NESC ACADEMY WEBCAST

Welcome…

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Multiple-Effector Control Allocation in Theory and Practice

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NESC ACADEMY WEBCAST

Multiple-Effector Control Allocation: Theory and Practice

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Introduction

- In many practical aerospace systems we encounter multiple, overlapping control effectors.
  - Aerosurfaces (aircraft, entry vehicles)
  - Thrust vectoring (launch vehicles, high-performance aircraft)
  - Reaction controls (spacecraft, entry vehicles)
- *Control allocation* schemes are intended to replace multiple controls with a fewer number of *virtual controls* in some optimal way.

Virtual Controls (I)

- Consider the simplified vehicle dynamics

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \\ \mathbf{y} &= \mathbf{h}(\mathbf{x})\end{aligned} \qquad \mathbf{x} \in \mathbb{R}^n,\ \mathbf{y} \in \mathbb{R}^n,\ \mathbf{u} \in \mathbb{R}^m$$

  with or without some set of actuator dynamics.

- Let's simplify the system to the form

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}_0, t_0) + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{h}(\mathbf{x})\end{aligned}$$

  where

$$\mathbf{B} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{bmatrix}$$

  in the usual sense.

Virtual Controls (II)

- The *input sensitivity matrix* $\mathbf{B}$ maps the actual controls $\mathbf{u}$ onto the controlled degrees of freedom, for example, the accelerations.
  - Usually, $\mathbf{B} \in \mathbb{R}^{n \times m}$ is not square!

- We would like to replace $\mathbf{Bu}$ with a virtual control $\mathbf{v}$, to "square up" the system with respect to some selected degrees of freedom.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}_0, t_0) + \mathbf{v}$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x})$$

- The *control allocation problem* is finding a unique $\mathbf{u}$ of all the possible solutions that satisfy $\mathbf{v} = \mathbf{Bu}$:

$$\min_{\mathbf{u}} H(\mathbf{u}) \text{ subject to } \mathbf{v} = \mathbf{Bu}$$

- The *constrained control allocation problem* introduces a constrained set of controls $\mathbf{u}$:

$$\min_{\mathbf{u}} \; H\left(\mathbf{v},\, \mathbf{Bu}\right) \; \text{subject to } c(\mathbf{u}) \leq 0.$$

Motivation

- Control allocation is desirable because it *eliminates redundant degrees of freedom* and *simplifies the design problem.*
  - General MIMO techniques can also be used, but
  - Human pilots and many practical/classical control laws are SISO (e.g., 3 separate axes with limited cross-connects).

- It is straightforward to solve the control allocation problem *if*
  - the moments [accelerations] are *linear* in the controls:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}_0, t_0) + \mathbf{B}\mathbf{u}$$

  - the controls are unconstrained; or
  - the constraints are geometrically simple; e.g., $|u_i| \leq c_i$.

- The formal theory of control allocation by W.C. Durham[1] and his students in the 1990s.
  - Ad-hoc and semi-formal methods were used in the 1960s and on.
  - Lallman[2] had introduced a pseudocontrols method in the 1980s.

[1] Durham, W., *Constrained Control Allocation*, J. Guidance, Control, and Dynamics, Vol. 16, No. 4, 1993, pp. 717-725.
[2] Lallman, F.J., *Relative Control Effectiveness Technique With Application to Airplane Control Cooridination*, NASA TP-2416, 1985.

Example – HARV

$n = 3,\ m = 10$

- **F/A-18 HARV Aircraft**
  - High angle of attack research vehicle operated at NASA AFRC* from 1987-1996
  - Simplified attitude dynamics† are given by:

$$\mathbf{J}\delta\dot{\boldsymbol{\omega}} + \left(\boldsymbol{\omega}_0^\times \mathbf{J} - (\mathbf{J}\boldsymbol{\omega}_0)^\times\right)\delta\boldsymbol{\omega} = \boxed{\boldsymbol{m}_{\text{aero}}\left(\bar{q}, \boldsymbol{u}_0, M_0, \alpha_0, \beta_0\right)} + \bar{q}Sl\mathbf{B}\mathbf{u}$$

Linearized flight condition

  - The control effectors are left and right horizontal tail, left and right aileron, combined rudder, left and right trailing edge flaps, and 3 thrust vectoring vanes.
  - The control system operates on the roll, pitch, and yaw rates.
  - The $\mathbf{B}$ matrix has units of normalized aero coefficients.
  - Limits are <u>asymmetric</u>.

F/A-18 High Angle-of-Attack Research Vehicle (HARV)

	Effector	min (deg)	max (deg)
u_1	R Horiz Tail	-24.0	+10.5
u_2	L Horiz Tail	-24.0	+10.5
u_3	Right Aileron	-30.0	+30.0
u_4	Left Aileron	-30.0	+30.0
u_5	Combined Rudder	-30.0	+30.0
u_6	R Trail Edge Flap	-8.0	+45.0
u_7	L Trail Edge Flap	-8.0	+45.0
u_8	Roll TVC Vane	-30.0	+30.0
u_9	Pitch TVC Vane	-30.0	+30.0
u_{10}	Yaw TVC Vane	-30.0	+30.0

HARV Control Limits, adapted from Bordignon [3]

*Formely NASA Dryden Flight Research Center (DFRC).
†Actuator dynamics are neglected in this example, and *l*=*b*=*c*.
[3] Bordignon, K., *Constrained Control Allocation for Systems with Redundant Control Effectors*, PhD Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1996.

