Orbital Mechanics Seminars, Module 3: Collision Probability

Roger Thompson
Systems Engineering Division
The Aerospace Corporation

15 September 2023

Abstract

The Orbit Mechanics Seminars series is a collection of introductory material covering orbit mechanics and related topics. It was assembled with an engineering audience in mind, though no prior knowledge of orbit mechanics is required. The series begins with an overview of orbits, geometry, and perturbations, and progresses to more advanced topics such as Maneuvers, Collision Avoidance, and Space Debris.

Guidance for using this material:
The material assumes a basic understanding of physics.
The content is best understood with the accompanying audio track, but the slides are useful as reference material and as points for discussion and further study.

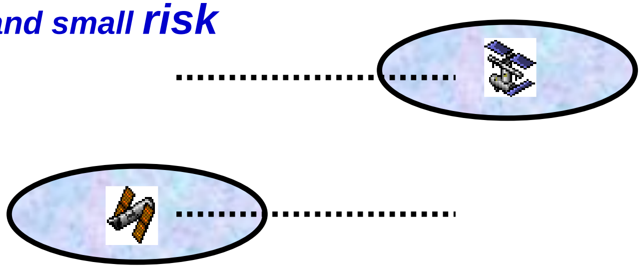
Outline of Presentation

- Basic Principles
- The Mathematics of Probability
- Encounter Geometry
- Close Approach Models
- Probability vs. Miss Distance vs. Ellipsoid Size
- Data Quality
- Examples
  - *Iridium/Cosmos*
  - *Other known collisions*

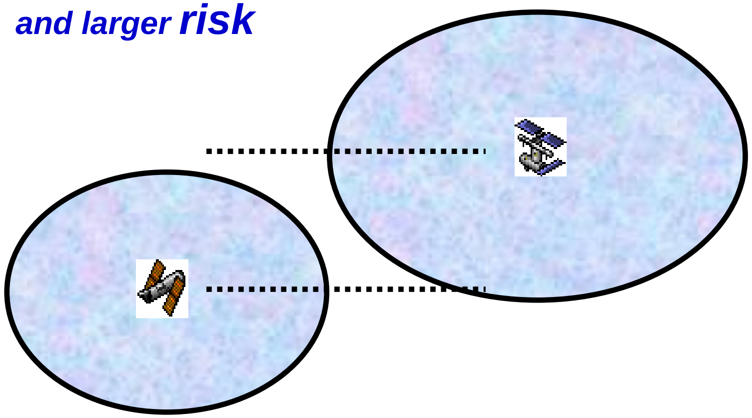
Basic Principles

- Probability & miss distance can both be utilized for risk assessment
- Miss distance alone does not accurately convey risk
- Probability of collision is a better measure
  - *Contributing factors are*
    - Miss distance
    - Position uncertainty (covariance or error ellipsoids)
    - Object size (hard-body radius)
- Probability-based assessments
  - *Generally reduce the number of events by eliminating conjunctions with low risk*
  - *Provides a relative risk assessment even when the absolute value is debatable*
  - *Can appear to be inconsistent*
  - *Very small risk can be more difficult to place in perspective*

Close miss distance, small ellipsoid, and small risk



Same miss distance, larger ellipsoid, and larger risk



The Mathematics of Probability

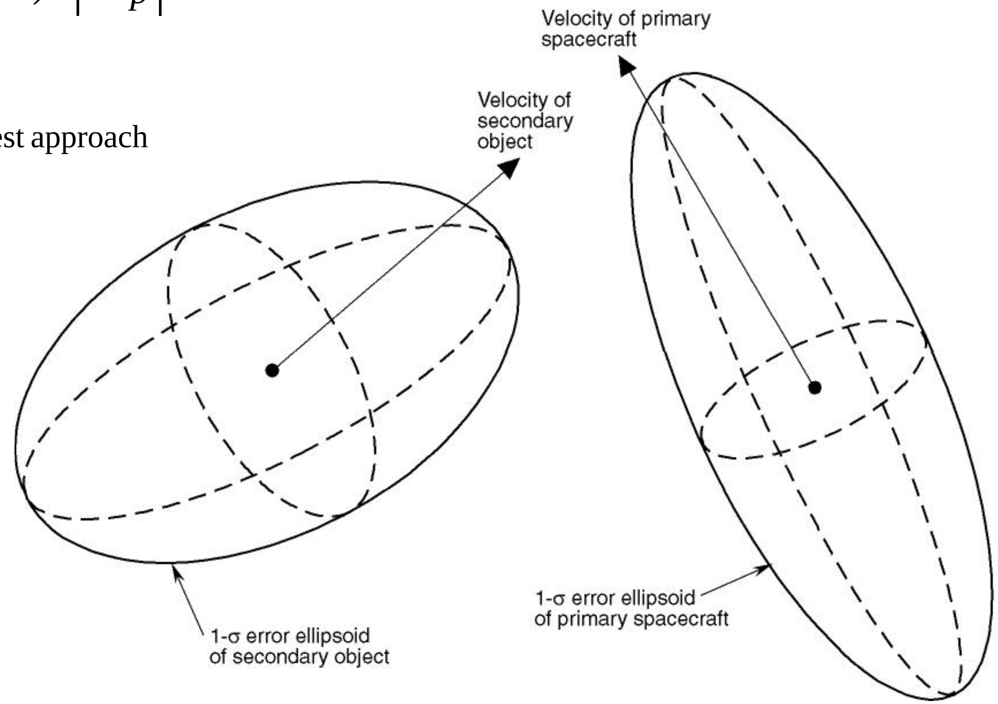
- Two objects with known positional uncertainties are passing by each other
- Assuming the uncertainties are a 3-dimensional Gaussian distribution
  - *Probability is a measure of the interaction of the uncertainties and is given by the volume integral*

$$P_c = \iiint_V \frac{1}{\sqrt{(2\pi)^3 |C_p|}} \exp\left[-\frac{1}{2}\left(\bar{r}^T C_p^{-1} \bar{r}\right)\right] dxdydz$$

P_c = probability

C_p = covarinace

$\bar{r}$ = relative position vector at closest approach



Ref: Chan, K., "Improved Analytical Expressions for Computing Spacecraft Collision Probabilities"

Simplifying Assumptions

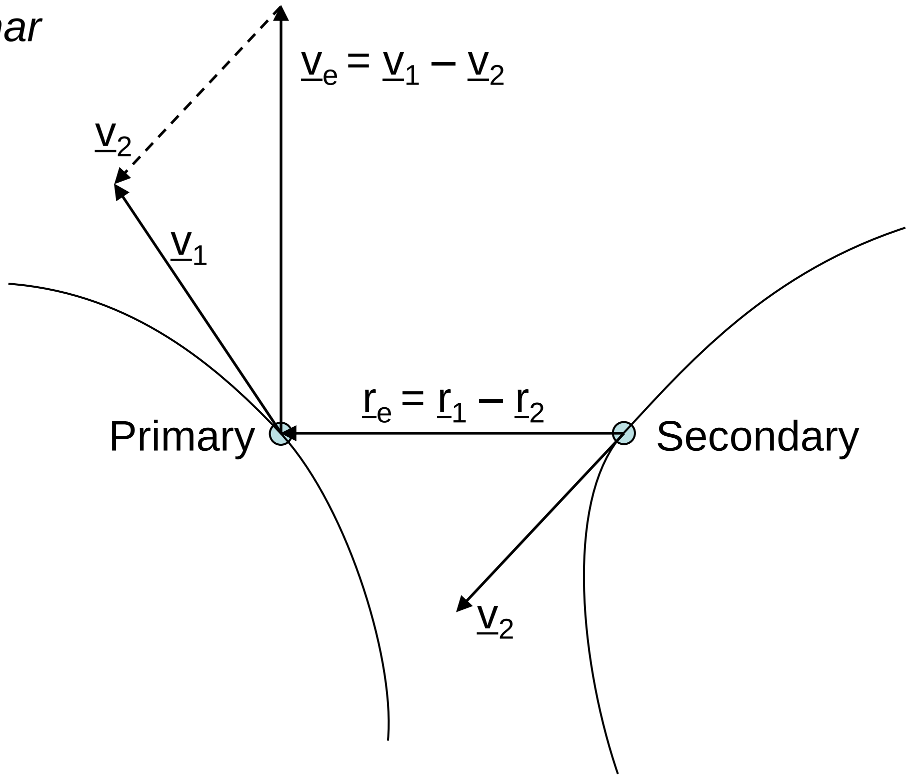
- Ellipsoids are independent of each other
  - *Uncertainties are unrelated (uncorrelated)*
  - *Mathematically, they can be added*
- Objects have spherical shape
  - *Otherwise, orientation (attitude) of objects must be known*
  - *Which dimension is used will cause under or over estimation of risk*
    - Largest dimension – more conservative results
- Relative velocity is high
  - *Relative motion is nearly rectilinear during encounter*
  - *Problem reduces to a 2-dimensional area integral*

$$P_c = \frac{1}{2\pi\left|C_p\right|}\iint_A \exp\left[-\frac{1}{2}\left(\bar{r}^T C_p^{-1}\bar{r}\right)\right]dxdz$$

Ref: Chan, K., “Improved Analytical Expressions for Computing Spacecraft Collision Probabilities”

