Analysis Cases Defined, Data Comparison Overview

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Many thanks to

Carol Wieseman, NASA, for processing:

• All of the spreadsheet data
• The ANSYS analysis team data files
• Most of the database plots that you are looking at
• Analysis Cases; Factors in choosing the cases
  • Benchmark Case
  • Availability of Experimental Data
  • Flow Physics

• Overview of Comparison Results Database
  • All data received by the data submittal deadline have been processed, along with format-preserving updates from those teams
  • Data from 8 teams have been processed into the comparison data bases (FIFO processing)
  • There are 14 separate comparison data bases
  • All data submitted in the template spreadsheets has been incorporated into the databases
## AePW-2 Analysis Cases

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach</td>
<td>0.7</td>
<td>0.74</td>
<td>0.85</td>
<td>.85</td>
<td>.85</td>
</tr>
<tr>
<td>Angle of attack</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Dynamic Data Type</td>
<td>Forced oscillation</td>
<td>Flutter</td>
<td>Unforced Unsteady</td>
<td>Forced Oscillation</td>
<td>Flutter</td>
</tr>
<tr>
<td>Notes:</td>
<td>• Attached flow solution.</td>
<td>• Unknown flow state.</td>
<td>• Separated flow effects.</td>
<td>• Separated flow effects.</td>
<td>• Separated flow effects on aeroelastic solution.</td>
</tr>
<tr>
<td></td>
<td>• Oscillating Turn Table (OTT) exp data.</td>
<td>• Pitch and Plunge Apparatus (PAPA) exp data.</td>
<td>• Oscillating Turn Table (OTT) experimental data.</td>
<td>• Oscillating Turn Table (OTT) experimental data.</td>
<td></td>
</tr>
</tbody>
</table>
14 Databases contain comparison data sets

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Steady or &quot;Overdamped&quot;</th>
<th>Time-accurate</th>
</tr>
</thead>
<tbody>
<tr>
<td>None, Unforced</td>
<td>1a</td>
<td>3aU</td>
</tr>
<tr>
<td></td>
<td>2aR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3aR</td>
<td></td>
</tr>
<tr>
<td>Forced Pitch Oscillation</td>
<td></td>
<td>1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3b</td>
</tr>
<tr>
<td>Aeroelastic, at common condition (qE = Experimental dynamic pressure)</td>
<td>2aSae_qE</td>
<td>2c_qE</td>
</tr>
<tr>
<td></td>
<td>3aSae_qE</td>
<td></td>
</tr>
<tr>
<td>Aeroelastic, at common condition (qF = Predicted flutter dynamic pressure; will be different for each analysis)</td>
<td>2aSae_qF</td>
<td>2c_qF</td>
</tr>
<tr>
<td></td>
<td>3aSae_qF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2c_qF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3c_qF</td>
<td></td>
</tr>
</tbody>
</table>

**Team Name**

- FOI
- EMBRAER
- NASAEast
- TECHNION
- UMich
- ZHAW
- ANSYS
- ATA
- NRC
- NLR
- ITA
- CDADAPCO
- MILANO
- RAFAEL
- STRASBOURG