Example: Apollo LM

$n = 6,\ m = 16$

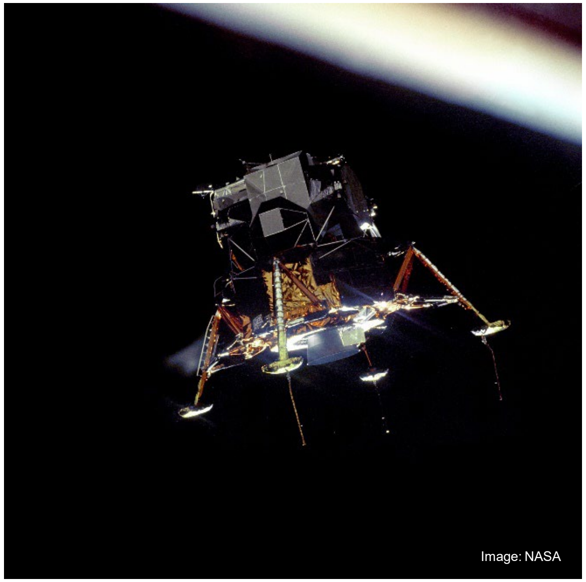
- **Apollo Lunar Module (LM)**
  - 16 RCS thrust chamber assemblies (TCAs) in 2 (A/B) strings and 4 quads
  - 6-DoF rigid-body dynamics:

Rotation
$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}\mathbf{J}\boldsymbol{\omega} = \sum_i \mathbf{r}_i^{\times} F_i \hat{\mathbf{a}}_i u_i = \mathbf{M}\mathbf{u}$$

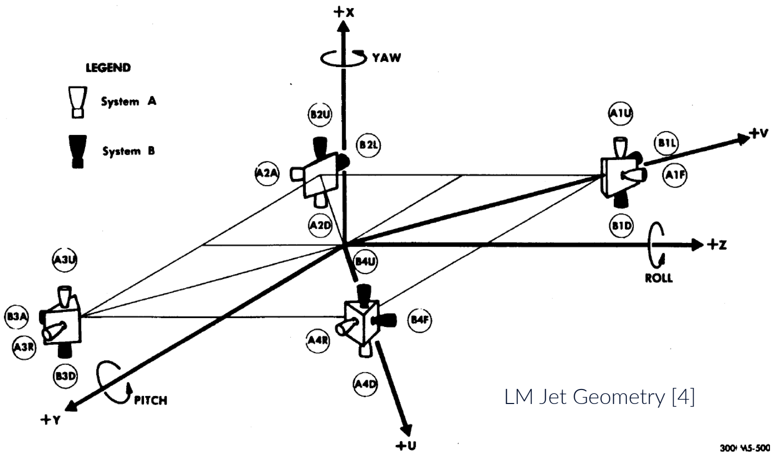
Translation
$$m\left(\dot{\mathbf{v}} + \boldsymbol{\omega}^{\times}\mathbf{v}\right) = \sum_i F_i \hat{\mathbf{a}}_i u_i = \mathbf{R}\mathbf{u}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{J}^{-1}\mathbf{M} & \\ & m^{-1}\mathbf{R} \end{bmatrix}$$

  - Firing commands given by $u_i \in \{0, 1\}$
  - Production autopilot used multiple firing tables with selectable 2-jet or 4-jet modes and fault cases.



Apollo 11 LM *Eagle*, July 20, 1969



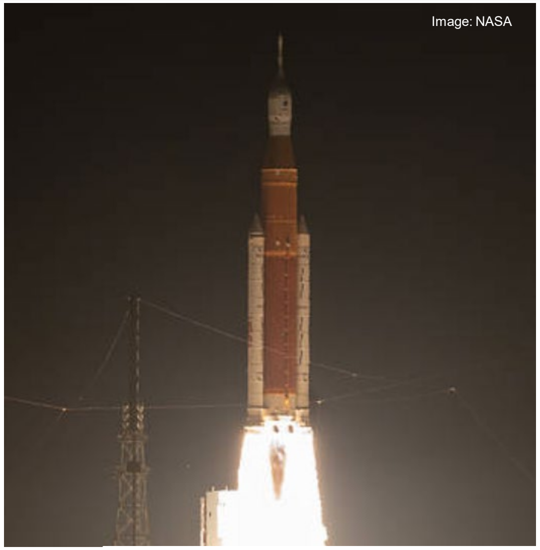
LM Jet Geometry [4]

[4] Apollo Operations Handbook, Lunar Module. LM 11 and Subsequent – Volume II: Operational Procedures, LM790-3-LM, Section 4.4.3, Grumman Aerospace, 1971.

Example: NASA SLS

$\boxed{n = 3,\ m = 12}$

- Human-rated launch vehicle for large-scale ("exploration-class") crew and cargo access
- First flight November 16, 2022
- 4 RS-25E core stage engines (CSEs) + 2 5-segment SRMs, 6 engines x 2 axes = 12 DoF
- Elliptical limits, on-line reconfiguration[5]
- Control-structure interaction and servoelasticity[6]