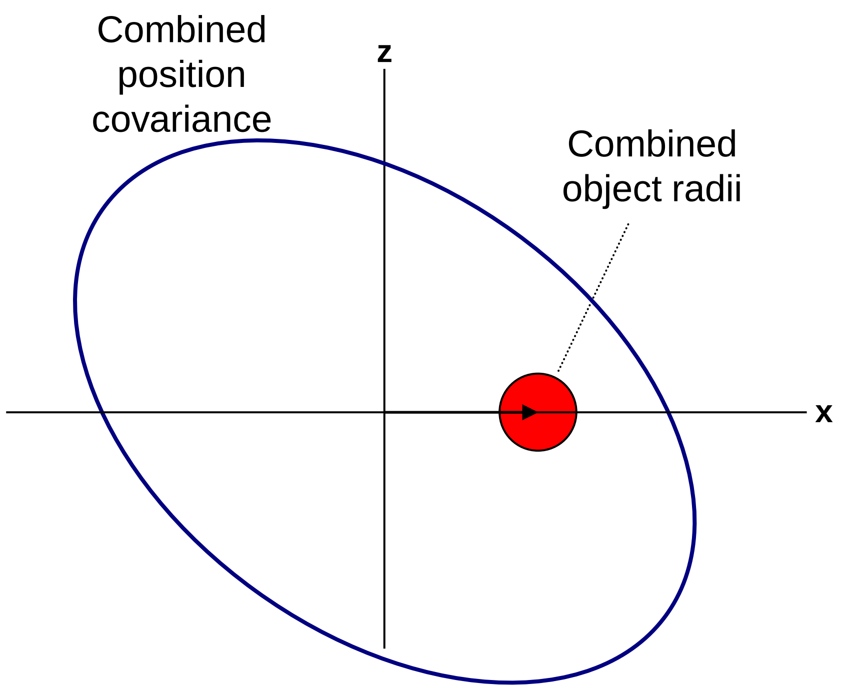
Point of Closest Approach

- Defined by relative position ($\underline{r}_e$) and velocity vectors ($\underline{v}_e$)
  - *Relative position and velocity vectors are perpendicular (definition of closest approach)*
- Intersecting planes
  - *Velocity vectors $\underline{v}_1$, $\underline{v}_2$, and $\underline{v}_e$ are coplanar*
  - *Position vectors $\underline{r}_1$, $\underline{r}_2$, and $\underline{r}_e$ are coplanar*
  - *Relative vectors $\underline{r}_e$ and $\underline{v}_e$ are coplanar*
    - *And perpendicular*
  - *But not necessarily the same planes*



The "Encounter Frame"

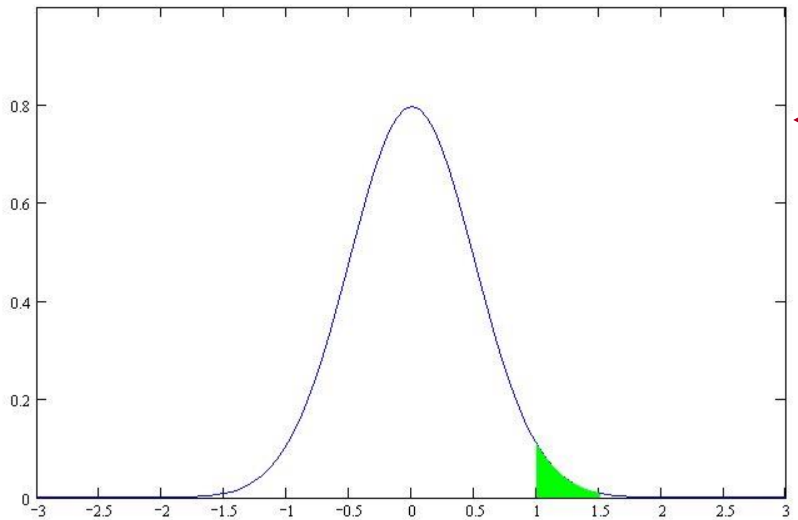
- At the point of closest approach
  - *Encounter plane is perpendicular to the relative velocity vector*
  - *Relative velocity is constant during conjunction duration*
    - Consequence of high-speed pass assumption
  - *Origin of axes is located at center of combined uncertainty ellipsoid*
- Additional conventions
  - *The y-axis is parallel to the relative velocity vector*
    - Relative velocity vector = $(0, v_e, 0)$
  - *The x-axis is parallel to the relative position vector*
    - Relative position vector = $(x_e, 0, 0)$
  - *Alternatively, in-plane axes can be aligned with principal axes of ellipse*



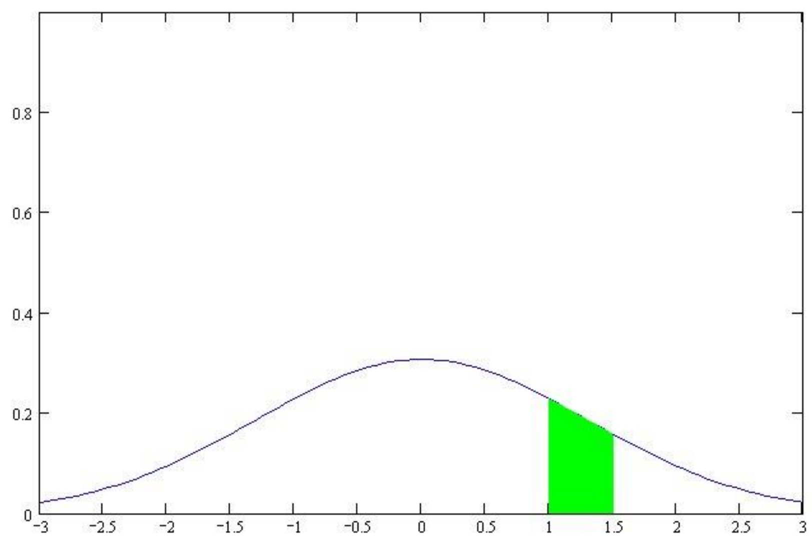
Available Models

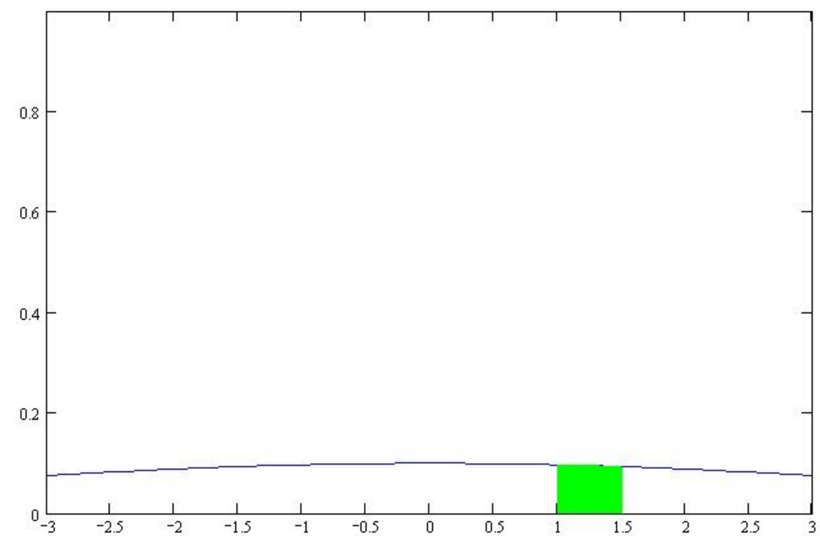
- All models solve the same problem but handle the math differently
  - *No closed-form solution exists, so different methods and approximations are employed*
- All models are based on same fundamental principles
  - *Rectilinear fast flyby*
  - *Dependency upon nominal close approach, position uncertainties, object sizes*
  - *Relative velocity at closest approach defines the encounter plane*
- A sampling of probability models that are available
  - *NASA (Foster & Frisbee): 2-d integration on encounter plane*
    - Used in Collision Vision and on-orbit COLA codes
  - *Patera: Contour integration around hard body radius*
    - Used in SOAP
  - *Chan: Equivalent circle formulation*
    - Used in STK
  - *Alfano: Polynomial expansion*
  - *Mains: Error function formulation*
- Comparisons between models show that they give the same answer to within 10% as long as:
  - *Magnitude of the relative velocity exceeds approximately 1% of the orbital speed*
  - *Aspect ratio of encounter plane ellipsoid < 100*

Probability Can Be a Counter-Intuitive



- Looking at a scalar case, for example
  - *Given uncertainty yields a certain probability*
  - *Uncertainty becomes worse (larger), probability can be higher*
  - *Uncertainty is very bad, probability is smaller*
    - “Big Sky” theory is ultimate extrapolation of greater uncertainty





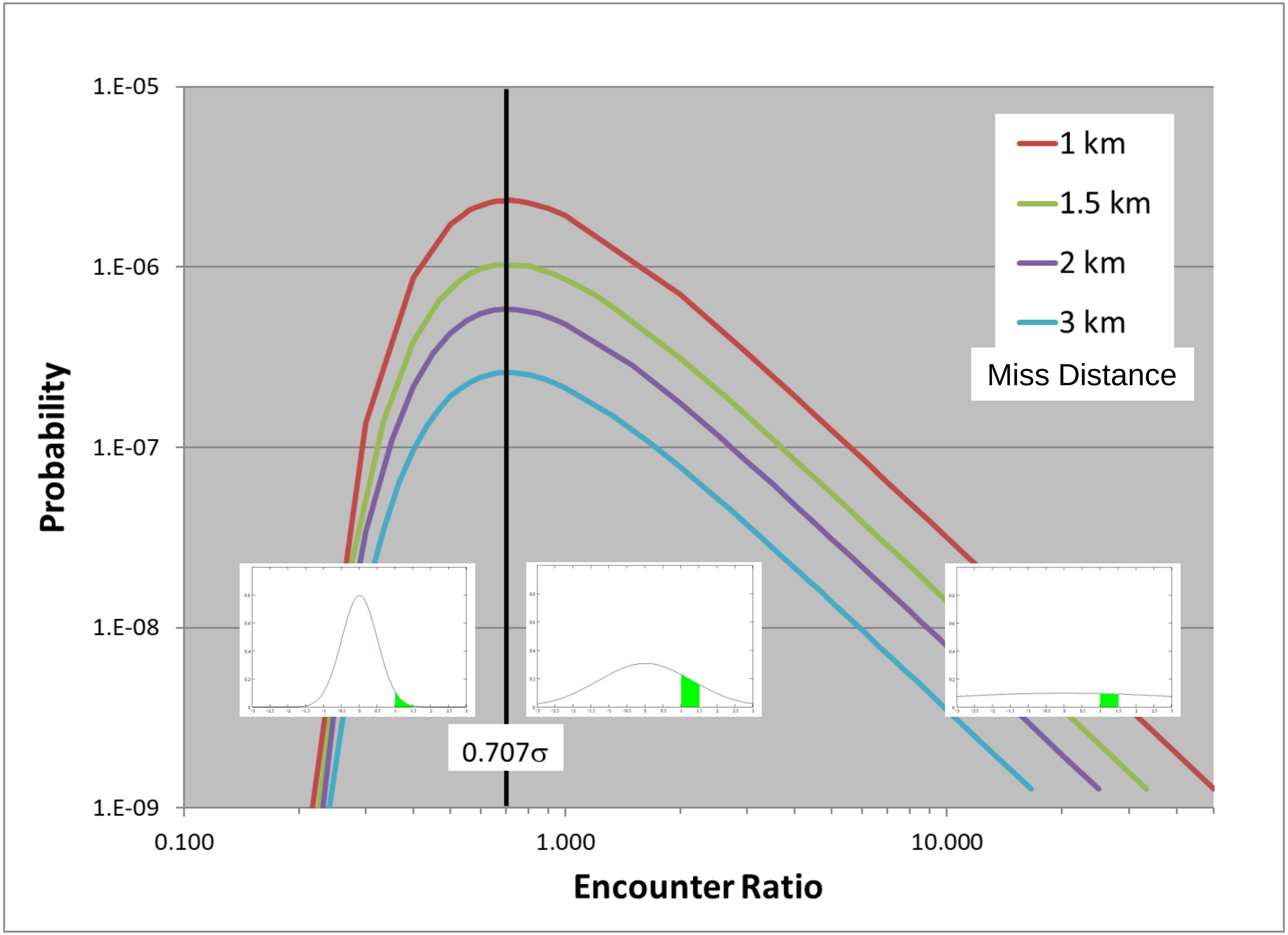
Ref: Alfano, S., “Relating Position Uncertainty to Maximum Conjunction Probability”

