
Multi-Output Auto-Regressive Modeling for Transonic Flutter Analysis

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MOAR Model Formulation

- Current output $y(k)$ is based on past outputs and inputs

$$y(k) = \sum_{i=1}^{na} \mathcal{A}_i y(k-i) + \sum_{i=0}^{nb} \mathcal{B}_i u(k-i)$$

- Assume $nb = na - 1$
- Coefficient $\mathcal{A}_i, \mathcal{B}_i$ found from LS fitting to reference data: CFD-based aerodynamic response to small-amplitude structural model

Aeroelastic Coupling

- Aeroelastic EOM in SS form with MOAR modeling of unsteady aerodynamics

$$\begin{Bmatrix} x_S(k+1) \\ x_A(k+1) \end{Bmatrix} = \begin{bmatrix} A_S + q_\infty B_S D_A C_S & q_\infty B_S C_A \\ B_A C_S & A_A \end{bmatrix} \begin{Bmatrix} x_S(k) \\ x_A(k) \end{Bmatrix}$$

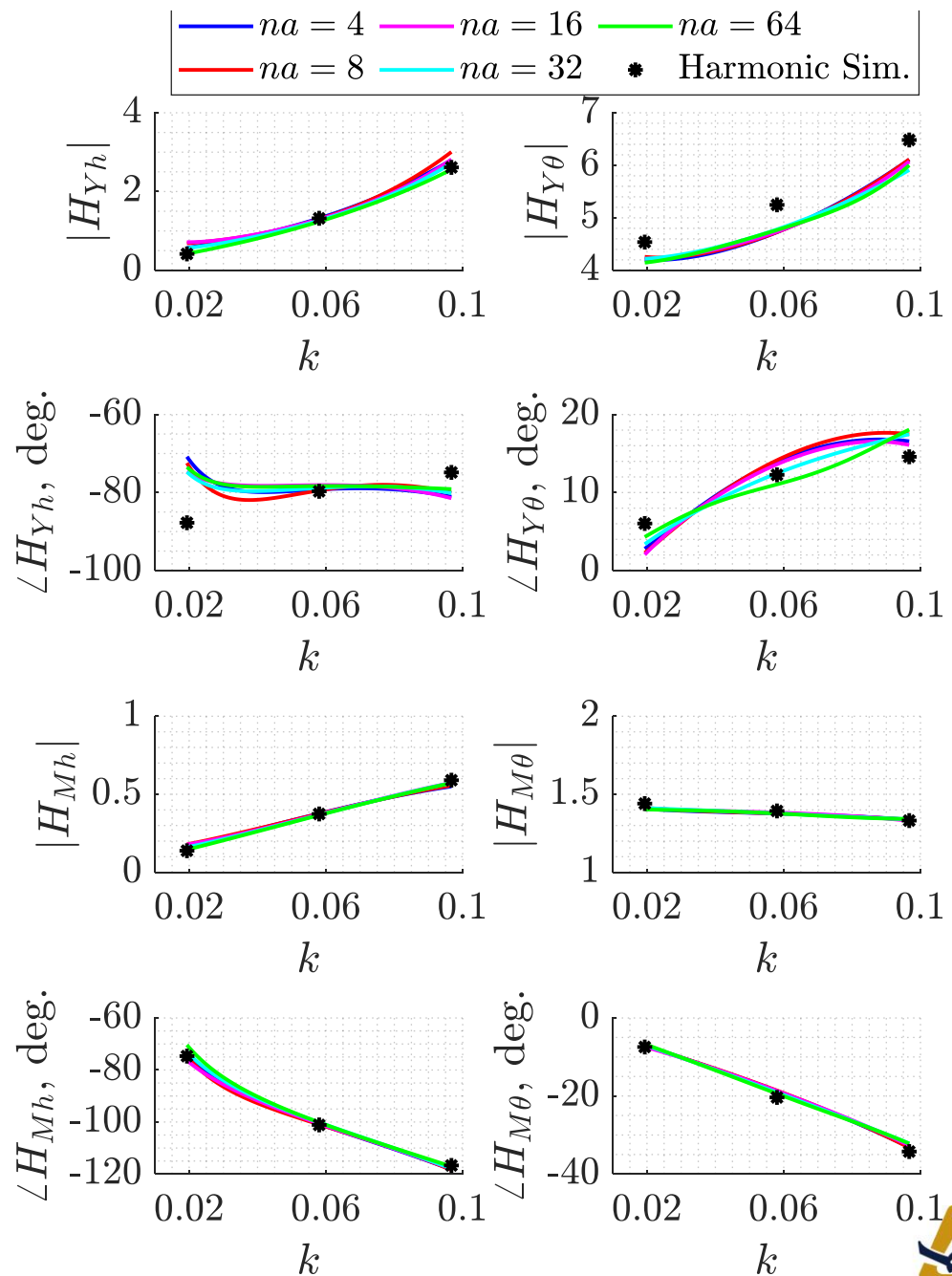
$$\xi(k) = [C_S \quad 0] \begin{Bmatrix} x_S(k) \\ x_A(k) \end{Bmatrix}$$