NASA Artemis I, Nov 16, 2022

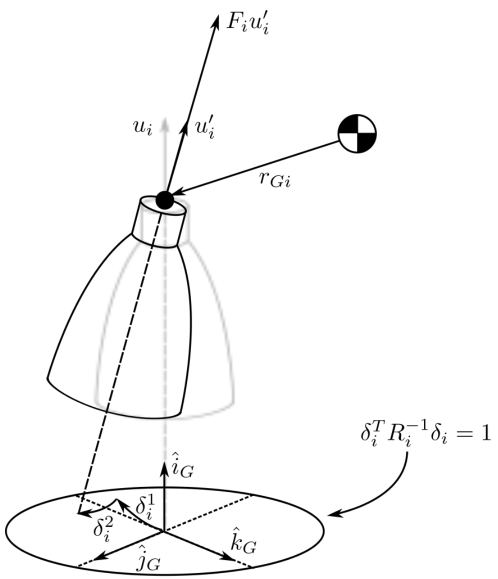
$$\mathbf{u}_i' = \mathbf{T}_i \mathbf{u}_i \qquad \mathbf{T}_i \approx \left[\mathbf{I} + \mathbf{\Theta}_i^{\times}\right]$$

TVC angle change in body frame

$$\mathbf{J}\delta\dot{\omega} = -\sum_{i=1}^{k} F_i \mathbf{r}_{Gi}^{\times} \mathbf{u}_i^{\times} \mathbf{T}_i^{G} \boldsymbol{\delta}_i = \mathbf{M}\boldsymbol{\Delta}$$

Dynamic moment

$$\mathbf{M} = \begin{bmatrix} -F_1 \mathbf{r}_{G1}^{\times} \mathbf{u}_1^{\times} \mathbf{T}_1^{G} & -F_2 \mathbf{r}_{G2}^{\times} \mathbf{u}_2^{\times} \mathbf{T}_2^{G} & \dots & -F_k \mathbf{r}_{Gk}^{\times} \mathbf{u}_k^{\times} \mathbf{T}_k^{G} \end{bmatrix}$$

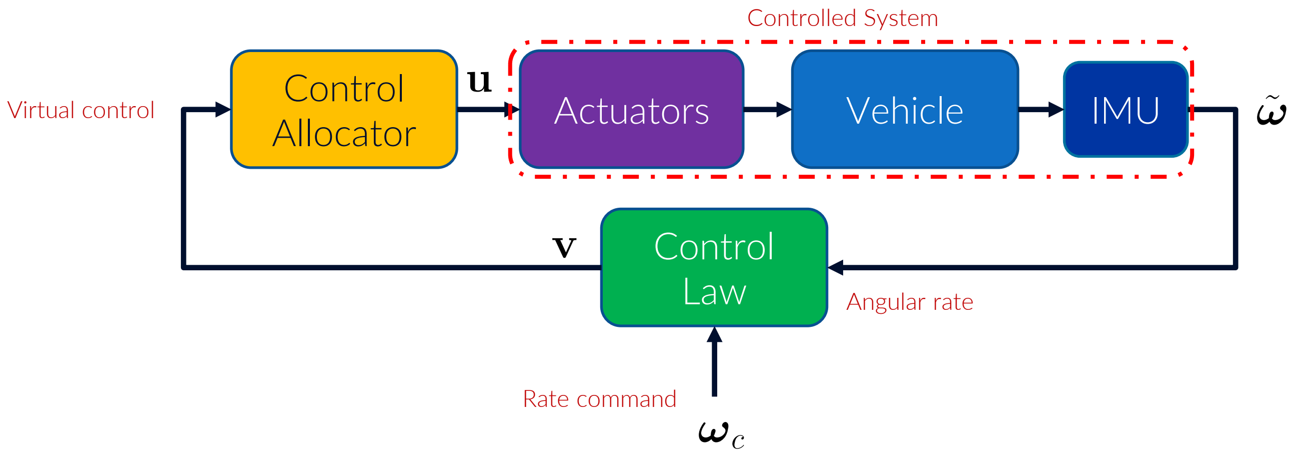


[5] Orr, J., Wall, J., VanZwieten, T., and Hall, C., *Space Launch System Ascent Flight Control Design*, American Astronautical Society Guidance, Navigation, and Control Conference, Breckenridge, CO, AAS 14-038, 2014.
[6] Orr, J., Wall, J., and Barrows, T., *Simulation-Based Analysis and Prediction of Thrust Vector Servoelastic Coupling*, American Astronautical Society Guidance, Navigation, and Control Conference, Breckenridge, CO, AAS 20-091, 2020.

Control Allocation

- Given a desired virtual control $\mathbf{v}$, find a $\mathbf{u}$ that is "optimal".

$$\min_{\mathbf{u}} H(\mathbf{u}) \text{ subject to } \mathbf{v} = \mathbf{B}\mathbf{u}$$



- This simplifies our design problem. Ideally, we can find a $\mathbf{u}$ such that $\mathbf{B}\mathbf{u} = \mathbf{v}$ (the command).
- There are usually n=3 controlled DoF.
- For the *unconstrained case*, a solution always exists for arbitrary $\mathbf{v}$ if

$$\mathrm{rank}(\mathbf{B}) = n$$

- The solution is not unique.

Pseudoinverse Solutions

- It is common to solve the equation $\mathbf{v} = \mathbf{B}\mathbf{u}$ using a *pseudoinverse.*
  - This is a linear, least-squares solution.
  - It does not have knowledge of the constraints.
  - There are many *right generalized inverses* $\mathbf{u} = \mathbf{P}\mathbf{v}$*.*

- A generalized inverse is any matrix $\mathbf{P}$ that satisfies $\mathbf{B}\mathbf{P} = \mathbf{I}$.
  - If $\mathbf{u} = \mathbf{P}\mathbf{v}_c$ and $\mathbf{B}\mathbf{P} = \mathbf{I}$, then $\mathbf{v} = \mathbf{B}\mathbf{u} = \mathbf{B}\mathbf{P}\mathbf{v}_c = \mathbf{v}_c$.