Probability vs. Encounter Ratio

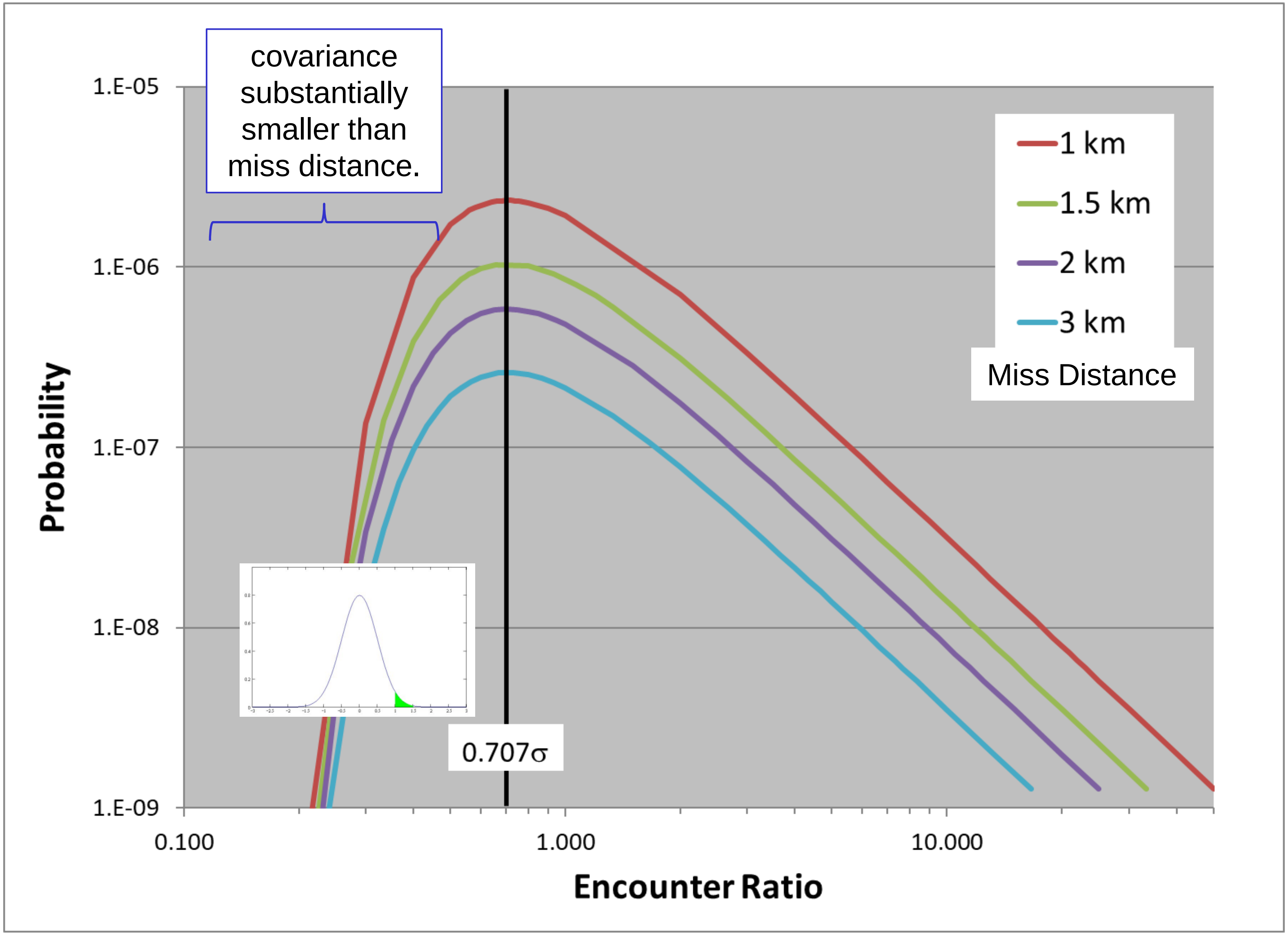
- Encounter is kept the same and covariance size is varied
  - *Spherical covariance*
  - *Object diameters are 1 meter*
- Encounter ratio = covariance size / by miss distance
- Probability will peak when encounter ratio = 0.707 (i.e. covariance = 0.707*miss distance)
  - *Below that point, the probability value is a good measure of the true risk*
  - *Beyond that, larger uncertainty reduces confidence in risk assessment*
    - Defined as "Dilution Region" (Ref: Alfano, S.)
    - Alfano recommends using maximum probability when "operating" in dilution region

Ref: Alfano, S., "Relating Position Uncertainty to Maximum Conjunction Probability"

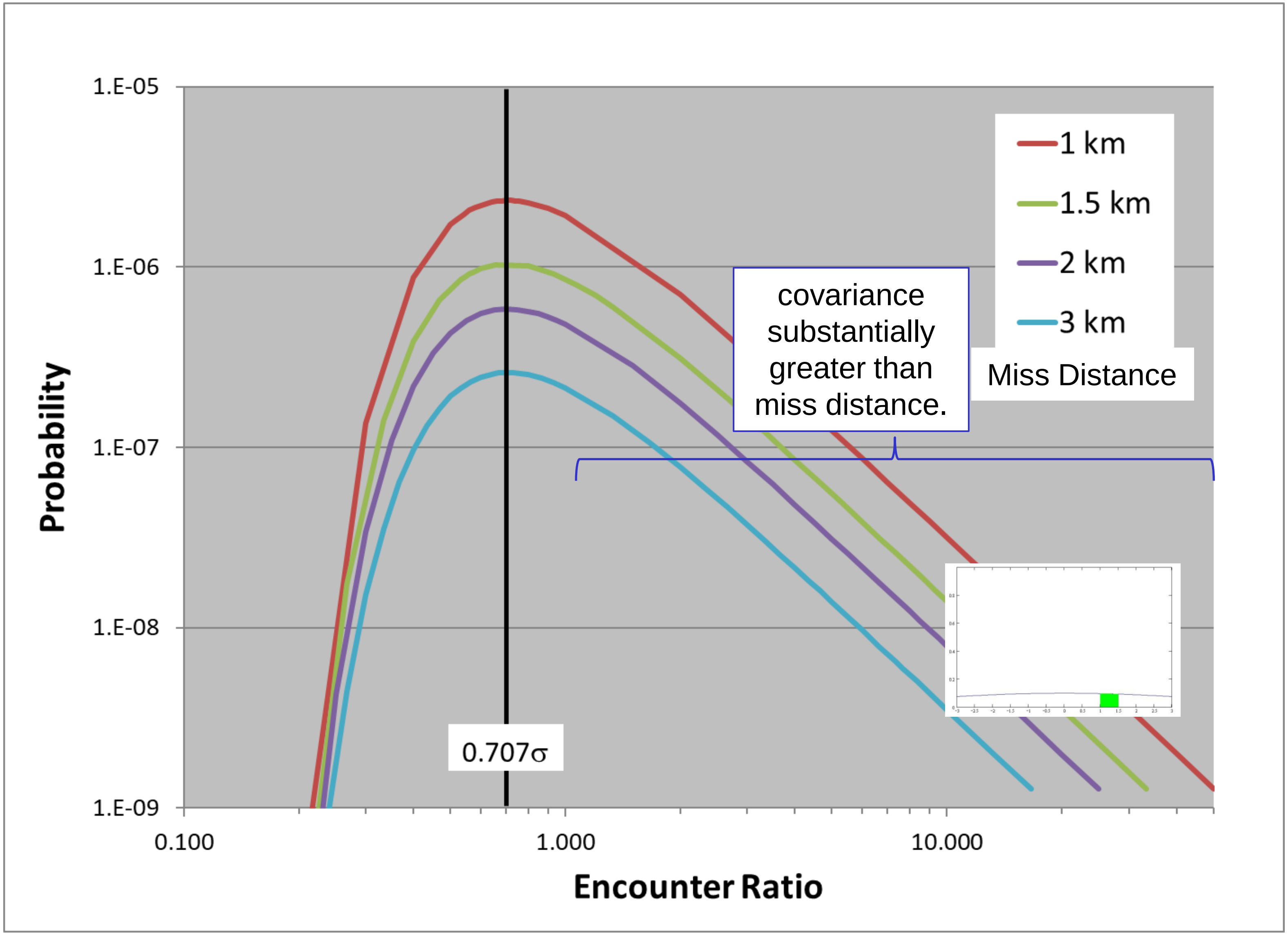
Probability vs. Encounter Ratio (cont'd)