- Damping and frequencies of the aeroelastic modes are obtained from the eigenvalues z_i of the discrete-time state matrix of the aeroelastic system

$$\omega_{n,i} = \frac{|\ln z_i|}{\Delta t}, \quad \zeta_i = -\frac{\ln|z_i|}{|\ln z_i|}$$

Aerodynamic FRF Identification Results

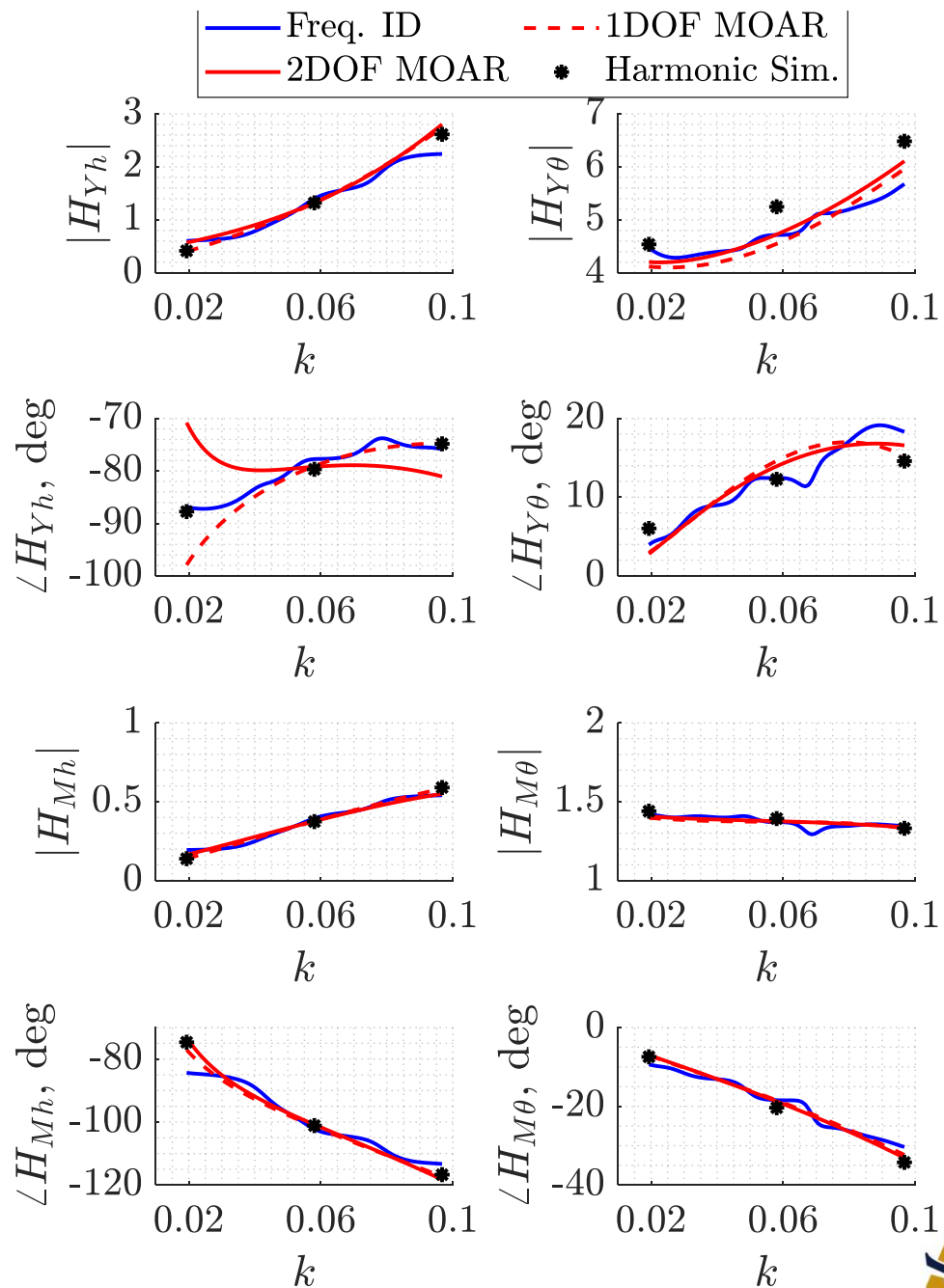
- Impact of model order on FRF ID
- Mach 0.8, AoA 1°