- A generalized inverse has at most $(m-n)n$ free parameters, but usually far fewer.

- This matrix is sometimes called the *allocator* matrix.

- A very common generalized inverse is the Weighted Least Squares (WLS) inverse.

The Weighted Least Squares WLS Allocator

- The WLS allocator minimizes the weighted 2-norm of the controls:

$$H(\mathbf{u}) = \frac{1}{2}\mathbf{u}^T\mathbf{S}\mathbf{u} + \boldsymbol{\lambda}^T(\mathbf{v} - \mathbf{B}\mathbf{u})$$

- The weight matrix $\mathbf{W} = \mathbf{S}^{-1}$ is used to penalize individual controls.
- The optimal solution is

$$\boxed{\mathbf{u} = \mathbf{W}\mathbf{B}^T\left(\mathbf{B}\mathbf{W}\mathbf{B}^T\right)^{-1}\mathbf{v} = \mathbf{P}\mathbf{v}}$$

- The weight matrix can be SPSD so long as $\mathbf{B}\mathbf{W}\mathbf{B}^T$ is nonsingular.

- A generalized inverse has n linearly independent columns $\mathbf{p}_i$.

- For $\mathbf{W} = \alpha\mathbf{I}$, the WLS allocator becomes the Moore-Penrose inverse.
  - This minimizes the norm of the control vector, $\|\mathbf{u}\|_2$.
  - The solution (columns $\mathbf{p}_i$) are orthogonal to the null space of $\mathbf{B}$.
  - This is an important feature for secondary objectives.

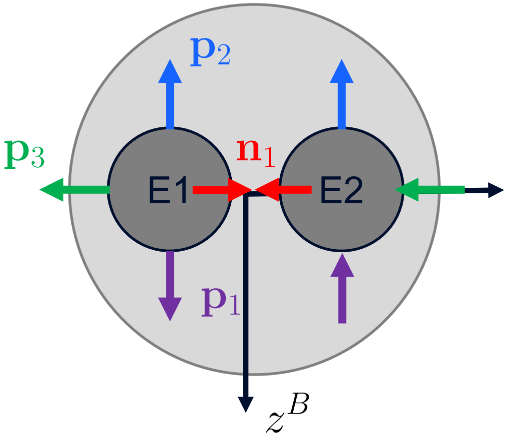
The Null Space – Titan II GLV

$\boxed{n = 3, \; m = 4}$

- Since $m > n$ there must exist a *null space* $\mathcal{N}(\mathbf{B})$ of dimension $m - n$.
  - This is the set of all vectors $\mathcal{N}(\mathbf{B}) = \{\mathbf{u} \in \mathbb{R}^m \mid \mathbf{B}\mathbf{u} = \mathbf{0}\}$
  - There are $m - n$ *waste directions* in the m*-space* that generate no net moment (but create internal loads).

- Sometimes the null space is geometrically obvious.
- The null space turns out to be very useful.

Titan II Gemini Launch Vehicle (GLV)

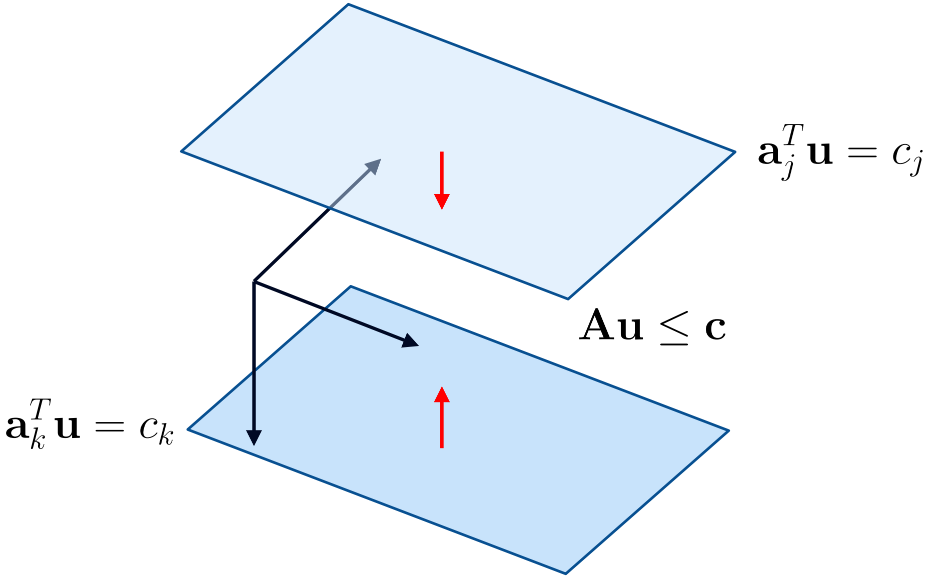


The Attainable Moment Set (AMS)

- The limits can be written in terms of the control vector $\mathbf{u}$:

$$\mathbf{a}_j^T \mathbf{u} < c_j$$

- Each of these expressions defines a *halfspace* in an *m*-dimensional space (the size of the controls), bounded by a *hyperplane.*
- Symmetric constraints $|u_i| \leq c_i$ are a special case.
- There are 2*m* inequalities, forming a *polytope* (a convex set).
- If there are no unlimited controls, the set is both *closed* and *bounded*.