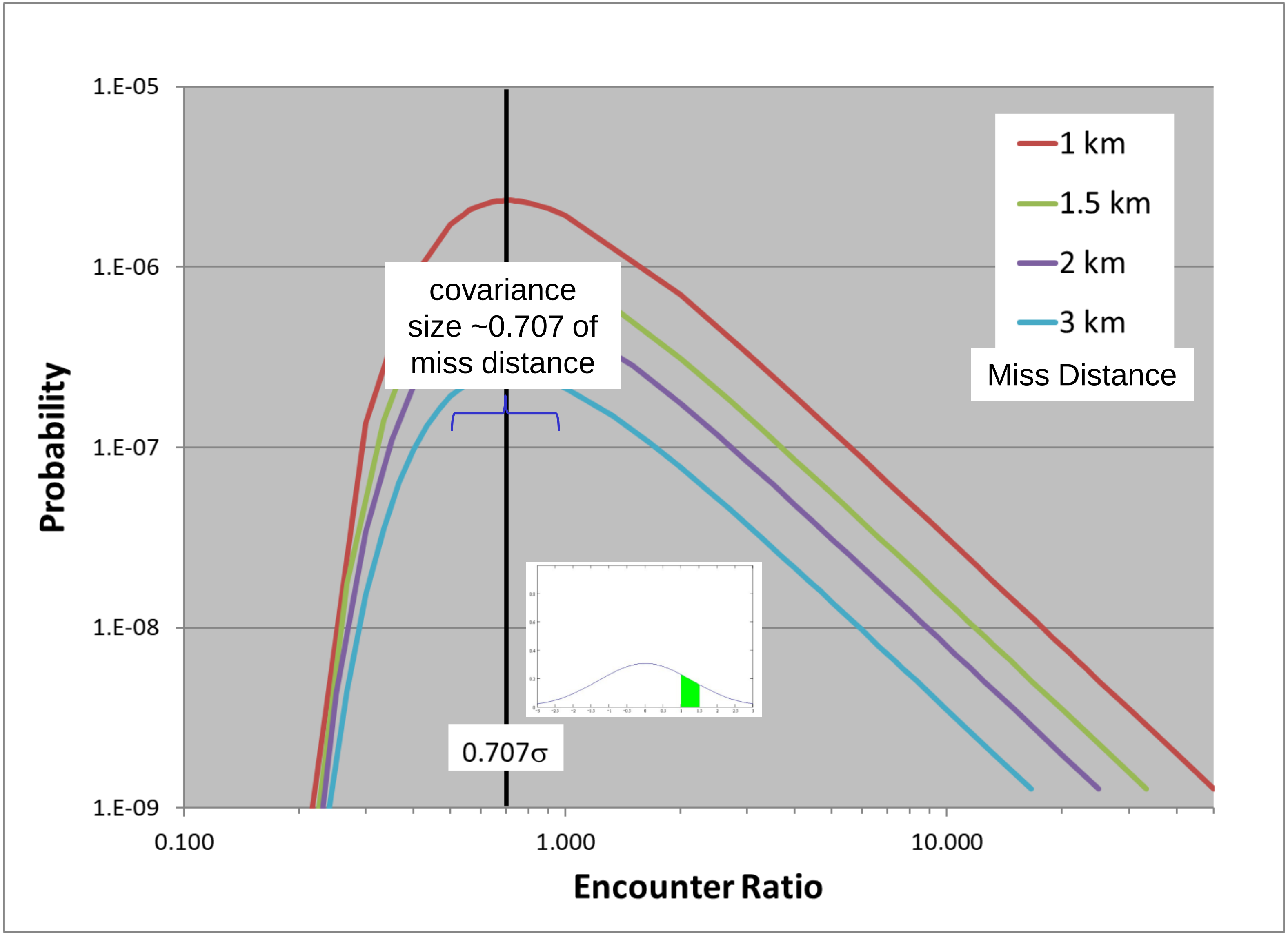
Probability vs. Encounter Ratio (cont'd)



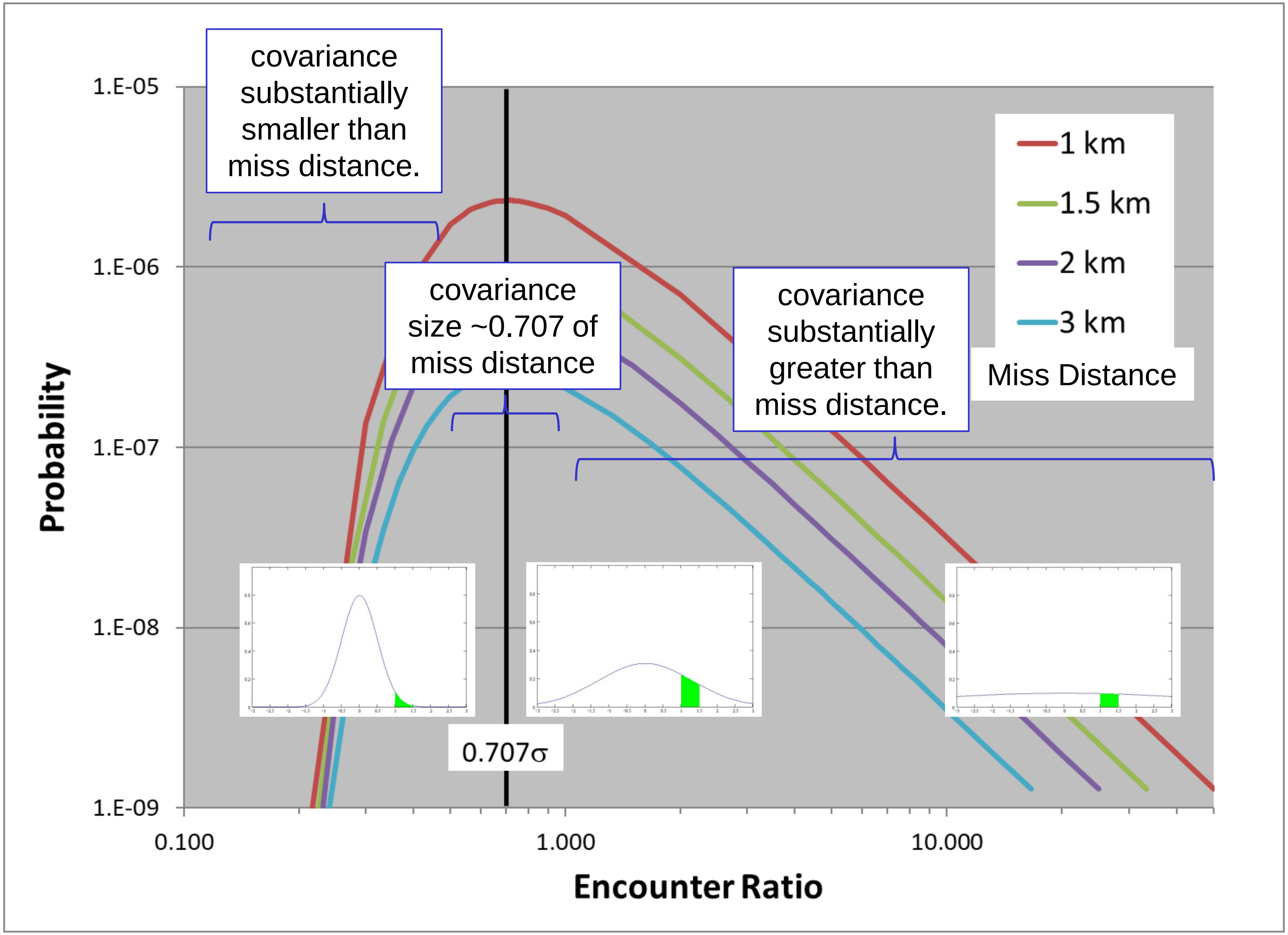
Probability vs. Encounter Ratio (cont'd)



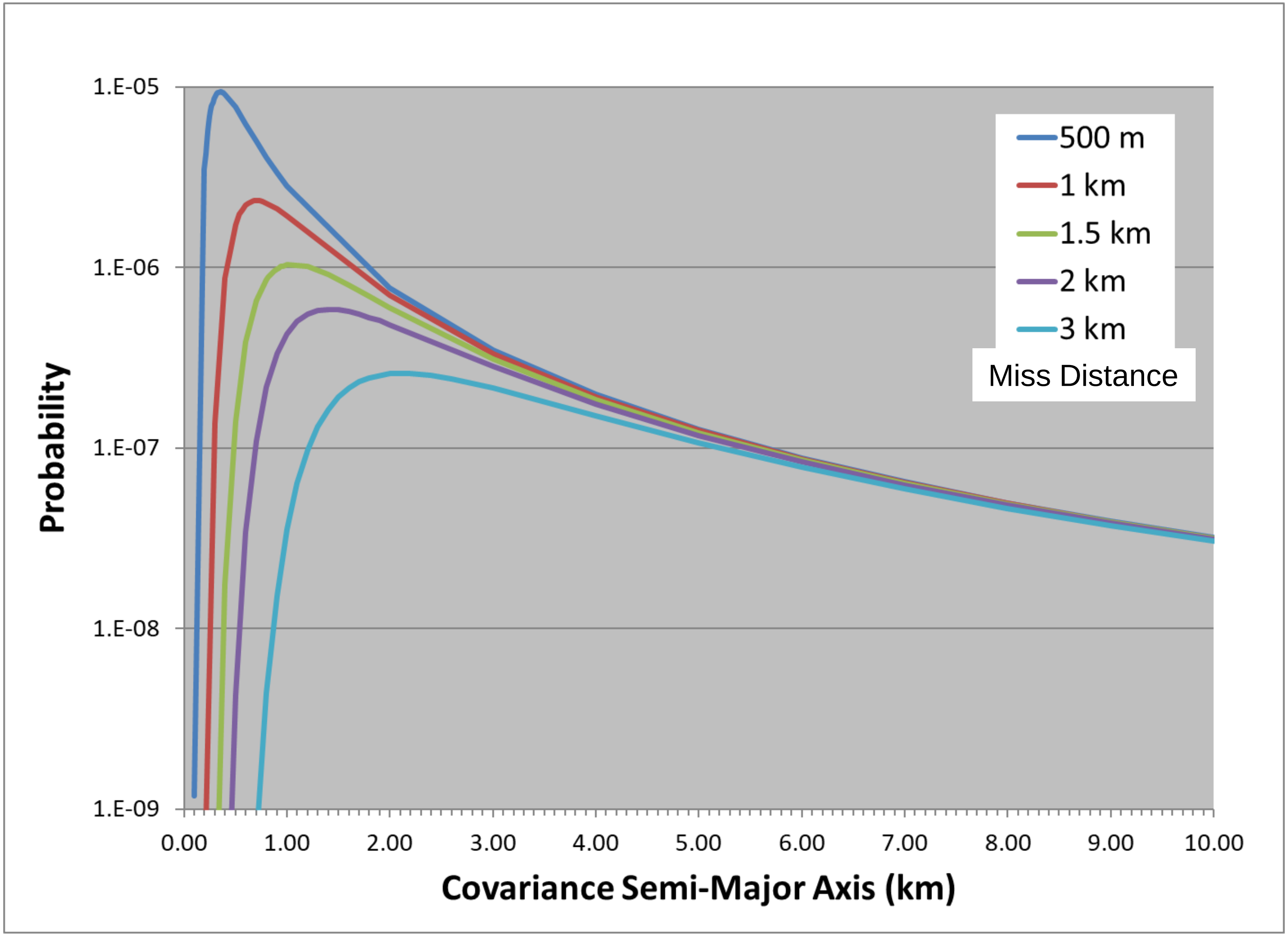
Probability vs. Encounter Ratio (cont'd)