Aerodynamic FRF Identification Results

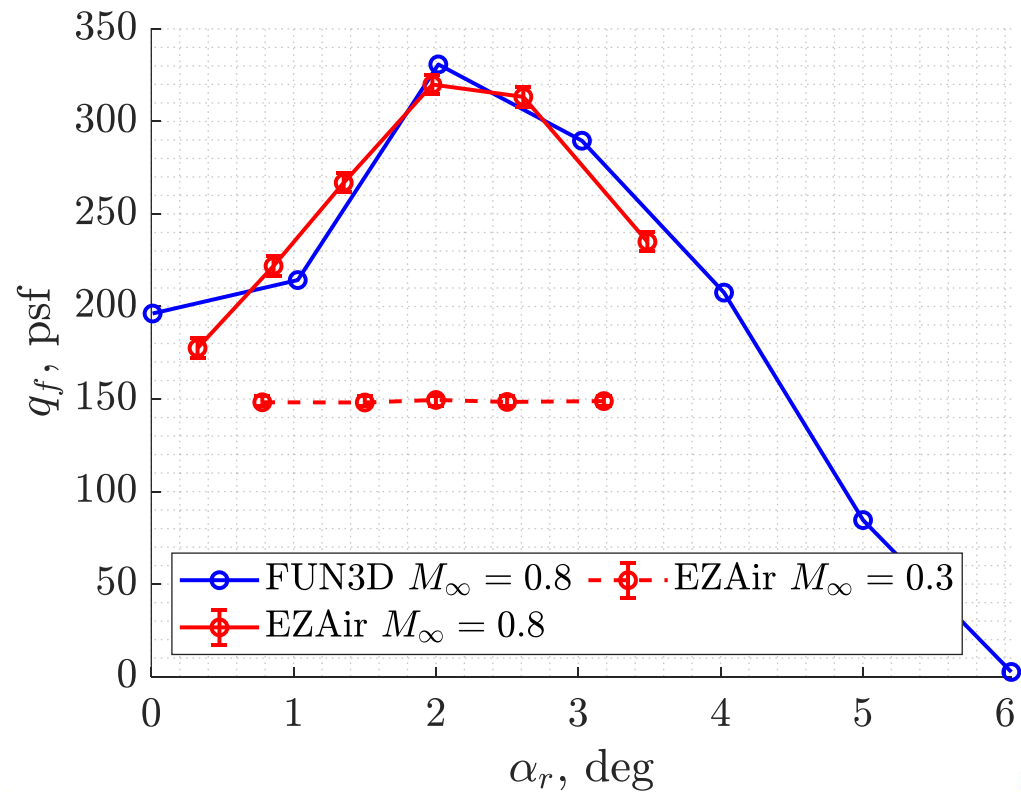
- Comparison with frequency-domain ID
- Mach 0.8, AoA 1°