Also includes Experimental Comparison Data
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>OTT Configuration</th>
<th>PAPA Configuration</th>
<th>OTT Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach</td>
<td>$M$</td>
<td></td>
<td>0.7</td>
<td>0.74</td>
<td>0.85</td>
</tr>
<tr>
<td>AoA</td>
<td>$\alpha$</td>
<td>deg</td>
<td>$3^\circ$</td>
<td>0$^\circ$</td>
<td>$5^\circ$</td>
</tr>
<tr>
<td>Reynolds number (based on chord)</td>
<td>$Re_c$</td>
<td></td>
<td>4.560x10$^6$</td>
<td>4.450x10$^6$</td>
<td>4.491x10$^6$</td>
</tr>
<tr>
<td>Reynolds number per unit length</td>
<td>$Re$</td>
<td>$Re_c/ft$</td>
<td>3.456x10$^6$</td>
<td>3.338x10$^6$</td>
<td>3.368x10$^6$</td>
</tr>
<tr>
<td>Dynamic pressure</td>
<td>$q$</td>
<td>psf</td>
<td>170.965</td>
<td>168.800</td>
<td>204.197</td>
</tr>
<tr>
<td>Velocity</td>
<td>$V$</td>
<td>ft/s</td>
<td>387.332</td>
<td>375.700</td>
<td>468.983</td>
</tr>
<tr>
<td>Speed of sound</td>
<td>$a$</td>
<td>ft/s</td>
<td>553.332</td>
<td>506.330</td>
<td>552.933</td>
</tr>
<tr>
<td>Static temperature</td>
<td>$T_{stat}$</td>
<td>$F$</td>
<td>85.692</td>
<td>89.250</td>
<td>87.915</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>slug/ft$^3$</td>
<td>0.00228</td>
<td>0.002392</td>
<td>0.001857</td>
</tr>
<tr>
<td>Ratio of specific heats</td>
<td>$\gamma$</td>
<td></td>
<td>1.113</td>
<td>1.136</td>
<td>1.116</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>$\mu$</td>
<td>slug/ft - s</td>
<td>2.58x10$^{-7}$</td>
<td>2.69x10$^{-7}$</td>
<td>2.59x10$^{-7}$</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>$Pr$</td>
<td></td>
<td>0.683</td>
<td>0.755</td>
<td>0.674</td>
</tr>
<tr>
<td>Test medium</td>
<td></td>
<td></td>
<td>R-134a</td>
<td>R-12</td>
<td>R-134a</td>
</tr>
<tr>
<td>Total pressure</td>
<td>$H$</td>
<td>psf</td>
<td>823.17</td>
<td></td>
<td>757.31</td>
</tr>
<tr>
<td>Static pressure</td>
<td>$p$</td>
<td>psf</td>
<td>629.661</td>
<td></td>
<td>512.120</td>
</tr>
<tr>
<td>Purity</td>
<td>$X$</td>
<td>%</td>
<td>95</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>Ref. molecular weight based on 100% purity</td>
<td>$M$</td>
<td>g/mol</td>
<td>102.03</td>
<td>120.91</td>
<td>102.03</td>
</tr>
<tr>
<td>Sutherland’s constant</td>
<td>$C$</td>
<td>$R$</td>
<td>438.07</td>
<td>452.13</td>
<td>438.07</td>
</tr>
<tr>
<td>Reference viscosity</td>
<td>$\mu_{ref}$</td>
<td>lb - sec/ft$^2$</td>
<td>2.332x10$^{-7}$</td>
<td>2.380x10$^{-7}$</td>
<td>2.332x10$^{-7}$</td>
</tr>
<tr>
<td>Reference temperature</td>
<td>$T_{ref}$</td>
<td>$R$</td>
<td>491.4</td>
<td>491.4</td>
<td>491.4</td>
</tr>
</tbody>
</table>
Two Experiments provide comparison data for AePW-2

TDT Test 548 (2000)
BSCW Testing on the Oscillating TurnTable (OTT) for Forced Oscillation Cases

TDT Test 470 (1992)
BSCW Testing on the Pitch And Plunge Apparatus (PAPA) for Flutter Cases
Shock-induced separation assessment lead to AePW-2 case selection

<table>
<thead>
<tr>
<th>Mach</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.85</th>
<th>0.87</th>
</tr>
</thead>
<tbody>
<tr>
<td>q (psf)</td>
<td>170</td>
<td>100</td>
<td>170</td>
<td>100</td>
<td>170</td>
</tr>
<tr>
<td>$\alpha^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
<td>1.27</td>
<td>1.28</td>
<td>1.28</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>1.28</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>1.21</td>
<td>1.21</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.20</td>
<td>1.21</td>
<td>1.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.07</td>
<td>1.29</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Orange circles indicate shock-induced separation.
- Orange circles with an X indicate shock-induced separation onset.
- Black circles indicate data unavailable.

Number value: Sub-critical, maximum local Mach

AePW-1 case

AePW-2 case
Overview of Comparison Data
Important Note

These comparisons are utilizing the preliminary data, as submitted prior to the AePW.