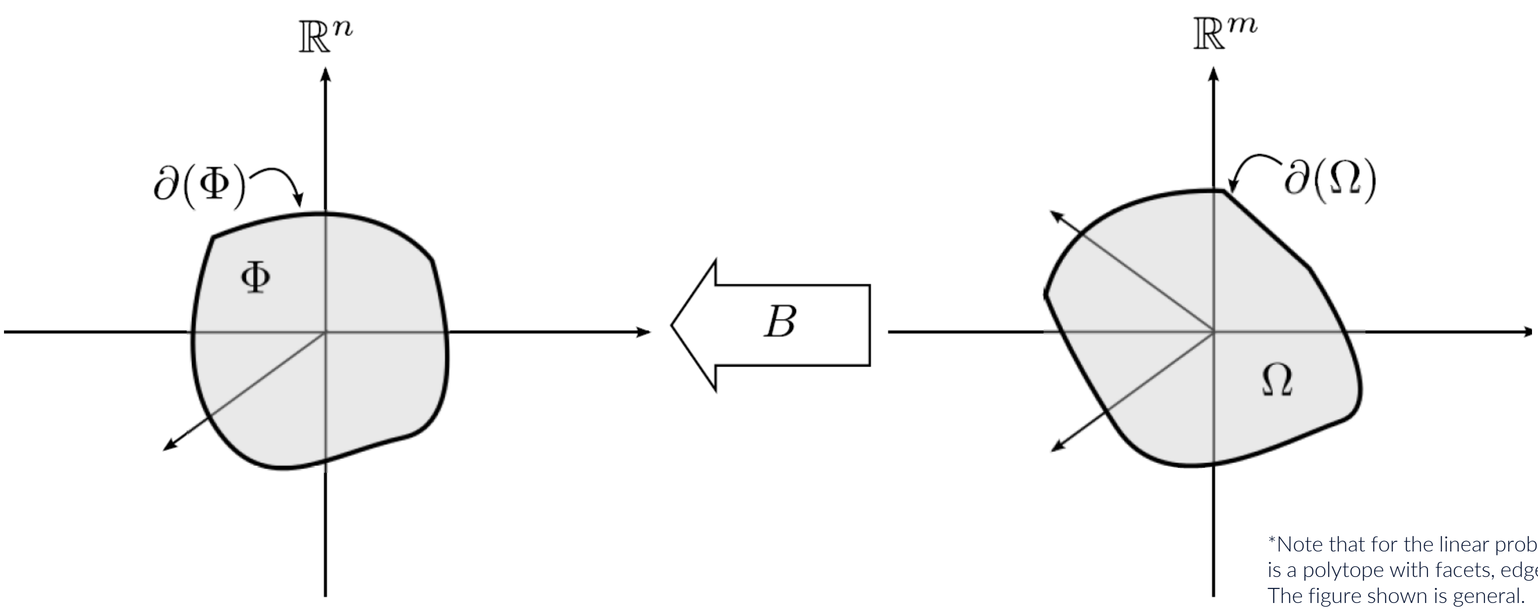
The Attainable Moment Set (AMS)

- The set of all possible controls is called the *admissible set*.

$$\Omega = \{\mathbf{u} \in \mathbb{R}^m \mid \mathbf{A}\mathbf{u} \leq \mathbf{c}\}$$

Linear inequality constraint (polytope)

- This set has an *image* in $\mathbb{R}^n$: $\Phi = \{\mathbf{B}\mathbf{u} \in \mathbb{R}^n \mid \mathbf{u} \in \Omega\}$
- Convexity is preserved under linear transformations.
  - The set Φ is called the *attainable moment set.*
- For any point within Φ, there exists a **u** that satisfies the constraints.



*Note that for the linear problem, the geometry is a polytope with facets, edges, and vertices. The figure shown is general.

Constrained Control Allocation

- Given a desired virtual control $\mathbf{v}$, find a $\mathbf{u}$ that is “close” to the desired moment, in some sense.

$$\min_{\mathbf{u}} H(\mathbf{v}, \mathbf{Bu}) \text{ subject to } c(\mathbf{u}) \leq 0.$$

  – Minimizing $\|\mathbf{v} - \mathbf{Bu}\|_2$ (a QP) does not guarantee collinearity!

- **Direct** control allocation finds $\alpha, \mathbf{u}$ such that

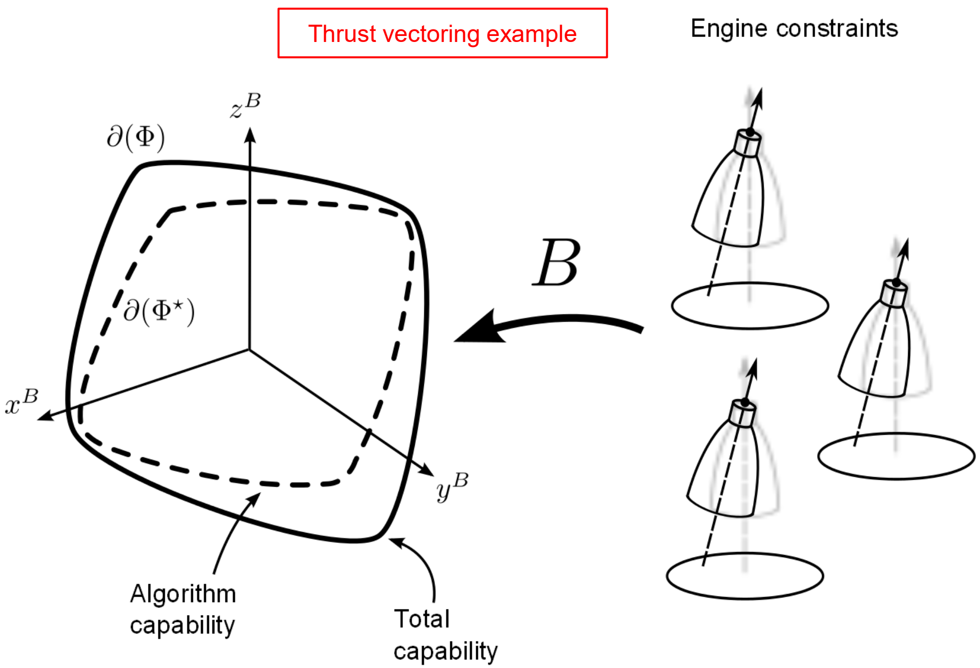
$$\alpha \mathbf{v} = \mathbf{Bu} \qquad c(\mathbf{u}) = 0$$