$$\text{FRF}(k) = \frac{P_{yx}(k)}{P_{xx}(k)}$$



Flutter Computation

- Time-accurate flutter results



Match Flutter Angle of Attack

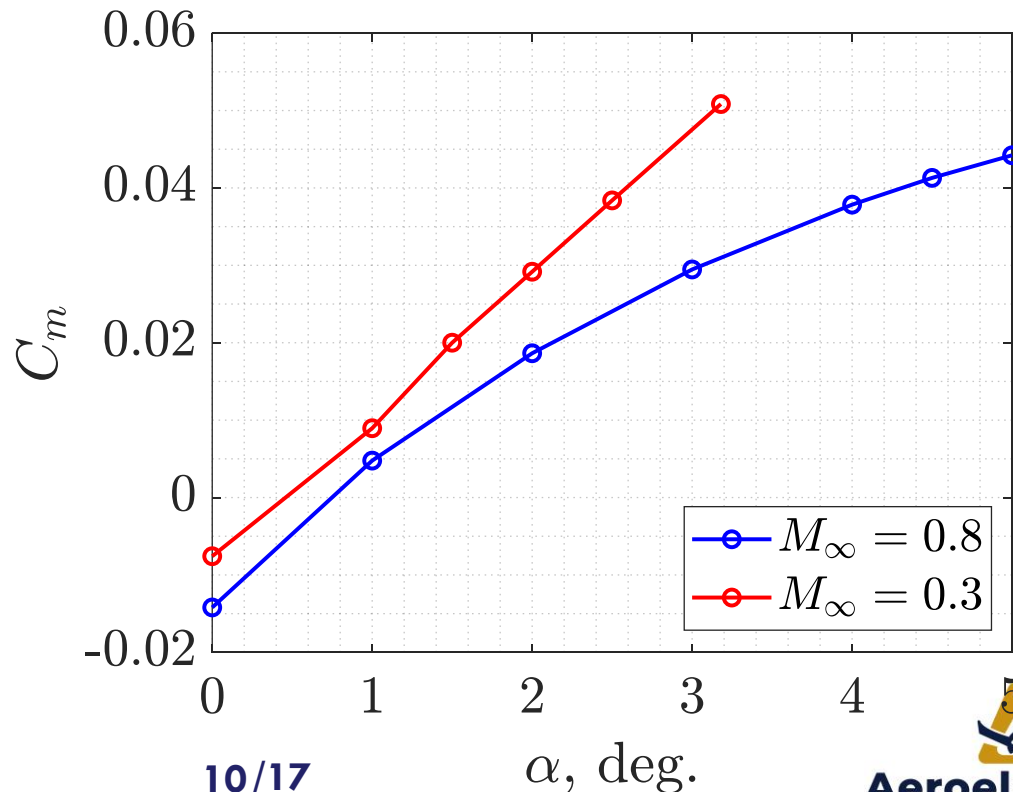
- Unsteady aerodynamic forces are linearized under small perturbations assumption about a non-linear state
- Prescribed structural motion is assumed **about static equilibrium**

$$M_{\text{elastic}} = M_{\text{aero}}$$

- Angle of attack at static equilibrium: α_e
- Related to rigid angle of attack α_r by elastic pitch angle θ
$$\alpha_e = \alpha_r + \theta$$
- q_f vs. α_e is computed, then transformed to the informative polar q_f vs. α_r

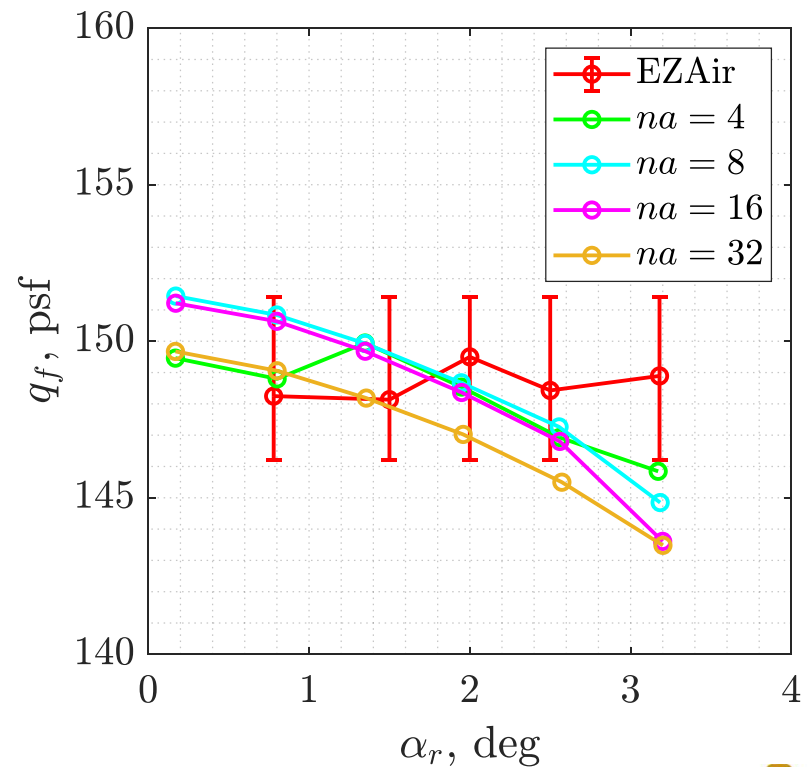
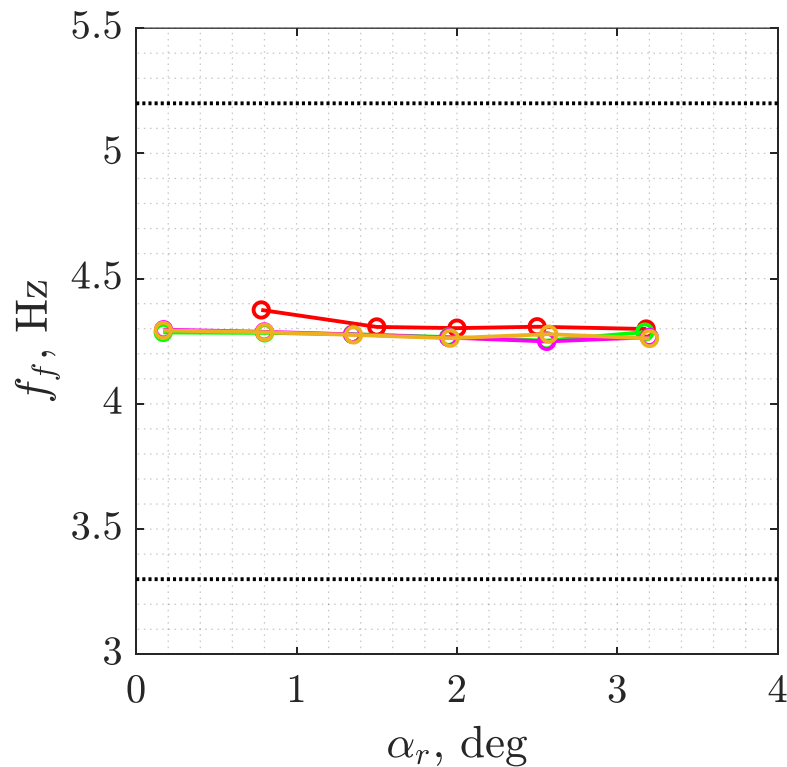
Match Flutter Angle of Attack