These are workshop results, not publication results.

Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.
Results from AePW-1: BSCW Mach 0.85 5°

Critique:
- Significant variation among computational results
- Inconsistent application of sign conventions led to uncertainty in phase angle definition
- No measure of the quality of the results; No coherence data
- Mean value characterization of experimental data artificially smears the shock (cants the pressure distribution, makes it less sharp than seen in instantaneous snapshots)
- Spacing of experimental data may lead to under-representing the magnitude peak
Primary data comparisons

- Steady rigid pressure coefficient distributions: statistics of the results

- Frequency response functions: $C_p/\theta$
  - At forced oscillation
  - At flutter condition, frequency
  - At prescribed, experimental condition

- Flutter conditions
  - Dynamic pressure at flutter onset
  - Frequency at flutter onset
  - Damping, frequency and static aeroelastic deformation at common analysis condition
Guide to reading file names

Steady Results

- Case_U/L_S1/2_SortBy
- U/L: Upper or Lower surface is designated by a single letter
- S1 / 2: Station 1 (60% span); Station 2 (95% span)
- SortedBy: string designating how the data is grouped on the plot
  - Groupings have different colors and symbols
  - Options currently plotted are Sorted By:
    - Analyst (specified by analysis team letter)
    - Plots are in directories named for other Sort options, but in the current versions they are not correctly designating the different sort parameters
      - Turbulence model
      - Grid Resolution
      - Flux Limiter
      - Grid Type (structured, unstructured, multiblock)
      - Software Name
- Currently only Mean Values of Cp are plotted (i.e. none of the other statistics have plots generated, although they are in the database files)
Guide to reading file names
Frequency Response Function Plots

- Case_U/L_C/M/P_S1/2_SortBy
- U/L: Upper or Lower surface is designated by a single letter
- C/M/P: Coherence, Magnitude or Phase, designated by a single letter
- S1 / 2: Station 1 (60% span); Station 2 (95% span)
- SortedBy: string designating how the data is grouped on the plot
  - Groupings have different colors and symbols
  - Options currently plotted are Sorted By:
    - Analyst (specified by analysis team letter)
    - Plots are in directories named for other Sort options, but in the current versions they are not correctly
designating the different sort parameters
      - Turbulence model
      - Grid Resolution
      - Flux Limiter
      - Grid Type (structured, unstructured, multiblock)
      - Software Name
Case 2 Flutter
Dynamic pressure
Case 3 Flutter
Dynamic pressure
Spatial convergence

Show CL, CM, CD as functions of Coarse, Medium and Fine Grids for the 3 analysis conditions.
Show sectional coefficients if sufficient data sets.

Coming Soon
Temporal convergence

Show some unsteady quantity, such as $CL/\theta$, $CM/\theta$, $CD/\theta$ as functions of time step size for the 3 analysis conditions.
Show sectional coefficients if sufficient data sets.

Show flutter condition as a function of time step size.

Coming Soon
Steady case results

60% span

**Case 1a**  
Mach 0.7, $\alpha = 3^\circ$

**Case 2a**  
Mach 0.74, $\alpha = 0^\circ$

**Case 3a**  
Mach 0.85, $\alpha = 5^\circ$

These comparisons are utilizing the preliminary data, as submitted prior to the AePW. These are workshop results, not publica-
tion results. Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.
Steady case results

Case 1a
Mach 0.7, \( \alpha = 3^\circ \)

Case 2a
Mach 0.74, \( \alpha = 0^\circ \)

Case 3a
Mach 0.85, \( \alpha = 5^\circ \)

These comparisons are utilizing the preliminary data, as submitted prior to the AePW. These are workshop results, not publication results. Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.