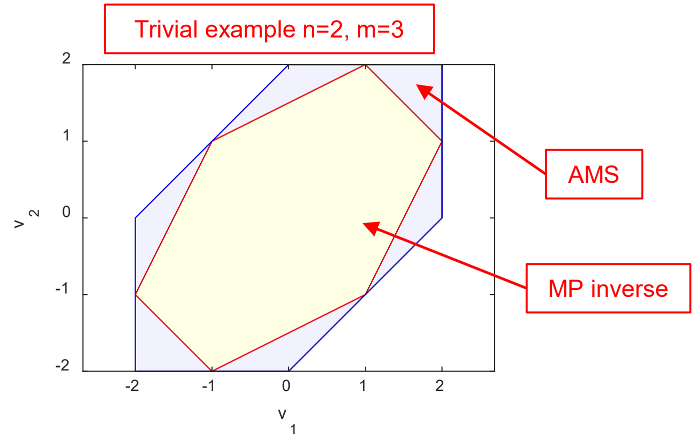
  – and α is maximized, i.e., the maximum admissible moment in the direction desired.
  – The solution is $\mathbf{u}^\star = \alpha^{-1}\mathbf{u}$ if $\alpha > 1$, or $\mathbf{u}$ otherwise.
  – This can be cast as an LP and solved using the simplex algorithm.

- There are numerous other algorithms, each with their own advantages and drawbacks.

Why Not Just Use Plain Old WLS?

- A generalized inverse cannot access all of the moments.
  - This result is counter-intuitive and was first proved by Durham[2].
  - This applies for <u>any</u> convex constraint with $n>1$.
  - An allocator that is *linear* is restricted to an n-dimensional subspace of the m controls.
  - This can be overcome using $\mathcal{N}(\mathbf{B})$ (via weighting, or "tailoring").





An *optimal* or *efficient* linear control allocator is one that makes $\mathbf{\Phi}^\star$ as large as possible in the desired controlled DoF.

There are multiple ways to define optimality or efficiency.

[2] Durham, W., *Constrained Control Allocation*, J. Guidance, Control, and Dynamics, Vol. 16, No. 4, 1993, pp. 717-725.

Example – HARV Dynamics

» System is linearized at $h = 10{,}000\,\mathrm{ft},\ M_0 = 0.3,\ \alpha_0 = 12.5\,\mathrm{deg},\ \beta_0 = 0.$

– There are *m*=10 effectors and 3 degrees of freedom.

– The $\mathbf{M}$ matrix is 3x10.

$$\mathbf{M} = \begin{bmatrix} -0.0438 & 0.0438 & -0.0584 & 0.0584 & 0.0167 & -0.0628 & 0.0628 & 0.0292 & 0.0000 & 0.0100 \\ -0.5330 & -0.5330 & -0.0649 & -0.0649 & 0 & 0.0623 & 0.0623 & 0.0000 & 0.3553 & 0.0000 \\ -0.0110 & 0.0110 & 0.0039 & -0.0039 & -0.0743 & 0 & 0 & 0.0003 & 0.0000 & 0.1485 \end{bmatrix}$$ [ft·lbf/radian]

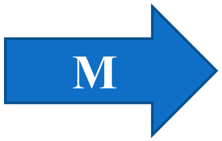
Data adapted from Bordignon [3]

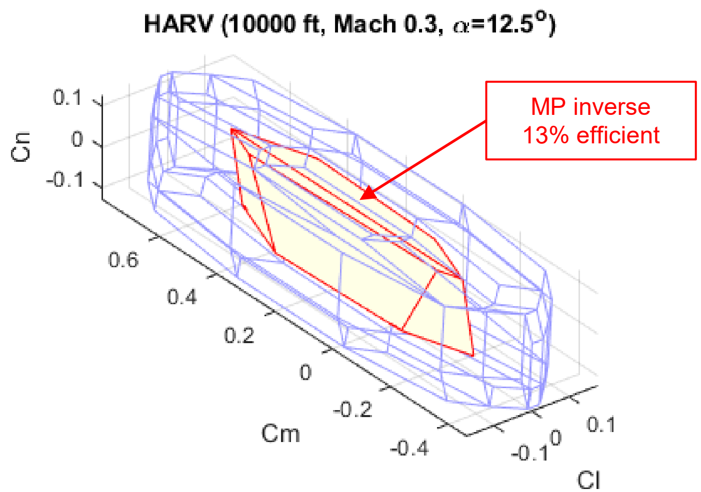
– In aircraft applications, it is common to work in "*moment" space* defined by $\mathbf{M}$.

– The $\mathbf{M}$ (or $\mathbf{B}$) matrix, combined with limits, gives the [linear] control capability.