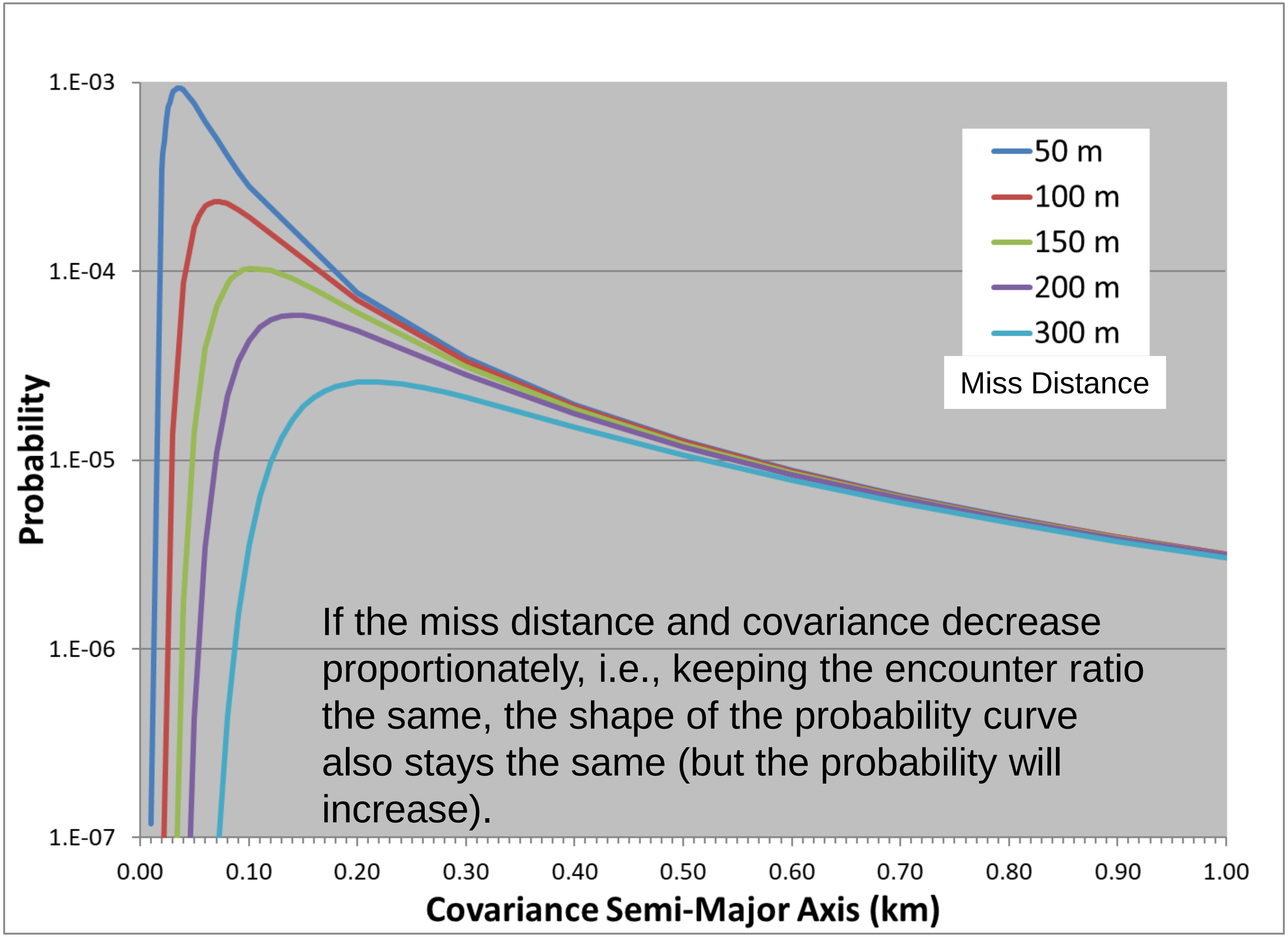
Probability vs. Encounter Ratio (cont'd)



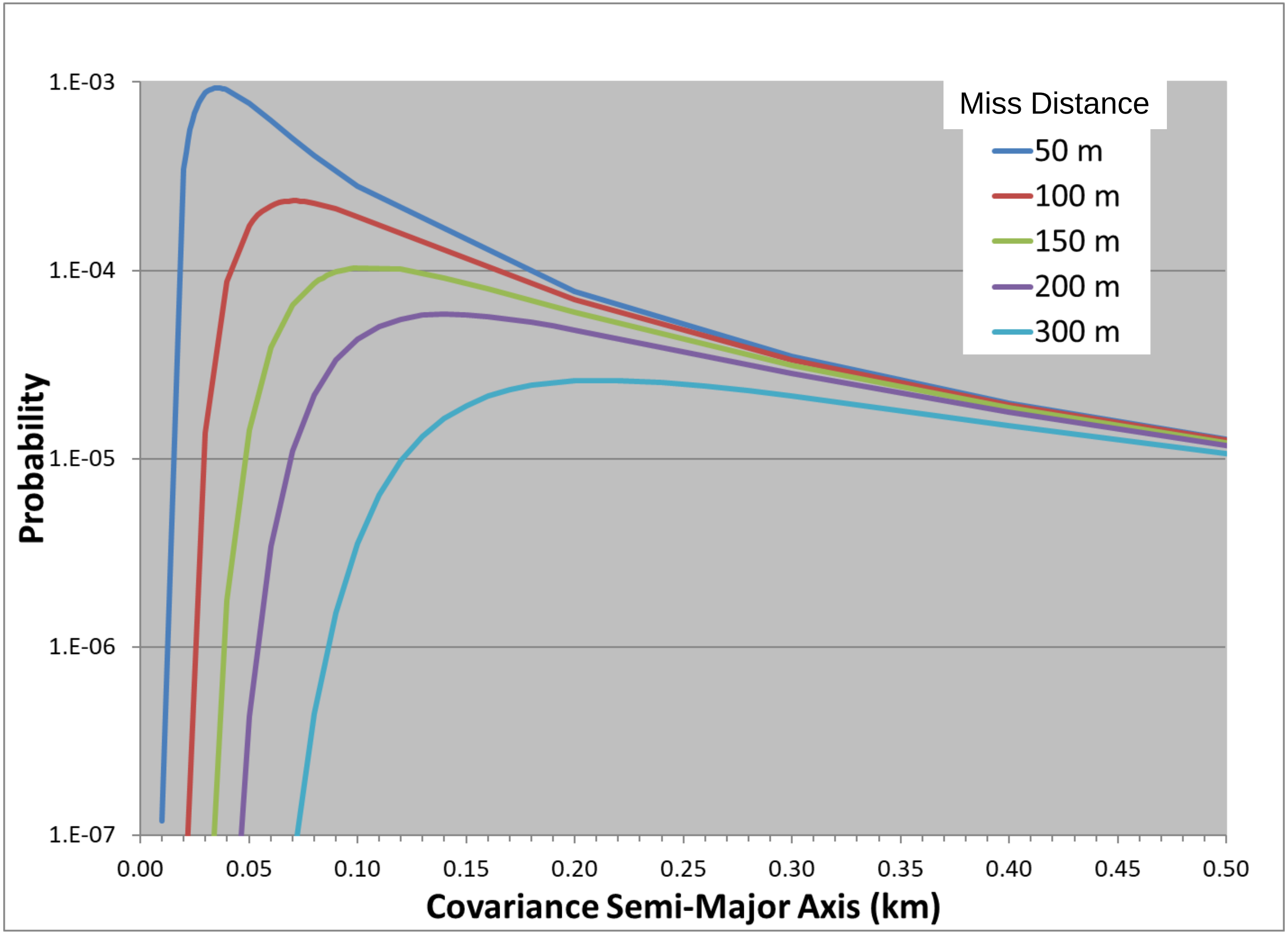
Probability vs. Covariance Size



Same curve, Different Scales



Probability vs. Covariance Size Closeup



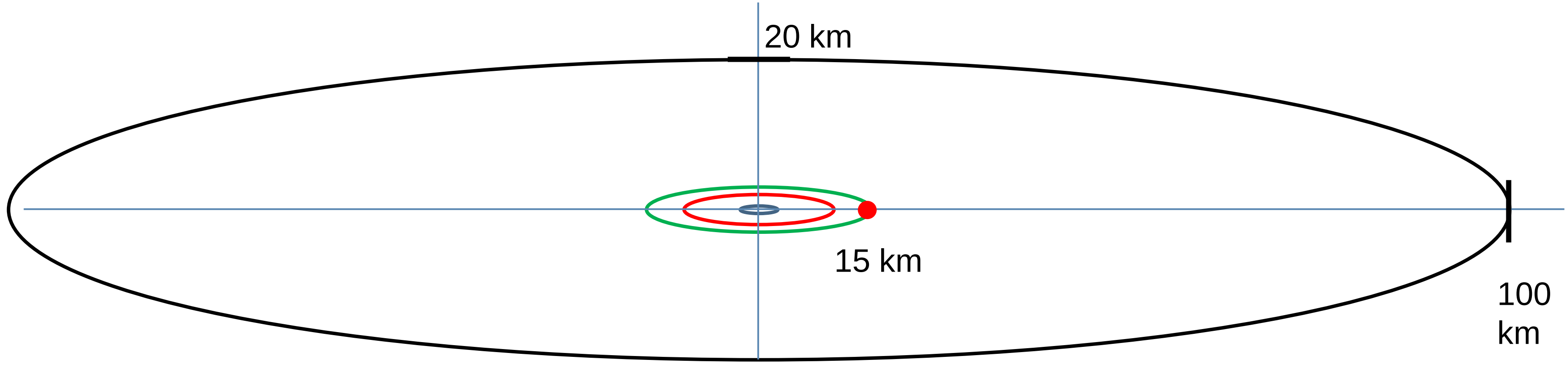
Probability vs. Miss Distance (15 km)