Colored lines with open symbols:
- Each analysis team shown by a separate color
- Each grid size shown by a different symbol
Steady case results

60% span

**Case 1a**
Mach 0.7, \( \alpha = 3^\circ \)

**Case 2a**
Mach 0.74, \( \alpha = 0^\circ \)

**Case 3a**
Mach 0.85, \( \alpha = 5^\circ \)

These comparisons are utilizing the preliminary data, as submitted prior to the AePW. These are workshop results, not publication results. Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.
These comparisons are utilizing the preliminary data, as submitted prior to the AePW. These are workshop results, not publication results. Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.
Case 2c_qE
Mach 0.74, $\alpha = 0^\circ$

These comparisons are utilizing the preliminary data, as submitted prior to the AePW. These are workshop results, not publication results. Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.
Forced oscillation

Case 1b
Mach 0.7, $\alpha = 3^\circ$

60% span

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Forced oscillation

**Case 3b**
**Mach 0.85, \( \alpha = 5^\circ \)**

Experimental data

Colored lines with open symbols:
- Each analysis team shown by a separate color
- Each grid size shown by a different symbol

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Flutter analysis FRFs

**Case 2c_qF**

Mach 0.74, $\alpha = 0^\circ$

These comparisons are utilizing the preliminary data, as submitted prior to the AePW. These are workshop results, not publication results. Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.

### Experimental data
- Colored lines with open symbols:
  - Each analysis team shown by a separate color
  - Each grid size shown by a different symbol

### Colored lines with open symbols:
- Experiment data
- Experiment bounds
Flutter analysis FRFs

**Case 3c, qE, 204 psf**
**Mach 0.85, \( \alpha = 5^\circ \)**

These comparisons are utilizing the preliminary data, as submitted prior to the AePW. These are workshop results, not publication results. Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.
Flutter analysis FRFs

**Case 3c_qF**
Mach 0.85, $\alpha = 5^\circ$

These comparisons are utilizing the preliminary data, as submitted prior to the AePW. These are workshop results, not publication results. Please use these results showing proper respect for the willingness of the analysts and data reduction team to share preliminary findings.

---

Colored lines with open symbols:
- Each analysis team shown by a separate color
- Each grid size shown by a different symbol

Experimental data

- Experimental data
- Experiment bounds
Sort by Turbulence Model
Considerations in data processing & interpretation computing FRFs

• Representing the average distributions
  • Statistic (mean, mode, median)
  • Unsteady effects

• Computing FRFs
  • Frequency to process for non-flutter dynamic aeroelastic points
  • Time history subset selection
    • Initial transient elimination
    • Large displacement solution elimination?
  • Fourier analysis parameters
    • Block size determination
    • Number of periodograms
  • Using the coherence information is important
Results from AePW-1: BSCW Forced Oscillation Mach 0.85 5°

Frequency Response Function at 10Hz

- Experimental data
- Bounds, ± 2 std

Colored lines with open symbols:
- Each analysis team shown by a separate color
- Each grid size shown by a different symbol
Steady Rigid Pressure Distributions

For the primary forced oscillation case, Case #1, disagreements with experimental data limited to the peak of the upper surface shock.

For the primary flutter case, Case #2, shows a well-matched rigid pressure distribution without much variation among the computational results.

The complexity of the Case #3 is indicated by the variation among the computational results & difference from the experimental data → Shock location, shock strength, aft loading especially on lower surface.

Case Comparisons
60% span,
Mean values of Cp
From AePW-1 Data Comparisons
Benchmark Supercritical Wing (BSCW)

- Chosen as a challenging test case, flow-wise, but simple geometry
  - Strong shock with suspected shock-induced separated flow
### Summary of Benchmark Supercritical Wing Entries

<table>
<thead>
<tr>
<th>Analyst</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TURBULENCE MODEL</strong></td>
<td>SA</td>
<td>SA</td>
<td>SA</td>
<td>SA</td>
<td>SST</td>
<td>SST-(k_\omega)</td>
</tr>
<tr>
<td><strong>GRID TYPE</strong></td>
<td>Str</td>
<td>Unstr</td>
<td>Str</td>
<td>Unstr</td>
<td>Str</td>
<td>Str</td>
</tr>
</tbody>
</table>