	Effector	min (deg)	max (deg)
u_1	R Horiz Tail	-24.0	+10.5
u_2	L Horiz Tail	-24.0	+10.5
u_3	Right Aileron	-30.0	+30.0
u_4	Left Aileron	-30.0	+30.0
u_5	Combined Rudder	-30.0	+30.0
u_6	R Trail Edge Flap	-8.0	+45.0
u_7	L Trail Edge Flap	-8.0	+45.0
u_8	Roll TVC Vane	-30.0	+30.0
u_9	Pitch TVC Vane	-30.0	+30.0
u_{10}	Yaw TVC Vane	-30.0	+30.0

HARV Control Limits, adapted from Bordignon [3]





HARV AMS, computed using *qcat* [7]

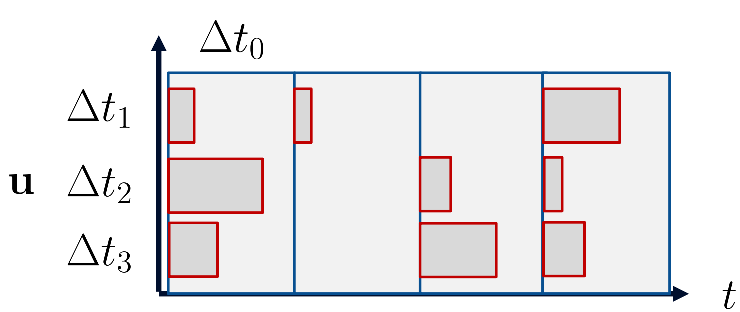
[7] Härkegård, O., *Quadratic Programming Control Allocation Toolbox for MATLAB*, v.1.21, Linköping University, Sweden, August 2004.

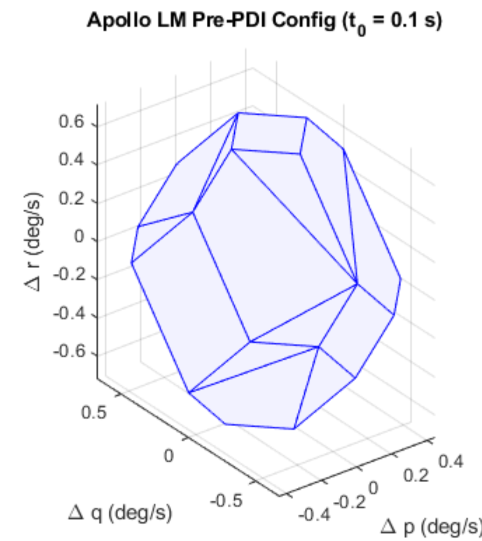
Example – Apollo LM (3-DoF)

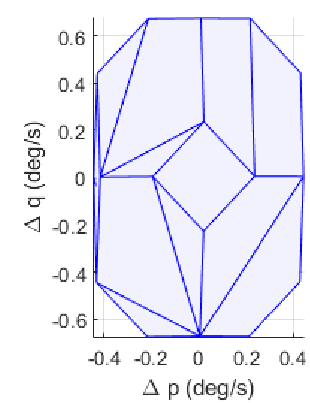
- Apollo 12 LM Pre-PDI configuration, all jets available
  - Consider only the rotational dynamics.
  - Let the control vector be jet on-times Δt_i :

$$\dot{\boldsymbol{\omega}}\Delta t_0 = \Delta\boldsymbol{\omega} \approx \mathbf{J}^{-1}\sum_i \mathbf{r}_i^{\times} F_i \hat{\mathbf{a}}_i \Delta t_i = \mathbf{B}\mathbf{u}$$

  - This solution is found using simplex[10] (LP) or NNLS[11].
  - Solutions with $\Delta t_i < t_{\min}$ (below MIB) are clipped or redistributed.
  - Minimum rate increment can be found using the same techniques used to find the AMS.