Effects of Ellipsoid Size

Ellipse Size (km)	Probability
2.5 x 0.5	6.10E-15
10 x 2	8.10E-09
15 x 3	6.70E-09
100 x 20	2.50E-10

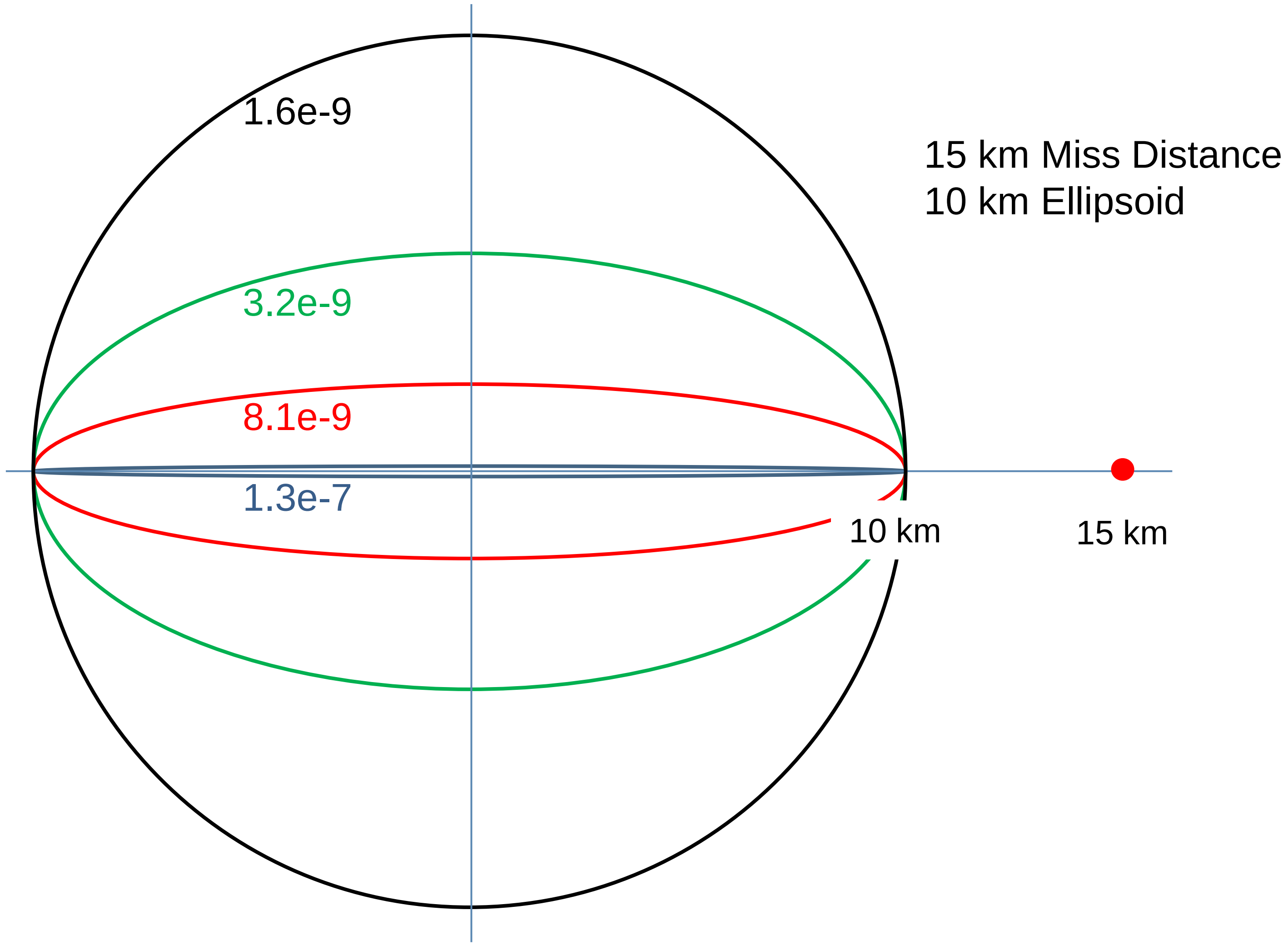
When the uncertainty ellipse is comparable in size to the miss distance, the probability is high.

A much larger or much smaller uncertainty ellipse actually results in lower probability because the overlapping area integrates to a smaller value.