- Static equilibrium relates the known α_e with the desired α_r
$$k_\theta \theta = q_\infty S c C_M(\alpha_e)$$
- $C_M(\alpha)$ - steady moment coefficient polar



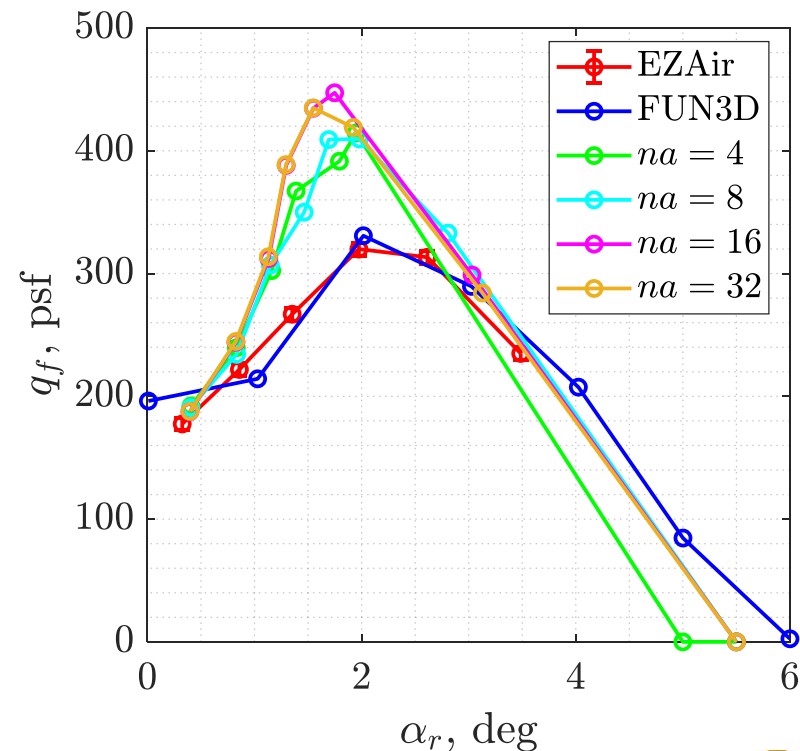
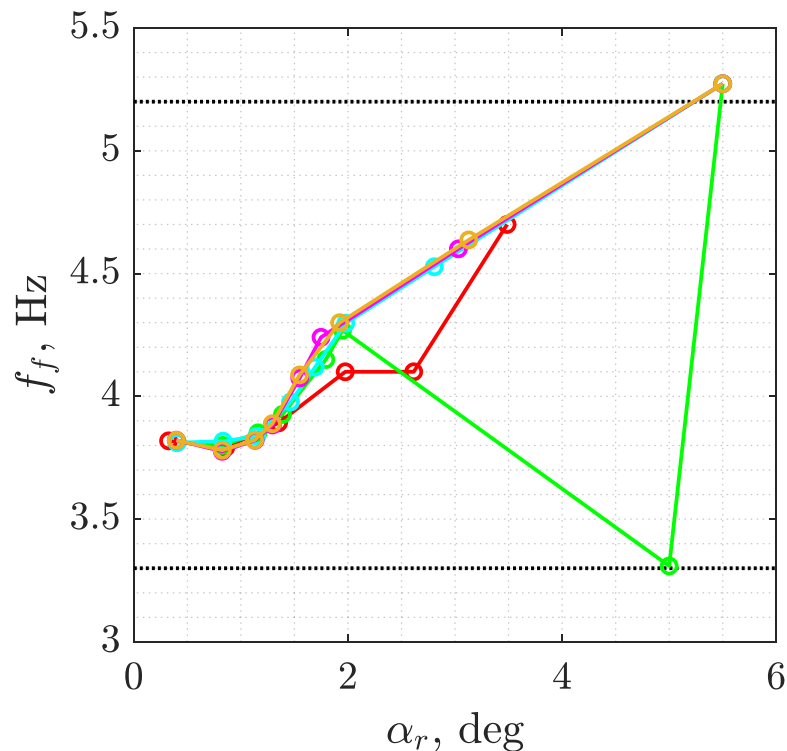
Flutter Computation at Mach 0.3