Str = Structured  
Unstr = Unstructured  

**Codes used:**  
- FUN3D  
- CFL3D  
- Overflow 2.2c  
- NSMB  
- NSU3D  
- ANSYS CFX
Frequency response functions (FRFs) calculation example

\[ FRF(\omega) = \frac{CSD_{x,y}(\omega)}{PSD_x(\omega)} = \frac{FFT(y) \cdot FFT(x)'}{FFT(x) \cdot FFT(x)'} \]

Here,
- \( x \) = displacement
- \( y \) = \( C_p \)

- **1 FRF for each pressure transducer**

Pressure / excitation:
At frequencies where there is no excitation, the calculation is dividing by 0’ish numbers, making the FRF a large amplitude noisy response

From AePW-1, using HIRENASD data for the example
Frequency response functions (FRFs) calculation example

![Graph showing FRF calculation example](image)

Here,  

\[ FRF(\omega) = \frac{CSD_{x,y}(\omega)}{PSD_x(\omega)} = \frac{FFT(y) \cdot FFT(x)' }{FFT(x) \cdot FFT(x)'} \]

From AePW-1, using HIRENASD data for the example

- \( x \) = displacement
- \( y \) = \( C_P \)
- 1 FRF for each pressure transducer
- Examine values only at the excitation frequency

Excitation frequency  
\(~ 80 \text{ Hz} \)
Frequency response functions (FRFs) calculation example

Here,

- \( x = \text{displacement} \)
- \( y = \text{Cp} \)
- 1 FRF for each pressure transducer
- Examine values only at the excitation frequency
- Plot the results for all transducers on a single plot, as a function of chord location

From AePW-1, using HIRENASD data for the example
Frequency response functions (FRFs) calculation example

Here,

\[ F(RF) = \frac{CSD_{x,y}(\omega)}{PSD_x(\omega)} = \frac{FFT(y) \cdot FFT(x)' \cdot FFT(x)''}{FFT(x)'} \]

\[ x = \text{displacement} \]
\[ y = C_p \]

- 1 FRF for each pressure transducer
- Examine values only at the excitation frequency
- Plot the results for all transducers on a single plot, as a function of chord location

From AePW-1, using HIRENASD data for the example
Uncertainty of FRFs (S4T DataGirl Paper)

• These methods were developed by various references using an assumption of Gaussian-distributed random data, rather than sinusoidal data. The methods were also developed based on non-overlapping segments (i.e. independent data records.). Other methods were examined but not utilized in this report (see references below for these other methods).


<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>frequency variable, (Hz)</td>
</tr>
<tr>
<td>$n$</td>
<td>number of segments, ensembles, in a Fourier coefficient calculation</td>
</tr>
<tr>
<td>$nfft$</td>
<td>Fourier analysis block size, (samples)</td>
</tr>
<tr>
<td>$samp$</td>
<td>sample rate (samples/sec)</td>
</tr>
<tr>
<td>$a$</td>
<td>statistical significance level</td>
</tr>
<tr>
<td>$\gamma_{xy}^2$</td>
<td>mean square coherence between an input, $x$, and an output, $y$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard deviation</td>
</tr>
<tr>
<td>$\omega$</td>
<td>frequency variable, (radians)</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
</tr>
<tr>
<td>$G$</td>
<td>Open loop plant frequency response function</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density function</td>
</tr>
<tr>
<td>$P_{xx}$</td>
<td>PSD of the input, $x$</td>
</tr>
<tr>
<td>$P_{yy}$</td>
<td>PSD of the output, $y$</td>
</tr>
<tr>
<td>$\hat{\cdot}$</td>
<td>approximate quantity based on a data sample</td>
</tr>
</tbody>
</table>
FRF uncertainty method #1

\[ \hat{\sigma}(\hat{G}(\omega)) = \frac{\sqrt{1 - \hat{\gamma}_{xy}^2(\omega)}}{|\hat{\gamma}_{xy}|\sqrt{2}} |\hat{G}(\omega)| \]  \hspace{1cm} (4)

\[ \hat{\sigma}(\text{angle}(\hat{G}(\omega))) = \sin^{-1}\left(\frac{\sqrt{1 - \hat{\gamma}_{xy}^2(\omega)}}{|\hat{\gamma}_{xy}|\sqrt{2}}\right) \]  \hspace{1cm} (5)

The confidence intervals or standard deviations of the magnitude and phase of the FRFs can be estimated using the coherence and FRF estimates, which are generated using a given number of data sets. For the S4T analysis case, each overlapped average segment is treated as a separate data set. The data analysis in the reference assumed that the influence of bias errors had been minimized, and the primary source of error was random error arising from unmeasured excitations. The magnitude and phase values at each given frequency were assumed to be Gaussian-distributed random variables.