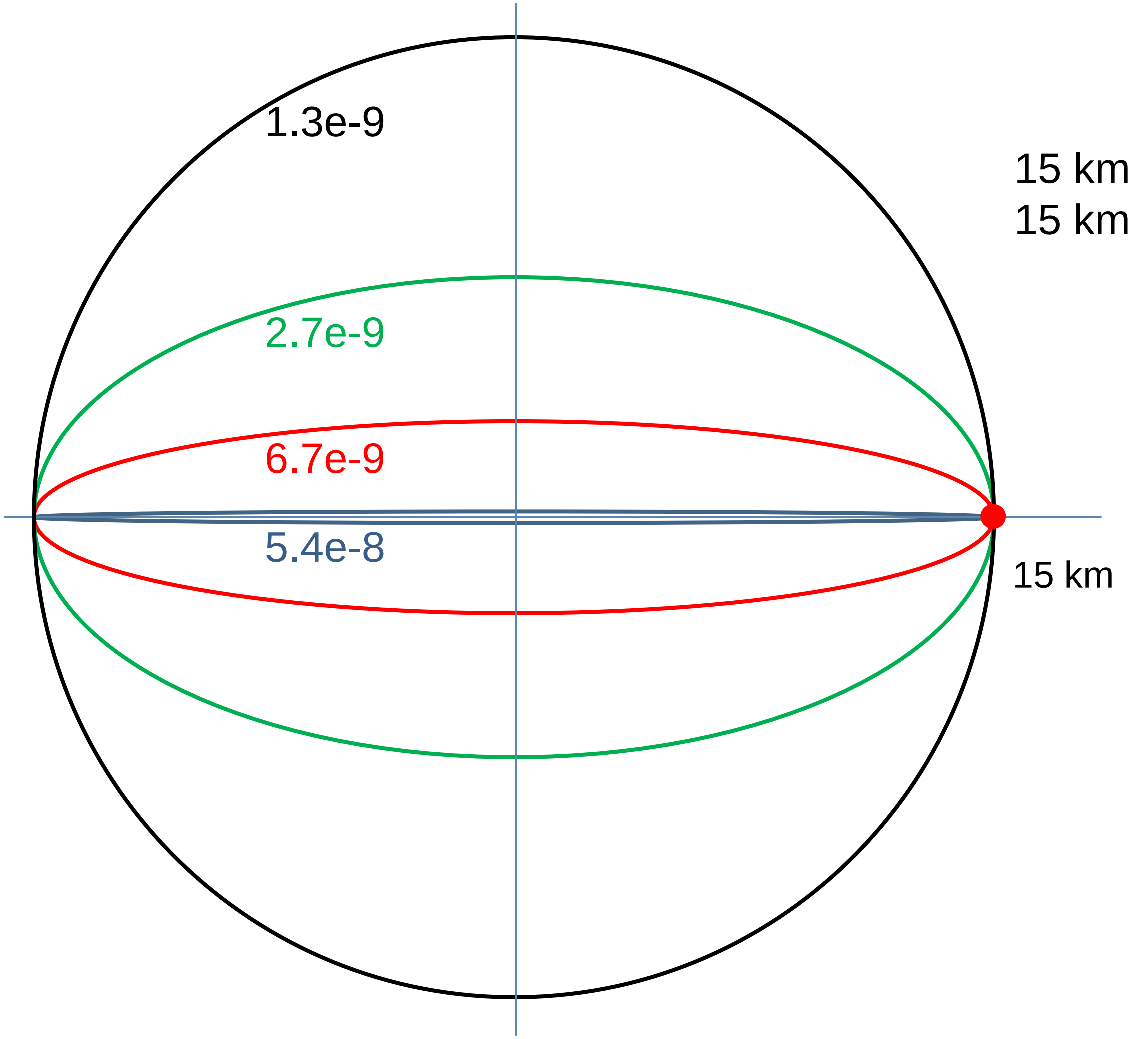
Probability vs. Miss Distance

Effects of Ellipsoid Shape



Probability vs. Miss Distance

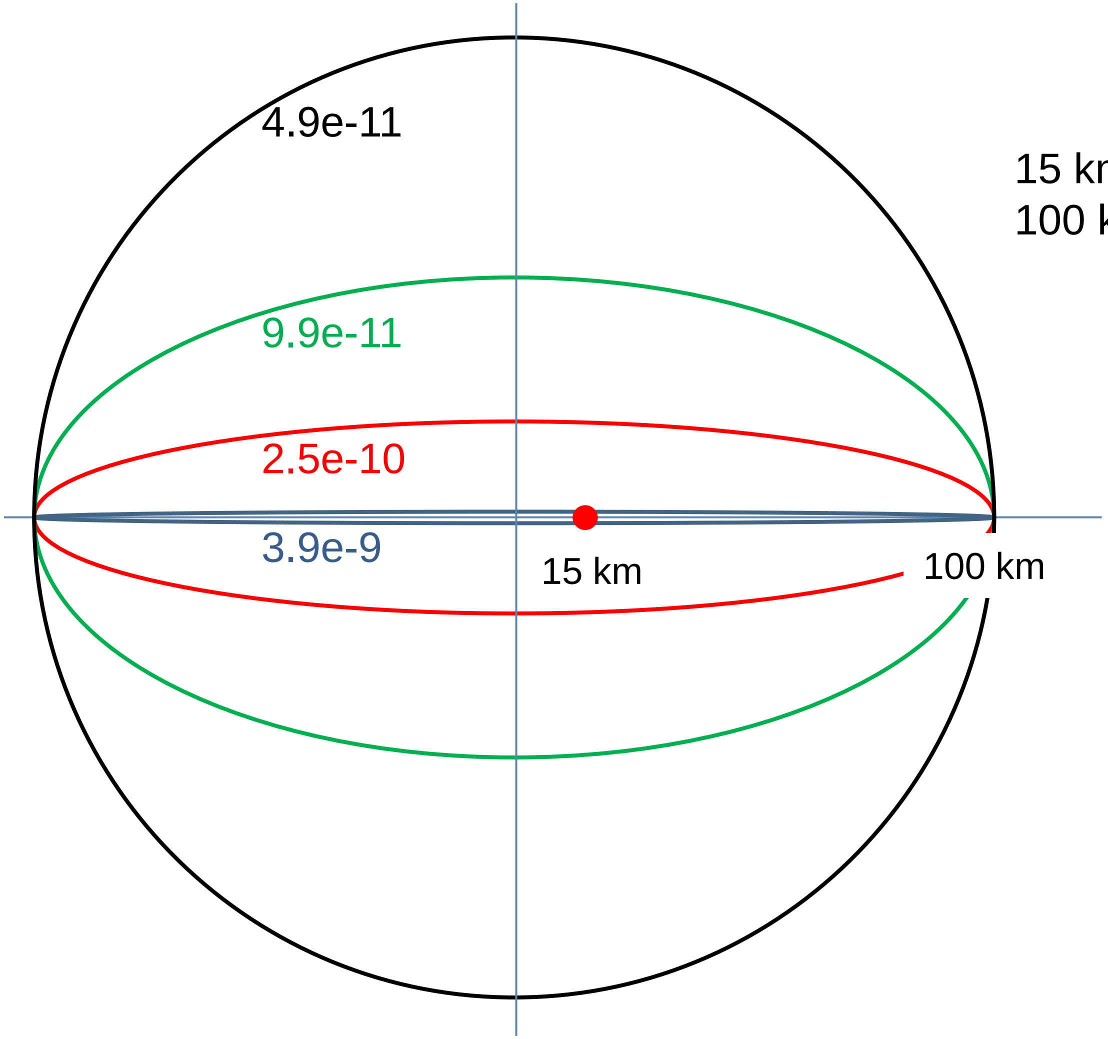
Effects of Ellipsoid Shape



15 km Miss Distance
15 km Ellipsoid

Probability vs. Miss Distance

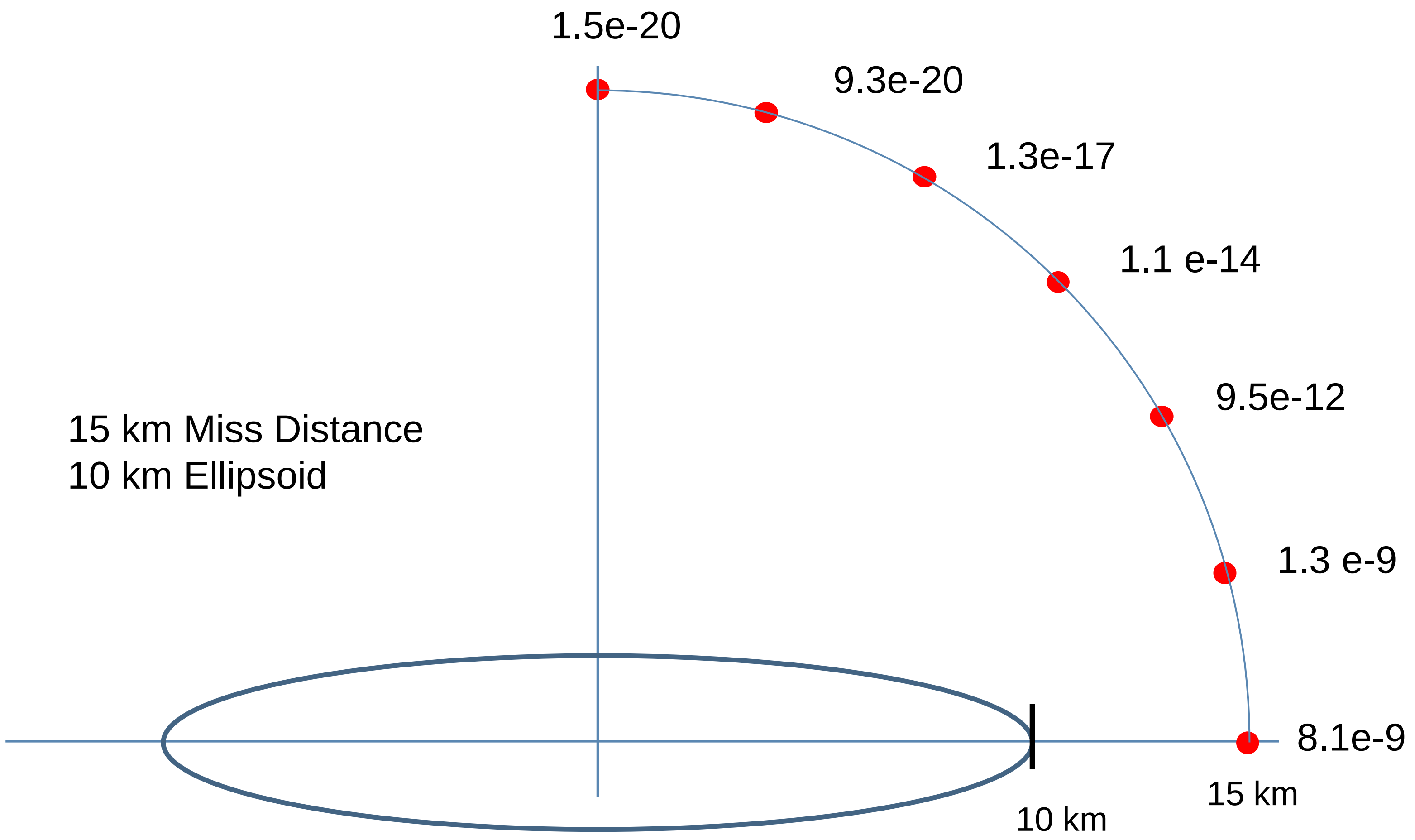
Effects of Ellipsoid Shape



15 km Miss Distance
100 km Ellipsoid

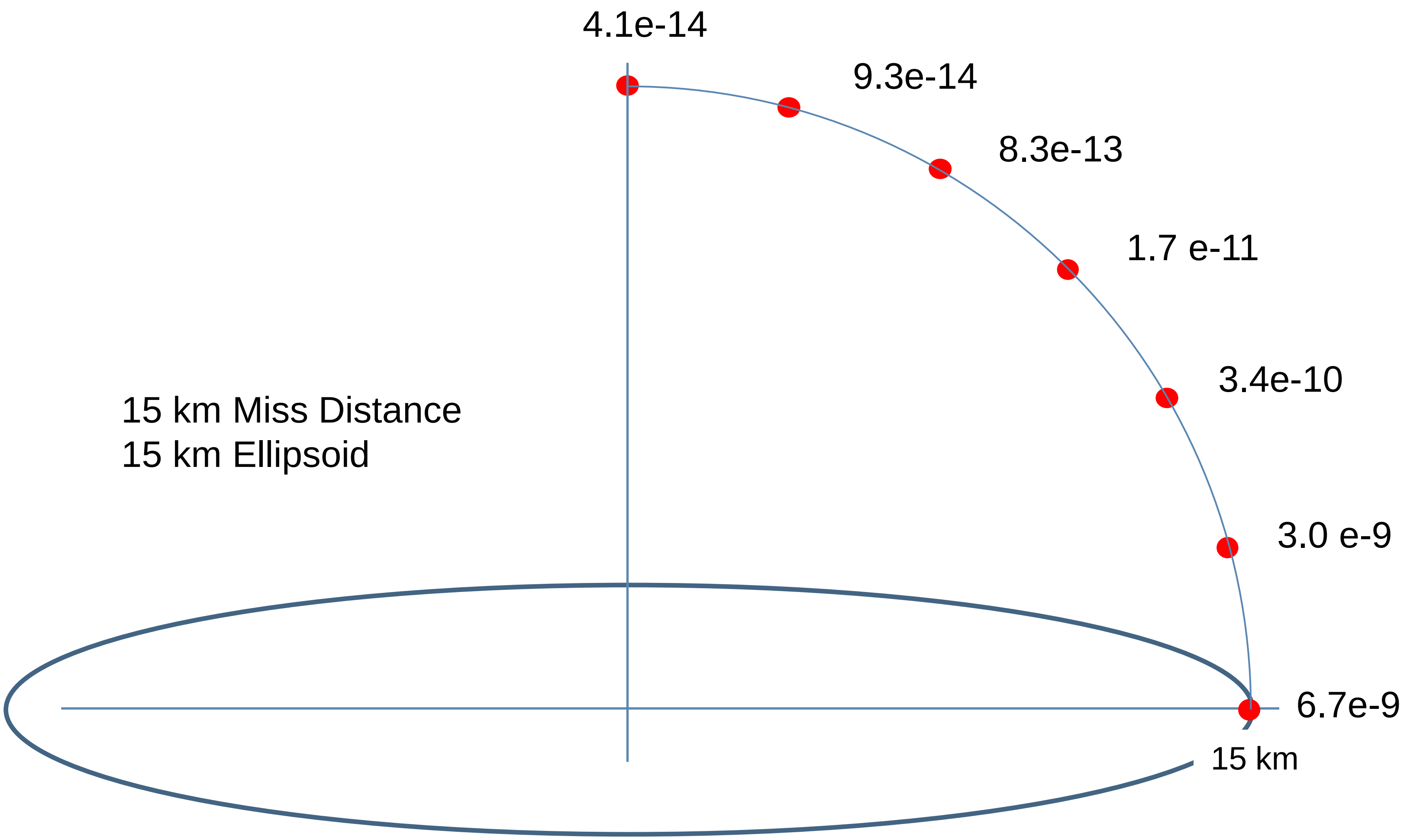
Probability vs. Miss Distance

Effects of Encounter Geometry – covariance orientation matters



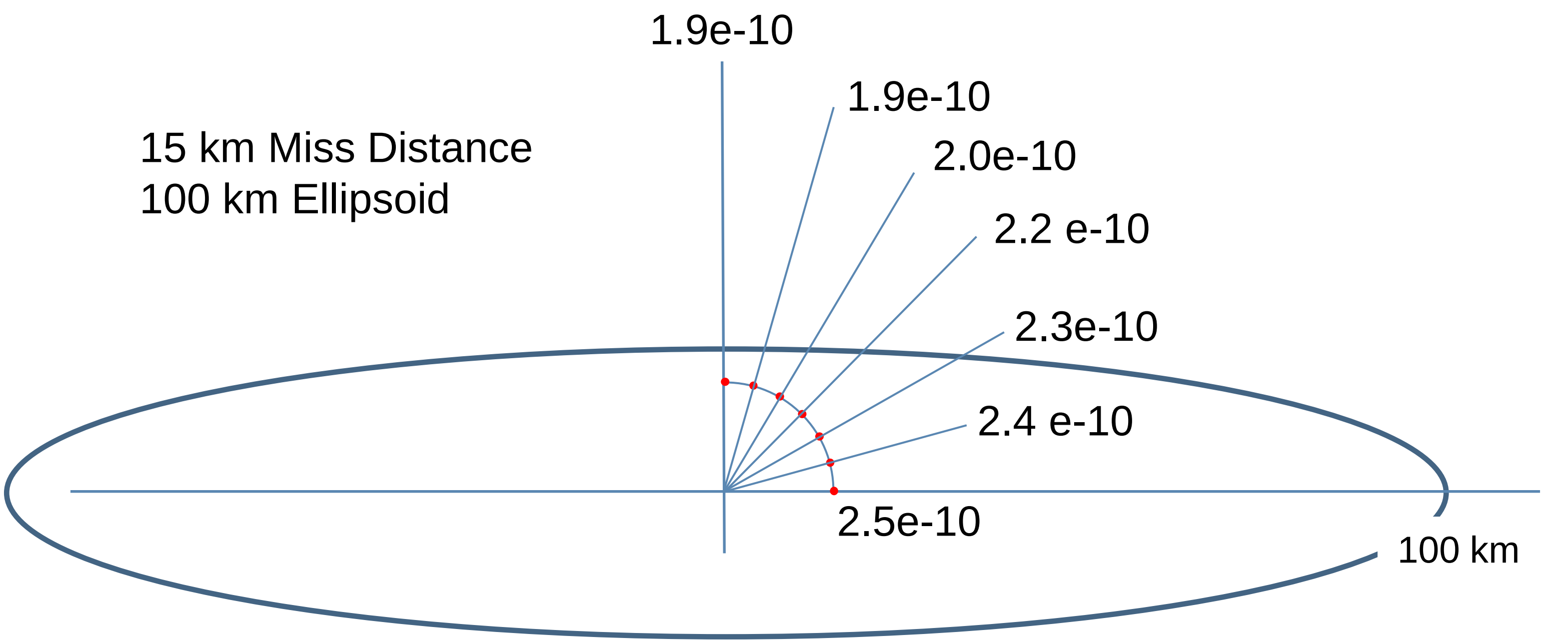
Probability vs. Miss Distance

Effects of Encounter Geometry



Probability vs. Miss Distance

Effects of Encounter Geometry



Accuracy Depends Upon Data Quality

- Positions and uncertainty are needed for all objects of interest
  - *Predicted ephemeris and covariance*
    - Daily for LEO, weekly for GEO
    - Must account for all predicted maneuvers and perturbations
      - *See Iridium/Cosmos Analysis*
  - *Daily estimates of all Resident Space Object (RSO) positions, from public or government sources*
  - *RSO positional errors via statistical modeling*
  - *Physical object size/orientations – directly scales probability*
- Consistency of data and terminology
  - *Need to be careful when comparing results and exchanging data from one propagation tool to another*
    - Example: Coordinate frame of the ephemeris

Iridium/Cosmos Analysis

- Iridium 33/Cosmos 2251 collision on 10 Feb 2009
  - *Iridium maneuvered approximately 8 hours prior to collision*
    - Extent of maneuver is unknown, but it was small
- SP analysis
  - *Represents the best data available*
  - *Iridium orbit data epoch was 8.5 hours prior to collision, Cosmos was 6.5 hours*
  - *Predicted miss distance 251 meters*
  - *Probability of collision* ***9.1E-128***
    - A very small encounter ratio
- GP analysis
  - *Predicted miss distance 584 meters*
  - *Probability of collision 1.3E-5*
    - Both ellipsoids less than 4 km

Four Known Collisions - GP Analysis

- Used validated process for launch and on-orbit collision avoidance
  - *1991 (discovered in 2005)*
    - Cosmos 1275 debris and Cosmos 1934
      - *Predicted miss distance 512 meters*
      - *Probability of collision 1.7E-5*
  - *1996*
    - Cerise active satellite and Ariane 1 Debris
      - *Predicted miss distance 882 meters*