- ROM flutter results – subsonic Mach number



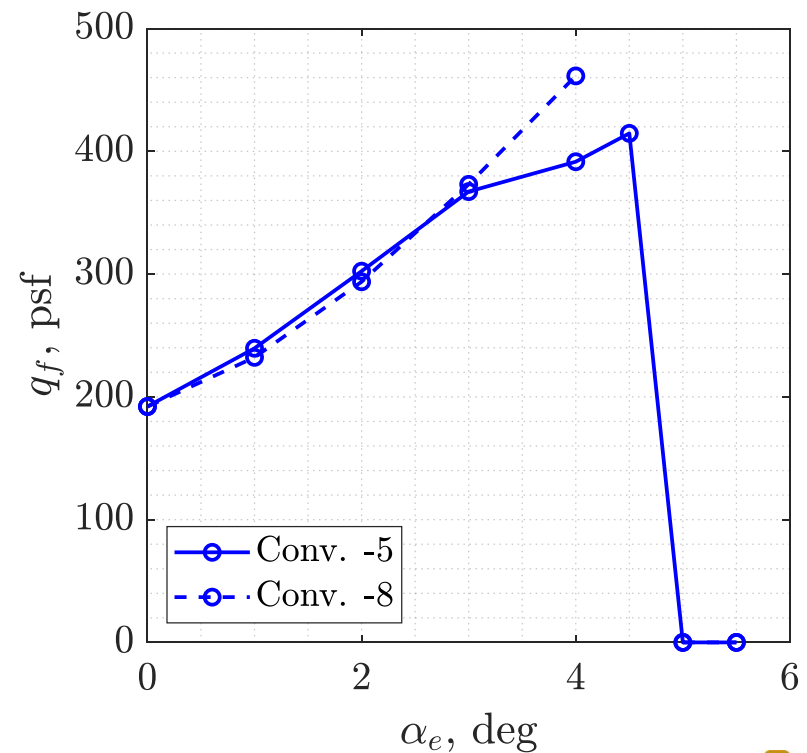
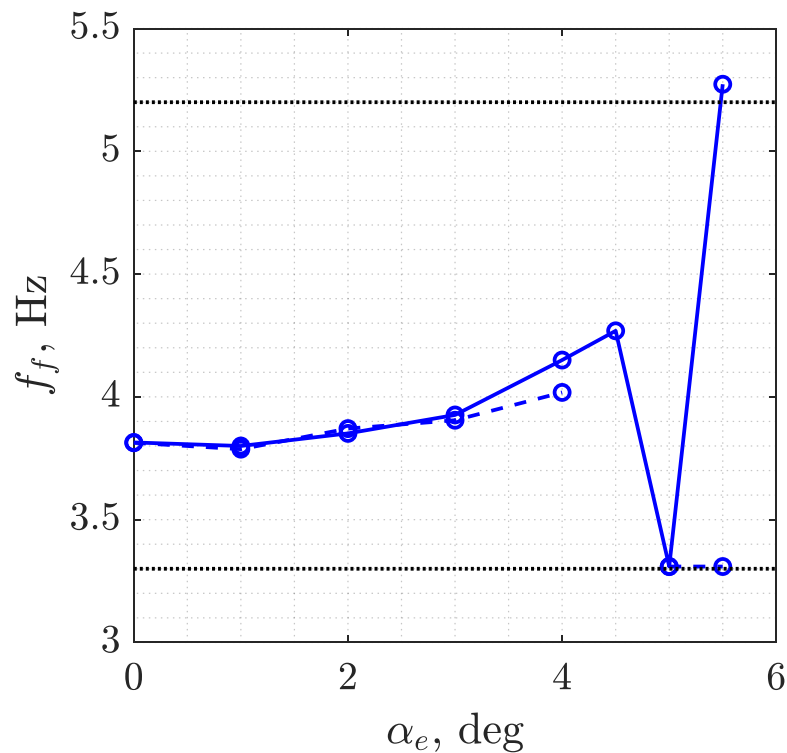
Flutter Computation at Mach 0.8