Error bounds on the FRF estimates are calculated following the method given in the reference, using Eqs (4) and (5). Results produced from this method are denoted FRF Method 1. The upper and lower bounds on the magnitude and phase were calculated using 3\( \sigma \) limits.

Ref: Doebling, S.W., and Farrar, C.R., Estimation of statistical distributions for modal parameters identified from averaged frequency response function data, Los Alamos National Laboratory, LA-UR-00-41, July 2000.
FRF uncertainty method #2

A second method to assess the uncertainties of the FRF- denoted FRF Method 2- is applied, following the process outlined in Bendar & Piersol. This method accounts for random error in the measured FRF. Specified in the reference by eqn 6.146, and reproduced here in Eqns (6) and (7), a confidence interval with confidence 100\(^*(1-\alpha)\%\) can be calculated. Equation (6) says that the difference between the estimated plant and the true plant is less than the bound given by (7). From (7), the error in FRFs is dependent on the degrees of freedom and the coherence. The uncertainty in the calculation decreases as the number of ensembles, \(n\), used in the averaging for computing the spectral estimates increases or as the coherence increases towards 1.

![Equation (6)](image)

\(\left|\hat{G}(f) - G(f)\right|^2 \leq \hat{r}(f)^2\)

![Equation (7)](image)

\[\hat{r}(f) = \frac{2}{(n - 2)} F_{2, n-2, \alpha} \left[1 - \gamma_{xy}(f)\right] \frac{\hat{p}_{yy}}{\hat{p}_{xx}}\]

As the number of ensembles increases, the coherence values will decrease. The confidence the coherence value, however, increases with the number of ensembles.

The S^4T simulation data, with 105 seconds in the time history, with 8192 points in the analysis block size, with 95\% overlap has 237 overlapped analysis segments. The associated value of the F-distribution, \(F_{2,235,0.05} = 3.03392\). Results from applying FRF Method 2 are shown in Figure 11. The uncertainty bounds are almost identical to those produced by FRF Method 1. There are small changes in the bounds near the zeros-valleys- of the FRFs.

Some advice from some experts

• As pointed out by Schoukens, Rolain and Pentelon “Measuring a periodic signal over an integer number periods removes the leakage problem completely, and we strongly advise the reader to apply periodic excitation signals whenever it is possible.” (Schoukens, J., Rolain, Y., and Pentelon, R, “Analysis of windowing/leakage effects in frequency response function measurements,” Automatica 42 (2006) P 27-38.)

• Using a rectangular window has its drawbacks, and one of them is leakage when the frequencies are not exactly windowed by the time length. From Bendat & Piersol, 1980, “… the large side lobes of (the the frequency domain representation of the window) allow leakage of power at frequencies well separated from the main lobe of the spectral window and may introduce significant anomalies in the estimated spectra, particularly when the data are sinusoidal…” This has been demonstrated to be the case in the analysis of these multisine data sets. (Bendat, J.S., and Piersol, A.G., “Engineering applications of correlation and spectral analysis,” John Wiley & Sons, New York, 1980.)
Some things recommended for future work (by myself in the DataGirl paper)

• Apply expressions for uncertainties of the FRF and coherence based on sinusoidal assumptions rather than on Gaussian random data assumptions.

• In the cases where the assumption has been made that the segments are non-overlapping, apply and/or develop the expressions for overlapping segments.

• Apply these same general methods to analysis of short time records pertinent to unsteady computational fluid dynamic simulation results.

• Investigate harmonic distortion through simulation results and analytical methods

• Investigate methods for properly representing and combining sources of uncertainty