      - *Probability of collision 5.6E-7*
  - *2005*
    - Thor Burner 2A debris and CZ-4 Debris
      - *Predicted miss distance 655 meters*
      - *Probability of collision 3.9E-7*
  - *2009*
    - Iridium 33 active satellite and Cosmos 2251
      - *Predicted miss distance 584 meters*
      - *Probability of collision 1.3E-5*

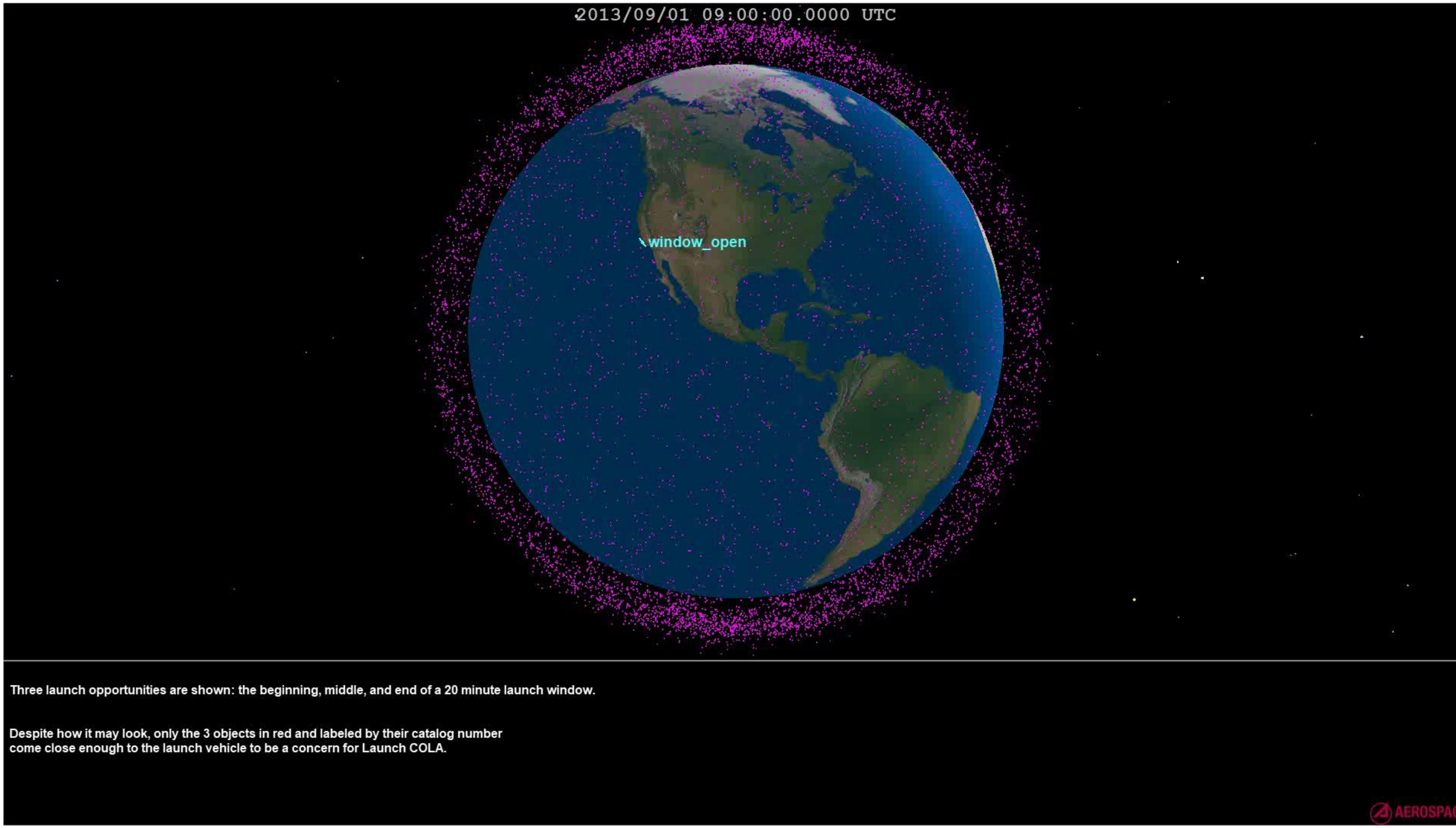
Animation and Analysis of 2005 Collision

Encounter
2005/01/17 02:14:35.1680 UTC
SOAP Encounter Report for View: Encounter
Start: 2005/01/17 00:14:00.00, Stop: 2005/01/18 02:14:00.00
Samples: 93601, Step: 1.000000 s, System: UTC
Range Threshold: 100.0000 km

HOST ID	TARG ID	TIME s	RANGE km	PROBABILITY	HOST_EPOCH	TARG_EPOCH	PROPAGATOR	HOST_X km	HOST_Y km
7219	26207	2005 JAN 17 02:14:37.168	0.6550	3.949414e-07	2005/01/16 13:11:36.260	2005/01/16 13:14:19.256	SGP4 5.x	-147.4045	1182.3201

Computation time: 0.02 minutes.
Number of conjunctions: 1
Number of errors: 0

Launch Collision Avoidance (LCOLA) Analysis



Summary

- Probability and miss distance (keep out zones) can both be used
- Probability gives greater insight, but miss distance easier to understand
- Probability modeling
  - *Both trajectories are propagated to point of closest approach*
  - *Uncertainty ellipsoids are combined and centered on one of the objects*
  - *Vehicle sizes are added (assumed spherical) and centered on the other object*
  - *Encounter frame is defined by relative velocity vector (x and z arbitrary)*
  - *Integration along relative velocity reduces the probability to a 2-d integral*
- Various models exist, but all can be used for >99% of conjunctions to within 10% accuracy
  - *Watch out for low relative velocities (<1% orbital speed) & high aspect ratios of the combined encounter plane covariance (>100)*
- Operational issues
  - *Data acquisition always a problem (timeliness, consistency, maneuver plans, uncertainties, sizes, etc.)*
  - *Maneuvers should be carefully considered for poor quality data*

Acronyms and Variables

- Cp = covariance / uncertainty
- Pc = probability of collision
- A = cross-sectional area
- r = relative position vector at closest approach
- LEO = low Earth orbit
- GEO = geosynchronous Earth orbit
- RSO = Resident Space Object

$$P_c = \text{probability}$$
$$C_p = \text{covarinace}$$
$$\bar{r} = \text{relative position vector at closest approach}$$