- Impact of model order on flutter prediction accuracy



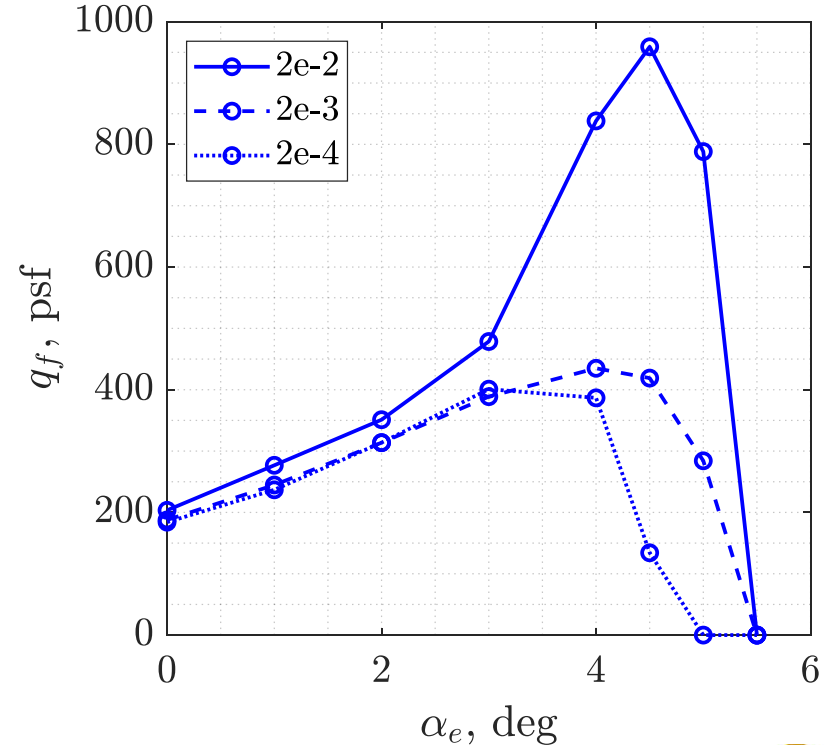
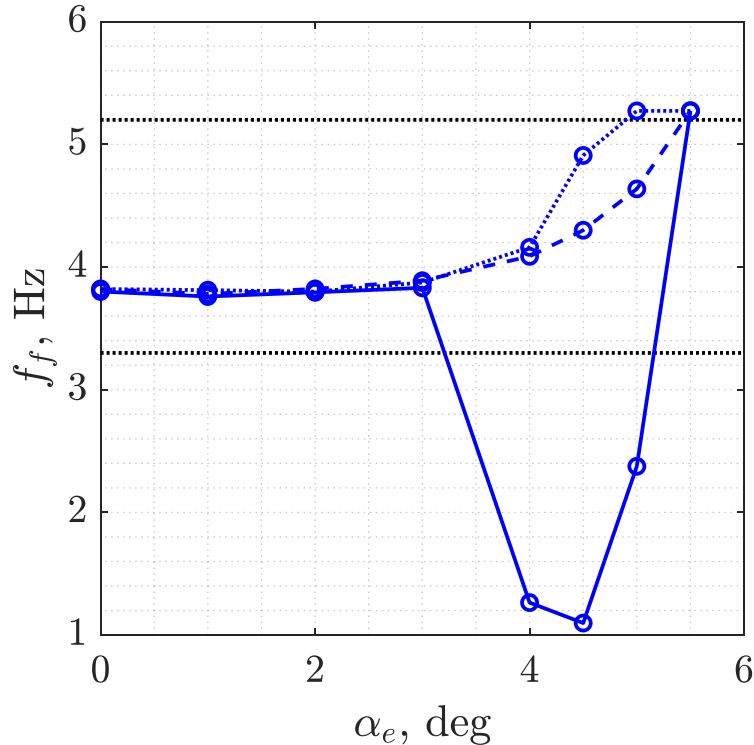
Flutter Computation

