative solution.

Frame	**Coord. type**	**Coordinates**	**Acronym**	**S-119 ID**
Inertial	Rectangular	X, Y, Z	XYZ	`ei`
Geocentric	Spherical	ψ, λ, r	ULR	`ge`
Geodetic	Spherical	ϕ, λ, h	LLH	`--`
Local	Rectangular	N, E, D	NED	`ll`
Body	Rectangular	x, y, z	xyz	`body`
Ground-relative	Rectangular	x_{fe}, y_{fe}, z_{fe}	--	`fe`
Orbit (LVLH)	Rectangular	x_o, y_o, z_o	--	`vo`

Reference models – Gravitation

1. Constant gravity – A fixed value at all locations.
   – Use unit g (9.80665 m/s^2 or 32.1740 ft/s^2) , which approximates free fall due to gravitation less centrifugal relief due to Earth's rotation.
2. Inverse-square law – Gravitation varies inversely with the square of the radius from the center of the Earth
   – Use WGS-84 $\mu = 3.986004418 \times 10^{14}\ m^3/s^2$ ($1.407644311 \times 10^{16}\ ft^3/s^2$)
3. J2 – includes the first zonal harmonic fluctuation of gravitation to approximate non-spherical shape of Earth
   – It is a function of geodetic latitude and geocentric radius.
   – Gravitation has two dimensions: radial (inward to Earth center) and latitudinal (Northward)
   – Use J2 = $1.08262982 \times 10^{-3}$, derived from WGS-84 value of $C_{2,0}$
4. GEM-T1 – Goddard Earth Model T1.
   – Full spherical harmonic expansion of the Earth's gravitation using GEM-T1 published coefficients.
   – Taken to degree and order of 4 x 4 or 8 x 8 in exo-atmospheric cases.

Reference models –Atmosphere

- 1976 U.S. Standard Atmosphere
  - Can be implemented as equations or tables
  - Normally given as a function of geometric height (Z)
  - Translated into S-119 model
- Marshall Engineering Thermosphere Model (MET)
  - Developed by Marshall Space Flight Center for engineering applications
  - modified Jacchia 1970 model that includes some spatial and temporal variation patterns of the Jacchia 1971 model
  - Computes thermospheric densities, temperatures, gravitational accelerations, and specific heats

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