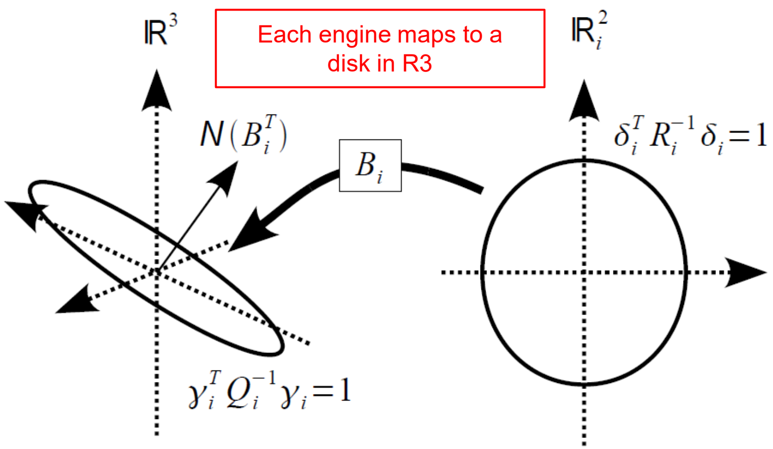


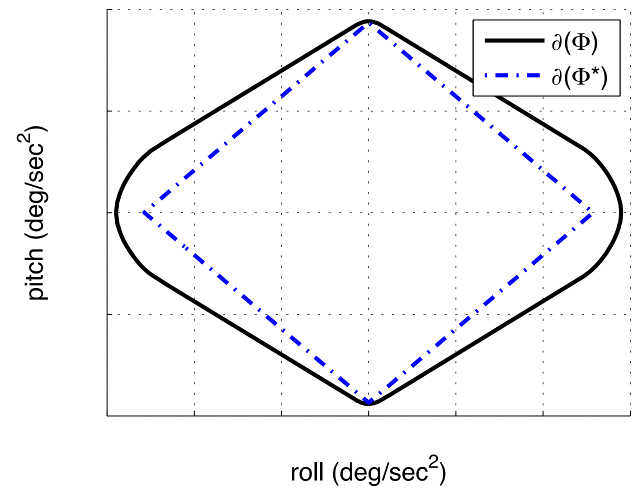


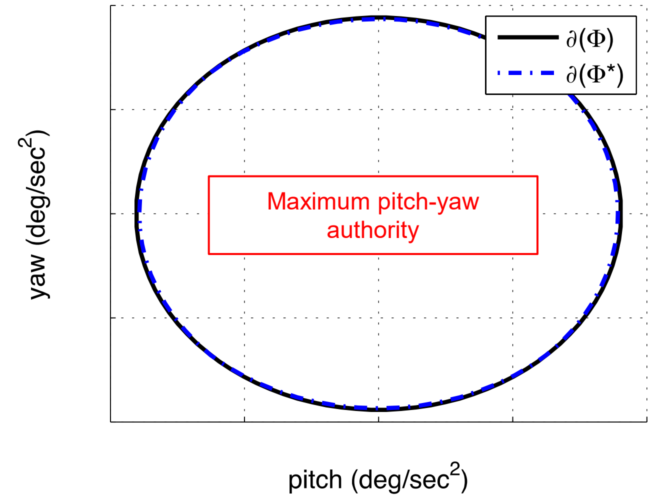
[8] "CSM/LM Spacecraft Operational Data Book, Volume III: Mass Properties, Rev. 2," NASA TM X-68968, 1969.
[9] Apollo Operations Handbook, Lunar Module. LM 10 and Subsequent – Volume I: Subsystems Data, LM790-3-LM, Section 2.4, Grumman Aerospace, 1971.
[10] Bodson, M., *Evaluation of Optimization Methods for Control Allocation*, AIAA Guidance, Navigation, and Control Conference, AIAA-2001-4223, August 2001.
[11] Lawson and Hanson, Solving Least Squares Problems, Prentice-Hall, 1974.

Example – Space Launch System

- Optimal-weighted online WLS allocator.[12]
- The problem is a QCQP but linearity is required for control-structure analysis.
- Elliptical (circular) constraints of differing sizes and control effectiveness.[13]
- Gives back unneeded roll capability to maximize pitch-yaw authority.
- Null space augmentation[14] was evaluated but found to be unnecessary.







[12] Wall, J., *Control Allocation for Launch Vehicles With Multiple Engines Using a Weighted Pseudo-Inverse Approach*, MSFC/EV40, January 26, 2011.
[13] Orr, J., and Wall, J., *Linear Approximation to Optimal Control Allocation for Rocket Nozzles With Elliptical Constraints*, AIAA 2011-6500, 2011.
[14] Orr, J., and Slegers, N., *High-Efficiency Thrust Vector Control Allocation*, AIAA JGCD, Vol. 37, No. 2, 2014, pp. 374-382.

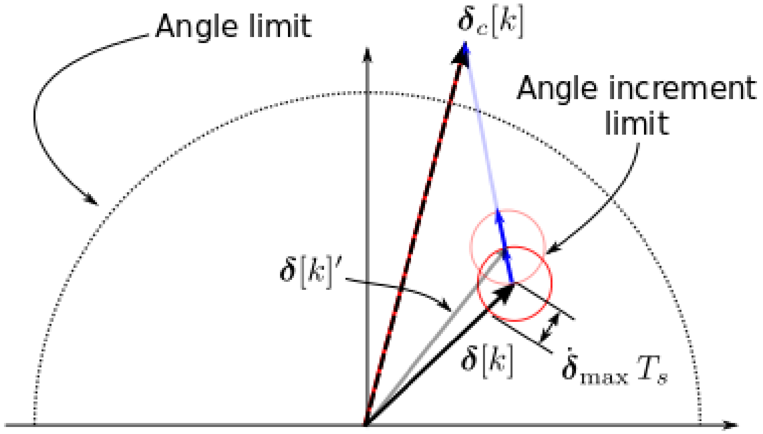
Practical Saturation/Rate Limiting

- The WLS allocator does not address saturation.
  - A change of one element of $\mathbf{u}$ leads to a change in direction of $\mathbf{v}$, in general.‡
  - A change in the response leads to undesirable cross-axis coupling.
- Since the allocator $\mathbf{P}$ is linear, compute a scale factor c such that

$$\mathbf{u} = c\mathbf{P}\mathbf{v} \in \partial(\Omega)$$

- The entire command is scaled linearly based on the largest effector value.
- The command is still formed from the range space of $\mathbf{P}$.
- Coordinated rate limiting can be implemented by applying a scaling limit over a minor frame timestep.

TVC Example
(SLS Algorithm)



‡Unless the change is in the null space. This is the idea for null projection control allocation (NPCA).

Summary

- Control allocation is a rich topic that can greatly improve capability of current and emerging aerospace systems.
  - Electric aircraft, UAS, novel spacecraft, etc.
  - Control of multiple objectives; e.g., structural loads, drag.

- This presentation has only summarized the theory and applications.
  - There are numerous algorithms, formal, ad-hoc, and semi-formal:
    - Cascaded inverses, null augmentation, minimum-variance weighting, preferred controls, daisy chaining…
  - Some have significant advantages or dangerous caveats, depending on application.

- *If the design doesn't close… make sure the control allocation is efficient before changing the vehicle.*

Questions?

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