- Sensitivity to **iterative convergence criterion**



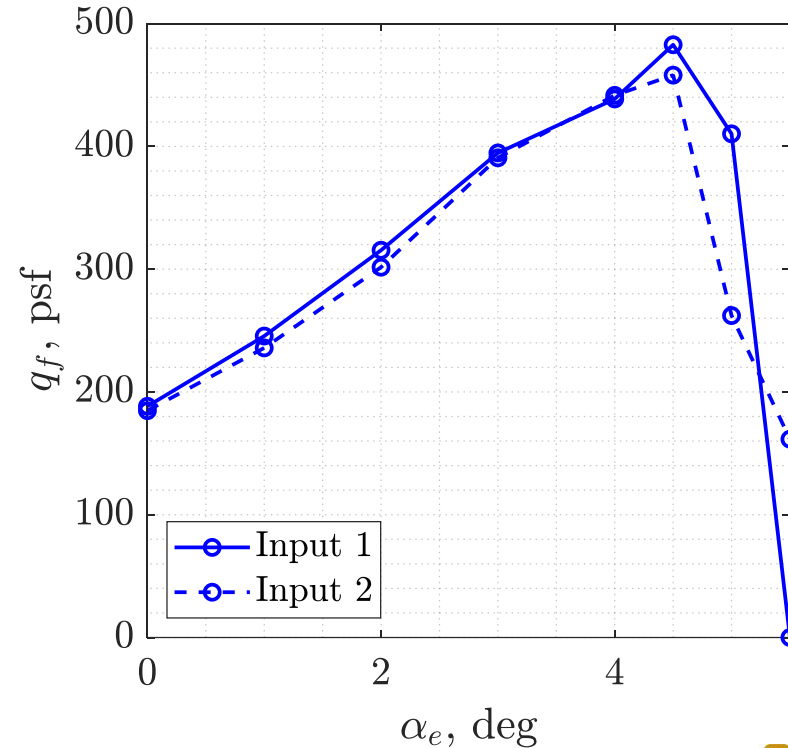
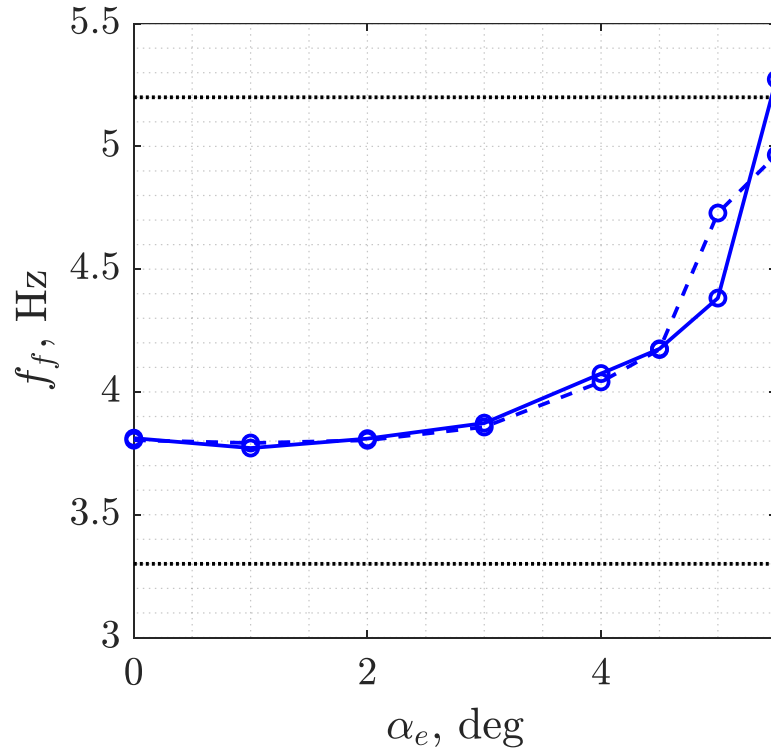
Flutter Computation

- Sensitivity to **prescribed motion amplitude**



Flutter Computation

- Sensitivity to **prescribed motion signal**



Conclusion

- Accurate identification of **linearized aerodynamic models**
 - Agreement with direct harmonic simulations
 - Linearization about a non-linear flow-field
- Accurate flutter prediction in **subsonic and attached-transonic flows**
 - Challenges arise in presence of flow separation
- Time-domain aerodynamic MOAR modeling is **highly efficient**
 - Small number of reference data samples
 - Model independent of dynamic pressure
 - Simultaneous excitation of multiple structural modes

Thank you!